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Book of Abstracts

Subadditivity, Generalized Jensen Inequalities and Superquadracity

Soshana Abramovich

This is Part 1 of a joint work which is still in progress. In this talk inequalities involving convex functions, subadditive functions and also superquadratic functions are presented. The obtained inequalities are related to results which generalize Jensen's inequalities via additive and subadditive functionals.

In Part 2 we will generalize the results described here for positive isotonic linear functionals and for isotonic subadditive functinals.

References:
S, Abramovich, L. E. Persson, J. Precaric and S. Varosanec, ``General inequalities via isotonic subadditive functionals", Mathematical Inequalities and Applications, **10** (2007), pp. 15-29.
S. S. Dragomir, Y. J. Cho and J. K. Kim, ``Subadditivity of some functionals associated to Jensen's inequality with applications", RGMIA, (2008).

Properties of a gradient of a hyperbolic radius for a doubly connected domain

A. Akhmetova

According to the Riemann mapping theorem there exists a conformal map $F : D \to E = \{\zeta : |\zeta| < 1\}$, where $F(z) = 0$ for a fixed point $z \in D$. The quantity

$$R(D, z) = \frac{1}{|F'(z)|} \tag{1}$$

is called *the conformal radius* of the domain D at the point $z = f(\zeta)$.

If D is a multiply-connected domain, then there exists a universal covering map of D onto E. The quantity (1) is called in this case *the hyperbolic radius.* We shall use the term "hyperbolic radius" in both cases.

When D is simply connected, the range of the gradient of the hyperbolic radius depends on the geometry of the boundary curve ∂D. For example, it is contained in the disk $\{w : |w| < 2\}$ for any convex domain $D \subset \mathbb{C}$. If the boundary curve has corners or rectilinear pieces, then new geometrical effects appear in the range of the gradient of the hyperbolic radius [2].

In $[1]$, a theorem is proved about the gradient image for a doubly connected domain for which point at infinity is an isolated boundary point and $\mathbb{C}\backslash D$ is a convex set. We prove the next result (and corollaries to it).

Theorem. *The gradient of the hyperbolic radius for the annulus $q<|\zeta|<Q$ maps it onto the degenerate Riemann surface realized as the disk $\{w:|w|<2\}$ attached at the center to the center of the disk $\{w:|w|<4\}$ and the latter is glued along the circumference to the corresponding boundary of the annulus $\{w:2<|w|<4\}$.*

References

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2. L.A. Aksent'ev, A.N. Akhmetova, *On Mappings Related to the Gradient of the Conformal Radius*, Mat. Zametki, 87(1)(2010), 3-12.

A Class Marcinkiewicz Integrals Related to Bochner-Riesz operators

Ahmad Al-Salman

In recent years the subject of integral operators with rough kernels has undergone a vast development. The key stone of this development is the substantial degree of interactions between maximal averages (maximal functions), Fourier transform, and oscillatory integrals. In this talk, we present one of the main features of this developments. Namely, we shall introduce a class of Marcinkiewicz integral operators related to Bochner-Riesz operators and Bochner-Riesz summability [4], [11].

Let Ω be a homogeneous function of degree zero on $\mathbf{R}^n$ that is integrable on $\mathbf{S}^{n-1}$ and satisfies

$$\int_{\mathbf{S}^{n-1}} \Omega(y)d\sigma(y) = 0. \tag{2}$$

The classical Marcinkiewicz integral operators on higher dimensions which was introduced by E. M. Stein in ([10]) and studied later by many authors ([1], [2], [3], [6], [9], among others) is defined by

$$\mu_\Omega f(x) = \left(\int_{-\infty}^{\infty} \left| \int_{|y|\leq 2^t} f(x-y)\,|y|^{1-n}\,\Omega(y)dy \right|^2 \frac{dt}{2^{2t}} \right)^{\frac{1}{2}}. \tag{3}$$

On the other hand, the Bochner-Riesz operator is the Fourier integral operator S^δ defined by

$$S^\delta(f)(x) = \int_{|\xi|<1} \hat{f}(\xi)(1-|\xi|^2)^\delta e^{2\pi i x\cdot\xi}d\xi = f*K^\delta(x) \tag{4}$$

where

$$K^{\delta}(y) = \pi^{-\delta}\Gamma(1+\delta)\,|y|^{-\frac{n}{2}-\delta}\,J_{\frac{n}{2}+\delta}(2\pi\,|y|), \delta > 0.$$

Motivated by the study of the classes of operators (3) and (4), we introduce the following class of operators.

For a suitable function $\Gamma : \mathbf{R}^n \to \mathbf{R}$ and real number $\nu > -1$**,** let $S_{\Gamma,\upsilon}$ be the square function defined by

$$S_{\Gamma,\nu}(f)(x) = \left(\int\limits_{-\infty}^{\infty}\left|\int\limits_{|y|<2^t} f(x-y)\frac{\Gamma(y)}{|y|^{n-1}}J_{\nu}(2\pi\,|y|)dy\right|^2\frac{dt}{2^{2t}}\right)^{\frac{1}{2}}. \tag{5}$$

By taking $\nu = \frac{n}{2}+\delta$ and $\Gamma_0(y) = \Gamma(y) = |y|^{\frac{n}{2}-\delta-1}$, we can show that the L^p boundedness of the corresponding Bochner-Riesz operator S^{δ} follows from the a priori L^p boundedness of the square function $S_{\Gamma_0,\frac{n}{2}+\delta}$. On the other hand, if

$$\Gamma_1(y) = \sqrt{\frac{\pi\,|y|}{2}}\cos 2\pi\,|y|\,\Omega(y) \;\text{ and }\; \Gamma_2(y) = \sqrt{\frac{\pi\,|y|}{2}}\sin 2\pi\,|y|\,\Omega(y), \tag{6}$$

then

$$\mu_{\Omega}f(x) \leq S_{\Gamma_1,-1/2}(f)(x) + S_{\Gamma_2,1/2}(f)(x). \tag{7}$$

Therefore, a priori L^p estimates of operators of the form (**??**) lead to corresponding L^p estimates of the Marcinkiewicz operator μ_{Ω}.

It is the aim of this paper to discuss the L^p boundedness of the operator $S_{\Gamma,\nu}$. We shall discuss the L^p mapping properties of this class of operators when their kernels are rough in $L(\log L)^{1/2}(\mathbf{S}^{n-1})$. The role played by the oscillation carried by the Bessel functions will be highlighted. We shall show that the global parts of the introduced operators are bounded on the Lebesgue spaces $L^p(1 < p < \infty)$ while the local parts are bounded on certain Sobolev spaces.

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Estimates of operator moduli of continuity

A. Aleksandrov

Let f be a uniformly continuous function on $\mathbb{R}$. Denote by ω_f the modulus of continuity of f. We define the operator modulus of continuity as follows

$$\Omega_f(\delta)\stackrel{\text{def}}{=}\sup\{\|f(A)-f(B)\|:A,B\ \text{ are self-adjoint},\quad \|A-B\|\le\delta\}.$$

It is well known that a Lipschitz function does not have to be operator Lipschitz. In other words, the condition $\omega_f(\delta)=O(\delta)$ does not imply that $\Omega_f(\delta)=O(\delta)$ in general.

Theorem. $\Omega_f\le\text{const}\delta\int_\delta^\infty\frac{\omega_f(t)}{t^2}\,dt.$

In particular, if f is a bounded Lipschitz function, then

$$\Omega_f(\delta)=O(\delta|\log\delta|)\quad\text{for small}\quad\delta>0.$$

We are going to improve these estimates for some special classes functions. For example, if f is a bounded piecewise linear function, then

$$\Omega_f(\delta) = O(\delta \log|\log\delta|) \quad \text{for small} \quad \delta > 0. \tag{8}$$

Moreover, we obtain lower estimates for Ω_f. In particular, we prove that the estimate cannot be improved if $f \neq \text{const}$.

Finally, we construct a C^∞ function f on $\mathbb{R}$ such that $|f| \le 1$, $|f'| \le 1$, and

$$\Omega_f(\delta) \ge \text{const}\,\delta\sqrt{|\log\delta|} \quad \text{for small} \quad \delta > 0.$$

The talk is based on a joint work with V. Peller.

Operator moduli of continuity for normal operators

A. Aleksandrov

Introduction. Let f be a uniformly continuous function on $\mathbb{C}$. Denote by ω_f the modulus of continuity of f. We define the operator modulus of continuity of f as follows

$$\Omega_f(\delta) \stackrel{\text{def}}{=} \sup\{\|f(M) - f(N)\| : M, N \text{ are normal}, \ \|M - N\| \le \delta\}.$$

Put

$$\omega^*(\delta) = \delta\int_\delta^\infty \frac{\omega(t)\,dt}{t^2} = \int_1^\infty \frac{\omega(\delta t)\,dt}{t^2}.$$

The following result was obtained in the paper [1] by A. Aleksandrov, V. Peller, D. Potapov, F. Sukochev.

Theorem 1. Let f be a uniformly continuous function on $\mathbb{C}$. Then $\Omega_f \le \text{const}\ \omega^*$. Moreover,

$$\|f(M)R - Rf(N)\| \le \text{const}\|R\|\ \omega_f^*\left(\frac{\max\left\{\|MR - RN\|, \|M^*R - RN^*\|\right\}}{\|R\|}\right)$$

for every bounded nonzero operator R and normal operators M and N.

Main results. Put

$$\omega^{**}(\delta) = (\omega^*)^*(\delta) = \delta\int_\delta^\infty \frac{\omega(t)\log(t/\delta)}{t^2}\,dt = \int_1^\infty \frac{\omega(\delta t)\log t}{t^2}\,dt.$$

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Theorem 2. Let f be a uniformly continuous function on $\mathbb{C}$. Then

$$\|f(M)R - Rf(N)\| \le \text{const}\|R\|\ \omega_f^{**}\left(\frac{\|MR - RN\|}{\|R\|}\right)$$

for every bounded nonzero operator R and normal operators M and N.

Corollary. Let f be a Hölder function of the order α, i. e. $|f(z)-f(w)| \le \text{const}|z-w|^\alpha$ for all $z, w \in \mathbb{C}$. Suppose that $\alpha < 1$. Then

$$\|f(M)R - Rf(N)\| \le \text{const}\|R\|^{1-\alpha}\|MR - RN\|^\alpha$$

for every bounded operator R and normal operators M and N.

It should be noted that if

$$\|f(M)R - Rf(N)\| \le \text{const}\|MR - RN\|$$

for some function $f : \mathbb{C} \to \mathbb{C}$ and all bounded operators R and normal operators M and N, then $f(z) = az + b$.

The talk is based on a joint work with V. Peller.

Sublinear operators on Herz spaces with variable exponents

A. Almeida

We consider both homogeneous and inhomogeneous Herz spaces where the two main indices are variable exponents. We present boundedness results for a wide class of sublinear operators acting on such spaces, which includes maximal, potential and singular type operators as particular cases.

This talk is based on joint work with D. Drihem from M'Sila University, Algeria.

A variational approach to multidimensional truncated moments problems

C.-G. Ambrozie

We consider truncated problems of moments in several real variables, concerned with the existence of representing densities for given moments on the whole Euclidian space. The existence of such solutions is characterized by the positive definiteness (in a generalize sense) of a polynomial associated with the prescribed moments. We also discuss various possibilities of approximating the coefficients of this polynomial in terms of the given data.

Supercharacters, set partitions and symmetric functions in noncommuting variables

Carlos A. M. André

Identifying structures in seemingly disparate fields is a basic task of mathematics. An example is the identification of the character theory of the symmetric group with symmetric function theory. We discuss a similar program for the supercharacter theory associated to the uppertriangular group and the symmetric functions in noncommuting variables. In particular, we will discuss an isomorphism of two Hopf algebras structures: supercharacters, which provide a useful way of doing Fourier analysis on the group of unipotent uppertriangular matrices with coefficients in a finite field, and symmetric functions in noncommuting variables.

The k-rank numerical radii

Aikaterini Aretaki

Let $\mathcal{M}_n(\mathbb{C})$ be the algebra of $n \times n$ complex matrices and $k \geq 1$ be a positive integer. The *k-rank numerical range* $\Lambda_k(A)$ of $A \in \mathcal{M}_n(\mathbb{C})$ is defined by

$$\Lambda_k(A) = \{\lambda \in \mathbb{C} : Q^* A Q = \lambda I_k \text{ for some } Q \in \mathcal{Q}_k\},$$

where $\mathcal{Q}_k$ is the set of all $n \times k$ isometries. For $k = 1$, $\Lambda_k(A)$ reduces to the classical *numerical range* of A, $\Lambda_1(A) \equiv F(A) = \{x^* A x : x \in \mathbb{C}^n,\ x^* x = 1\}$.

Initially in our work, $\Lambda_k(A)$ is proved to coincide with the indefinite intersection of numerical ranges $F(M^* A M)$, $M \in \mathcal{Q}_{n-k+1}$. This statement is proved by using the Courant-Fisher Theorem, without taking into account [2,Cor4.9]. Further, elaborating this expression, we present $\Lambda_k(A)$ as *the countable intersection*

$$\Lambda_k(A) = \bigcap_{\nu \in \mathbb{N}} F(M_\nu^* A M_\nu), \quad \text{where } \{M_\nu\}_{\nu \in \mathbb{N}} \subseteq \mathcal{Q}_{n-k+1}. \tag{9}$$

Equality (9) motivates our study of the *k-rank numerical radius* $r_k(A) = \max\{|z| : z \in \partial\Lambda_k(A)\}$ and *inner k-rank numerical radius* $\widetilde{r}_k(A) = \min\{|z| : z \in \partial\Lambda_k(A)\}$, where $\partial(\cdot)$ is the boundary of a set. This is accomplished by denoting $\mathcal{J}_\nu(A) = \bigcap_{p=1}^{\nu} F(M_p^* A M_p)$, with $M_p \in \mathcal{Q}_{n-k+1}$, and then the radius $r_k(A)$ is identified with the limited formula $r_k(A) = \lim_{\nu\to\infty} \sup\{|z| : z \in \mathcal{J}_\nu(A)\}$. Also, if $0 \notin \Lambda_k(A)$, it follows $\widetilde{r}_k(A) = \lim_{\nu\to\infty} \inf\{|z| : z \in \mathcal{J}_\nu(A)\}$. The results developed draw attention to the k-rank numerical range $\Lambda_k(L(\lambda))$ of a matrix polynomial $L(\lambda) = \sum_{i=0}^{m} A_i \lambda^i$ $(A_i \in \mathcal{M}_n)$, introduced in [1].

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Joint work with J. Maroulas, Department of Mathematics, National Technical University of Athens, Greece

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On the extended and Allan spectra and topological radii

H. Arizmendi

In this paper we prove that the extended spectrum $\Sigma(x)$, defined by W. Żelazko, of an element x of a pseudo-complete locally convex unital complex algebra A is a subset of the spectrum $\sigma_A(x)$, defined by G. R. Allan. Furthermore, we prove that they coincide when $\Sigma(x)$ is closed. We also establish some order relations between several topological radii of x, among which are the topological spectral radius $R_t(x)$ and the topological radius of boundedness $\beta_t(x)$.

Exponential decay and localization of the spectrum of a quadric pencil with strong damping operator

Nikita Artamonov

In Hilbert space H we consider a quadric pencil

$$L(\lambda) = \lambda^2 I + \lambda D + A$$

with self-adjoint positive definite operator A. By H_s denote a collection of Hilbert spaces generated by operator $A^{1/2}$, $\|\cdot\|_s$ is a norm on H_s. We will assume that D is a bounded accretive operator acting from H_1 to H_{-1} and

$$\inf_{x\in H_1, x\neq 0} \frac{\mathrm{Re}(Dx,x)_{-1,1}}{\|x\|_1^2} > 0 \quad \text{or} \quad \inf_{x\in H_1, x\neq 0} \frac{\mathrm{Re}(Dx,x)_{-1,1}}{\|x\|^2} > 0$$

(here $(\cdot,\cdot)_{-1,1}$ is a duality pairing on $H_{-1}\times H_1$). We obtain a localization of the spectrum of $L(\lambda)$ and an exponential decay rate for associated linear differential equation

$$u''(t) + Du'(t) + Au(t) = 0 \quad t \geq 0.$$

The work is supported by the Russian Fund for Basic Research (grant No. 11--01--00790).

Operators and maximal regularity on tent spaces

Pascal Auscher

We discuss progress on comprehension of operator theory on tent spaces, with a focus on the maximal regularity operator, which looks like the analog of the Hilbert transform in this context. This talk is based on collaborations with A. Rosé9n, S. Monniaux, P. Portal and C. Kriegler.

The method of alternating projections revisited

C. Badea

We present several new results related to the method of alternating projections and its variations. This method is based on the following theorem of von Neumann-Halperin : the iterates of a product of several orthogonal projections in Hilbert space are strongly convergent.

An atomic decomposition and maximal characterization of the Hajłasz Sobolev space M_1^1 on manifolds

Nadine Badr

Let X be a metric measure space with a positive Borel measure. For $1 \leq p \leq \infty$, the (homogeneous) Hajłasz Sobolev space $\dot{M}_p^1$ is defined as the set of all functions $u \in L_{1,loc}$ such that there exists a measurable function $g \geq 0$, $g \in L_p$, satisfying

$$|u(x) - u(y)| \leq d(x,y)(g(x) + g(y)) \quad \mu - a.e.$$

Assume now that X is a Riemannian manifold satisfying the doubling volume property and admitting a Poincaré inequality (P_p). Then the Hajłasz Sobolev space and the usual Sobolev space coincide if $1 < p \leq +\infty$. This is not the case for $p = 1$: $\dot{M}_1^1$ is a strict subspace of $\dot{W}_1^1$. In this talk, we prove that $\dot{M}_1^1$ is identified with a Hardy-Sobolev space defined in terms of atoms. We also give a maximal characterization of this space. This is a joint work with G. Dafni.

The intertwining of function theory and systems engineering

Joseph A. Ball

It is now well known how intertwined are the two areas of function theory on the unit disk (including e.g. zero-pole interpolation, Beurling-Lax representations, Nevanlinna-Pick interpolation, inner-outer and Wiener-Hopf factorization, Sz.-Nagy-Foias model theory) and the systems theory coming from control engineering and signal processing (including e.g. input/state/output linear systems, transfer-function realization, control and observation operators, cascade connection of linear systems). In recent years there has been steady progress in understanding how these ideas extend to multivariable function theory and multidimensional systems. While the best understood examples are the Schur-Agler class over the polydisk (where the validity of the appropriate von Neumann inequality is taken as an axiom rather than a theorem) and the Drury-Arveson space over the unit ball in multivariable complex Euclidean space (where the Drury-Arveson commutative shift-tuple plays the role of the unilateral shift in the dilation theory for commutative row contractions), more recent results give system-theory connections and interpretations for function theory on the Bergman space which are new even for the single-variable setting. The plan of the talk is to give an overview of all these diverse developments and to suggest future directions in this area.

Test functions, kernel functions, and matrix-valued Schur-Agler class

Joseph A. Ball

Test functions, kernel functions, and matrix-valued Schur-Agler class The classical Schur class consists of holomorphic functions on the unit disk with values equal to Hilbert-space contraction operators. Elements of this space have a number of equivalent characterizations: contractive multipliers on Hardy spaces over the unit disk, positivity of the associated de Branges-Rovnyak kernel function, realization as the transfer function of a dissipative (or even conservative) discrete-time input/state/output linear system. There has been much interest of late in extensions of these ideas to more general settings. One such setting is the test-function approach, explored by Agler-McCarthy and Dritschel-Marcantognini-McCullough, where one defines a generalized Schur class as the intersection of the contractive multiplier algebras over the collection of kernels for which each function in a preassigned collection of test functions is a contractive multiplier. We indicate extensions of this test-function approach to the case where the test functions, kernel functions, and Schur-class functions are allowed to be be matrix- or operator-valued. As an application we consider the matrix-valued Schur class of a finitely-connected planar domain and indicate connections with the recent negative solution of the spectral set question for such domains (having at least three holes) due to Agler-Harland-Raphael and Dritschel-McCullough. This is joint work with Moisés Guerra-Huaman of Virginia Tech.

Spectral Regularity and Families of Homomorphisms in Noncommutative Gelfand Theory

H. Bart

Let $\mathcal{B}$ be a Banach algebra with unit element. If D is a bounded Cauchy domain in the complex plane and f is an analytic $\mathcal{B}$-valued function taking invertible values on the boundary ∂D of D, the contour integral

$$\frac{1}{2\pi i}\int_{\partial D} f'(\lambda)f(\lambda)^{-1}d\lambda \tag{10}$$

is well-defined. By Cauchy's theorem, it is equal to the zero element in $\mathcal{B}$ when f has invertible values on all of D. The Banach algebra $\mathcal{B}$ is said to be *spectrally regular* if the converse of this is true. This means that (10) can only vanish in the trivial case where f takes invertible values on all of D. Matrix algebras are always spectrally regular, Banach algebras of bounded linear operators on an infinite dimensional Banach space generally not.

The talk focusses on criteria for Banach algebras to be spectrally regular. These involve new aspects of non-commutative Gelfand theory using families of matrix homomorphisms. The crucial notion turns out to be that of a *radical-separating* family, i.e., one where the intersection of the kernels of the homomorphisms in question is contained in the radical of the underlying Banach algebra. This notion generalizes both the concept of a sufficient family and that of a weakly sufficient family, familiar from the existing literature.

The talk is based on joint work with T. Ehrhardt (Santa Cruz) and B. Silbermann (Chemnitz).

Algebraic properties and the finite rank problem for Toeplitz operators on the Segal-Bargmann space

W. Bauer

In this talk we discuss different problems in the area of Toeplitz operators T_f with symbol f acting on the Segal-Bargmann space of Gaussian square integral entire functions on $\mathbb{C}^n$. We calculate the commutant of T_f whenever f is a radial function or a monomial. Moreover we address the *"zero product problem"* and the *"finite rank problem"* of Toeplitz operators in this framework. Except for the study of zero products of Toeplitz operators these problems have been quite well understood in the case of compactly supported symbols (Bergman spaces over bounded domains). Here we do not make any assumption on the the support of f and we even deal with unbounded operator symbols with a controlled behaviour at infinity. We point out that new effects arise from the unboundedness of the domain and the symbols in all of the above problems. This presentation is based on three recent joint papers together with T. Le (U. Toledo, USA), H. Issa (U. Göttingen, Germany) and Y.J. Lee (Chonnam U., Korea), respectively.

Sectorial Stieltjes functions and their realizations by L-systems

S. Belyi

We consider a class of sectorial Stieltjes functions. It is shown that a function belonging to this classes can be realized as the impedance function of a singular L-system with a sectorial state-space operator. We provide an additional condition on a given function from this class so that the state-space operator of the realizing L-system is α-sectorial with the exact angle of sectoriality α. Then these results are applied to L-systems based upon a non-self-adjoint Schrödinger operator.

The talk is based on a joint work with Yu. Arlinskiĭ and E. Tsekanovskiĭ.

Some applications for the spectral theory for the pencil of operators in hilbert spaces

Mehdi Benabdallah

The aim of this research is to generalize the famous General Theorem of Lyapounov of the stability for some implicit systems in Hilbert spaces, using the spectral theory of the corresponding pencil of operators.

Additive transformations compressing certain spectral functions of operators

M. Bendaoud

In this talk we present new results concerning additive transformations compressing some spectral functions or preserving certain spectral quantities such as the local spectrum, the convexified local spectrum, and the reduced minimum modulus of operators. Part of this talk is based on a joint work with M. Sarih.

Orbits of operators commuting with the Volterra operador

Sergio Bermudo Navarrete

This talk is concerned with two classes of operators that commute with the classical Volterra operator V. The first one consists of operators of the form $T = V^r(I + W)$, where $r > 0$ and W is a bounded quasi-nilpotent operator that commutes with V. The second class of operators is of the form $I + V^r(\lambda I + W)$, where W is a bounded quasinilpotent operator that commutes with the Volterra operator, $r > 0$, and λ is a nonzero complex number.

Bilinear Strichartz estimates and bilinear multipliers

F. Bernicot

The so-called linear Strichartz estimates are efficient ways to control the size of solutions to a linear problem in terms of the size of the initial datum. Usually this notion of ``size" is given by a suitable functional space $L^q_t L^r_x$. So for an initial data f, it is now well-known that the unitary group $e^{it\Delta}$ satisfies the following inequality:

$$\|e^{it\Delta} f\|_{L^p L^q(\mathbb{R}^+ \times \mathbb{R}^d)} \lesssim \|f\|_{L^2(\mathbb{R}^d)}$$

for every admissible exponents (p, q) which means: $2 \leq p, q \leq \infty$, $(p, q, d) \neq (2, \infty, 2)$ and $\frac{2}{p} + \frac{d}{q} = \frac{d}{2}$.

Recently bilinear (and more generally multilinear) analogs of such inequalities have appeared. They correspond to controlling the size of the (pointwise) product of two linear solutions, for instance:

$$\|u\|_{L^p L^q(\mathbb{R}^+ \times \mathbb{R}^d)} \lesssim \|f\|_{L^2(\mathbb{R}^d)} \|g\|_{L^2(\mathbb{R}^d)} \quad \text{with} \quad \begin{cases} i\partial_t v + \Delta v = 0, \quad v(t=0) = f \\ i\partial_t w + \Delta w = 0, \quad w(t=0) = g \\ i\partial_t u + \Delta u = vw, \quad u(t=0) = 0. \end{cases} \tag{11}$$

The relevance of these estimates for the analysis of the nonlinear problem is obvious (and have appeared in numerous works).

In this context, p and q are typically chosen equal to 2 (which is related to the use of the Bourgain $X^{s,b}$ spaces); and f and g are chosen with vastly different frequency supports, the focus being to understand the effect on the implicit constant in (11).

This is to be contrasted with our aim in the present talk: we want to allow any Lebesgue indices p and q, and to obtain estimates similar to the above for f and g typically sharing the same compact support in frequency. Also, we shall present results valid for general dispersion relations. This new kind of bilinear Strichartz estimates will allow us to obtain stability results for such nonlinear PDEs. Aiming that, we will use the framework and the point of view of space-time resonnances making appear bilinear operators, associated to a symbol supported around a singular set in the frequency space. We will also present some general results concerning such bilinear multipliers.

The talk is based on a joint work with P. Germain (Courant Institute - New York).

Variation operators for semigroups and Riesz transforms in L^p and BMO spaces in the Schrödinger setting

J. Betancor

In this talk we present L^p- and BMO- boundedness properties of the variation operators associated with the heat semigroup, Riesz transforms and commutators between Riesz transforms and multiplication by $BMO(\mathbb{R}^n)$-functions in the Schrödinger setting. The talk is based on joint works with J.C. Fariña, E. Harboure and L. Rodríguez-Mesa.

Asymptotics of individual eigenvalues of a class of large Hessenberg Toeplitz matrices

J. Bogoya

We study the asymptotic behavior of individual eigenvalues of the $n \times n$ truncations of certain infinite Hessenberg Toeplitz matrices as n goes to infinity. The generating function of the Toeplitz matrices is supposed to be of the form $a(t) = t^{-1}(1-t)^{\alpha} f(t)$ ($t \in \mathbb{T}$), where α is a positive real number but not an integer and f is a smooth function in H^{∞}. The classes of generating functions considered here and in a recent paper by Dai, Geary, and Kadanoff are overlapping, and in the overlapping cases, our results imply in particular a rigorous justification of an asymptotic formula which was conjectured by Dai, Geary, and Kadanoff on the basis of numerical computations.

The talk is based on a joint work with A. Böttcher, S. M. Grudsky, and E. A. Maksimenko.

On the Reduction of Matrices of Linear Functional Operators

M.S. Boudellioua

Matrices whose elements are linear functional operators arise from the treatment of multidimensional linear systems such as systems of partial differential equations and delay-differential equations. In this paper we present necessary and sufficient conditions under which such matrices can be reduced by unimodular equivalence to a simpler form. We will establish the exact connection between the original matrix and the reduced form. The method is illustrated by examples appearing in the literature.

Unbounded subnormal weighted shifts on directed trees

P. Budzyński

We provide a criterion for subnormality of unbounded weighted shifts on directed trees. It is written in terms of consistent systems of probability measures. The proof is based on a new approach to subnormality of unbounded Hilbert space operators. We discuss this approach as well. We also supply some necessary conditions for subnormality of weighted shifts on directed trees.

The talk is based on a joint work with Z. J. Jabłoński, I. B. Jung and J. Stochel.

Sharp weighted L^p estimates for spectral multipliers without Gaussian estimates

The Anh Bui

Let L be a non-negative self-adjoint operator on $L^2(\mathbb{R}^n)$. Assume that operator L generates the analytic semigroup $\{e^{-tL}\}_{t>0}$ which satisfies full-off diagonal estimates $L^{q_0} - L^{q_0'}$ for some $q_0 \in (1,2)$. The aim of this paper is studying the weighted norm inequalities for spectral multipliers $F(L)$ and their commutators with BMO functions.

Kernels of asymmetric Toeplitz operators and applications to almost periodic factorization

M. C. Câmara

A Riemann-Hilbert approach to almost periodic factorization is presented, based on finding particular solutions to a related homogeneous Riemann-Hilbert problem. The space of solutions of the latter is described in terms of the kernels of certain associated asymmetric Toeplitz operators, which are introduced and discussed. The results are applied to a class of triangular 2×2 matrix functions for which the factorization can be explicitly obtained.

The talk is based on a joint work with Yu. I. Karlovich and I. M. Spitkovsky.

Heisenberg uniqueness pairs and Perron-Frobenius operators

F. M. Canto

Let Γ be a curve in the plane and Λ a set in the plane. Let $M(\Gamma, \Lambda)$ be the space of finite Borel measures in the plane supported on Γ, absolutely continuous with respect to arc length and whose Fourier transform vanishes on Λ. We say that (Γ, Λ) is a Heisenberg uniqueness pair if $M(\Gamma, \Lambda)$ is trivial. In this talk we will consider non Heisenberg uniqueness pairs. The curve Γ will be a hyperbola and the set Λ will be the lattice cross

$$\Lambda = (\alpha\mathbb{Z} \times \{0\}) \cup (\{0\} \times \beta\mathbb{Z}),$$

where $\alpha\beta > 1$. We will show that (Γ, Λ) fails to be a Heisenberg uniqueness pair in a very drastic way, in the sense that the corresponding space of measures $M(\Gamma, \Lambda)$ is infinite-dimensional. Some interesting related problems will also be discussed. Dynamical systems, and more specifically Perron-Frobenius operators, play a key role and a brief exposition about the main tools that are needed will be presented. The talk is based on a joint work with H. Hedenmalm and A. Montes.

Sharpened inequalities for operators and applications

Gilles Cassier

We show how sharpened forms of the von Neumann inequality can be derived from mapping theorem for rho-contractions. We introduce a Harnack ordering between such operators and define the corresponding Harnack parts. It involves some operatorial Harnack inequalities as well as von Neumann inequalities. Also, some sharpened forms of Schwarz inequality are given. We present a general Julia's lemma for operators whose spectrum is contained in the closed unit disc. Finally, some applications in three directions are indicated. The first one is concerned with the hyperbolic operatorial metric, the second one is devoted to inequalities of coefficients of rational functions positive on the torus and the last one gives estimates for various norms of generalized derivations.

Closed-range composition operators on Dirichlet-type spaces

G. R. Chacón

We study the boundedness of composition operators mapping one Dirichlet-type space into another. We also study when close composition operators have closed range and give a geometric characterization.

C*-Algebras Generated by Spherical Hyperexpansions

S. Chavan

Let T be a spherical completely hyperexpansive m-variable weighted shift on a complex, separable Hilbert space $\mathcal{H}$ and let $T^{\mathfrak{s}}$ denote its spherical Cauchy dual. We obtain the hyperexpansivity analog of the structure theorem of Olin-Thomson for the C^*-algebra $C^*(T)$ generated by T, under the natural assumption that $T^{\mathfrak{s}}$ is commuting. If, in addition, the defect operator $I - T_1T_1^* - \cdots - T_mT_m^*$ is compact then we ensure exactness of the sequence of C^*-algebras

$$0 \longmapsto \mathcal{C}(\mathcal{H}) \hookrightarrow C^*(T) \overset{\pi}{\longmapsto} C(\sigma_{ap}(T)) \longmapsto 0,$$

where $\mathcal{C}(\mathcal{H})$ stands for the ideal of compact operators on $\mathcal{H}$, and $\pi : C^*(T) \to C(\sigma_{ap}(T))$ is the unital $*$-homomorphism defined by $\pi(T_i) = z_i$ $(i = 1, \cdots, m)$. This unifies and generalizes the results of Coburn, and Arveson in case $m = 2$.

We further illustrate our results by exhibiting a one parameter family $\mathcal{F}$ of spherical completely hyperexpansive 2-tuples T_{ν_λ} acting on $P^2(\mu_\lambda)$ $(1 \leq \lambda \leq 2)$, where $d\mu_\lambda := d\nu_\lambda d\sigma$, ν_λ is a probability measure on $[0, 1]$, and σ is the normalized surface area measure on the unit sphere $\partial\mathbb{B}$. Interestingly, within the family $\mathcal{F}$, the Szegö 2-shift T_{ν_1} and the Drury-Arveson 2-shift T_{ν_2} occupy the extreme positions. We would like to emphasize that T_{ν_λ} is unitarily equivalent to the multiplication operator tuples in $P^2(\mu_\lambda)$ if and only if $\lambda = 1$.

Composition operators on the Fock-Sobolev spaces

B. R. Choe

Linear combinations of composition operators acting on the Fock-Sobolev spaces of several variables are studied. We show that such an operator is bounded only when all the composition operators in the combination are bounded individually. So, cancelation phenomenon is not possible on the Fock-Sobolev spaces, in contrast to what have been known on other well-known function spaces over the unit disk. We also show the analogues for compactness and the membership in the Schatten classes. In particular, compactness and the membership in some/all of the Schatten classes turn out to be the same.

This talk is based on a joint work with H. Cho and H. Koo.

Computing Cauchy type Singular Integrals with *Mathematica* software

Ana C. Conceição and José C. Pereira

We constructed an algorithm, [SInt] (see [2]), for computing some classes of Cauchy type singular integrals on the unit circle. The design of [SInt] was focused on the possibility of implementing on a computer all the extensive symbolic and numeric calculations. Furthermore, we show how the factorization algorithm described in [1] allowed us to construct and implement the [SIntAfact] algorithm (see [2]) for calculating several interesting singular integrals that cannot be computed by [SInt]. All the above techniques were implemented using the symbolic computation capabilities of the computer algebra system *Mathematica*. Several examples of nontrivial singular integrals computed with both algorithms are presented.

The talk is based on a joint work with Viktor G. Kravchenko.

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Invariant Subspaces of Composition Operators

C. C. Cowen

If ϕ is an analytic map of the disk into itself and H is a Hilbert space of analytic functions on the disk, the composition operator C_ϕ on H is the operator defined by $(C_\phi f)(z) = f(\phi(z))$ for z in the disk and f in H.

In 1984, Nordgren, Rosenthal, and Wintrobe observed that, if ϕ is a hyperbolic automorphism of the disk, then C_ϕ^* acting on the Hardy Hilbert space is a `universal operator' in the sense that every bounded operator on a Hilbert space is unitarily equivalent to a non-zero multiple of the restriction of C_ϕ^* to one of its invariant subspaces. Since that time, especially over the past decade, there has been growing interest in the invariant subspaces of composition operators. This talk will be an incomplete survey of what is known about invariant subspaces of composition operators with the goal of pointing out that this a rich area for future study.

The most striking result up to now has been the characterization by Montes-Rodríguez, Ponce-Escudero, and Shkarin (2010) of the lattice of invariant subspaces of a composition operator C_ϕ on the Hardy Hilbert space when ϕ is a linear fractional map of the disk into itself with $\phi(1) = \phi'(1) = 1$ but not an automorphism of the disk.

The most work has been done on invariant subspaces of composition operators that are also invariant for the operator of multiplication by z (and therefore the algebra of operators of multiplication by an analytic function). We will see that a promising beginning has been made in understanding these subspaces. In this context, a theorem identifying the Hermitian weighted composition operators on standard weight Bergman spaces and computing their spectral resolutions has led to identifying the extremal functions for the subspaces associated with the usual atomic inner functions for these weighted Bergman spaces and getting explicit formulas for the projections of the kernel functions to these subspaces.

Structural stability of smoothness of the maximal solution to the geometric eikonal equation

Jaime Cruz-Sampedro

We investigate the structural stability of smoothness of the maximal solution to the geometric eikonal equation

$$\nabla S(x) G^{-1}(x) (\nabla S(x))^T = 1, \qquad S(0) = 0, \qquad\qquad x \in \mathbb{R}^d \setminus 0.$$

For a subclass of metrics of order zero G on $\mathbb{R}^d$ we show existence, stability as well as precise asymptotics for the derivatives of the maximal solution. Our results generalize similar results of Barles and Lions for the standard eikonal equation and are applicable to Schrödinger operator theory. This is joint work with E. Skibsted from the University of Århus, Denmark.

Hyponormality and subnormality of block Toeplitz operators

Raúl Curto

I will discuss hyponormality and subnormality of *block* Toeplitz operators acting on the vector-valued Hardy space $H^2_{\mathbb{C}^n}$ of the unit circle.

In joint work with I.S. Hwang and W.Y. Lee, we first establish a tractable and explicit criterion to determine the hyponormality of block Toeplitz operators having bounded type symbols; we do this via the triangularization theorem for compressions of the shift operator.

Secondly, we consider the gap between hyponormality and subnormality for block Toeplitz operators. This is closely related to Halmos's Problem 5: Is every subnormal Toeplitz operator either normal or analytic ? We show that if Φ is a matrix-valued rational function whose co-analytic part has a coprime decomposition then every hyponormal Toeplitz operator T_Φ whose square is also hyponormal must be either normal or analytic.

Next, we apply our results to solve the following ``Toeplitz completion" problem: Find the unspecified Toeplitz entries of the partial block Toeplitz matrix

$$A := \begin{bmatrix} U^* & ? \\ ? & U^* \end{bmatrix}$$

so that A becomes subnormal, where U is the unilateral shift on H^2.

Condition spectrum results that generalizes usual spectrum results

D. Sukumar

The condition spectrum of an element a in a Banach algebra $\mathcal{A}$ is defined as

$$\sigma_\epsilon(a) := \left\{ \lambda \in \mathbb{C} : a - \lambda I \text{ is not invertible or } \|a-\lambda\|\|(a-\lambda)^{-1}\| \geq \frac{1}{\epsilon} \right\}.$$

This spectrum satisfies the axioms of generalized spectrum defined by Ransford. As the definition directly involves condition number, it concerns about the computational stability aspect of solving a system $Tx = y$. It has similar properties that the spectrum has. Recently a sufficient condition for a linear function to be almost multiplicative was given in terms of condition spectrum. Here, certain results which are generalizing the point spectra results and the pseudo spectra results will be presented. Also bounds of the condition spectrum for special classes of operators will be discussed.

Szegoś theorem for matrix orthogonal polynomials

Maksym Derevyagin

We will discuss extensions of some classical theorems in the theory of orthogonal polynomials on the unit circle to the matrix case. In particular, a matrix analogue of Szegő's theorem will be presented. It should be mentioned that the developed theory allows us to obtain an elementary proof of the distance formula by Helson and Lowdenslager.

In fact, our approach is based on the adaption of the Khrushchev scheme to the matrix case.

This talk is based on a joint work with O. Holtz, S. Khrushchev, and M. Tyaglov.

Canonical polynomial representations for semi-separable matrices

Patrick Dewilde

Canonical polynomial representations (Popov canonical forms) are classical for rational matrix functions. We show how such representations can be generated for the more general class of semi-separable matrices, thereby generalizing the notion and opening the way for a new type of representation in semi-separable matrix theory. Unfortunately, the Popov construction based on the so called ``column canonical form" does not seem to generalize, but another construction based on so called ``deadbeat control" does, and leads even to Bezout relations and stepping stones towards the solution of a semi-separable version of Löwner interpolation. Interestingly enough, not all semi-separable systems have such a canonical representation, due to the fact that it prevents the potential occurrence of doubly invariance characteristic subspaces, a case that cannot occur in the rational case, making semi-separable operators substantially different from rational ones.

Semigroups of composition operators and integral operators

Santiago Díaz Madrigal

We study the maximal spaces of strong continuity on $BMOA$ and the Bloch space $\mathcal{B}$ for semigroups of composition operators. Characterizations are given for the cases when these maximal spaces are $VMOA$ or the little Bloch $\mathcal{B}_0$. These characterizations are in terms of the weak compactness of the resolvent function or in terms of a specially chosen symbol g of an integral operator T_g. For the second characterization we prove and use an independent result, namely that the operators T_g are weakly compact on the above mentioned spaces if and only if they are compact.

Fredholm properties of operator matrices

Dragan S. Djordjević

We present new result on Fredholm properties of 2×2 operator matrices over Banach or Hilbert spaces.

Riesz projection of a Hilbert space operator

Slaviša V. Djordjević

If a Hilbert space operator T is polar of order k for some $k \in \mathbf{N}$ at a point $\lambda \in \mathrm{iso}\sigma(T)$ and $(\lambda - T)^{-k}(0) \subseteq ((\lambda - T)^*)^{-k}(0)$, then the Riesz projection $P_T(\lambda)$ associated with λ is self-adjoint.

Title A Commutator Approach to Absolute Continuity

J. Dombrowski

Commutator equations provide a useful technique for studying the absolute continuity of spectral measures associated with both bounded and unbounded self-adjoint operators. This technique, which has its roots in the work of C. R. Putnam, will be used to study the spectral properties of certain subclasses of unbounded self-adjoint Jacobi matrix operators. A Jacobi matrix operator can be represented by a tridiagonal matrix with diagonal sequence $\{b_n\}$ and subdiagonal sequence $\{a_n\}$, acting on a dense subset of $\ell^{2.}$. Given such an unbounded Jacobi matrix operator, with appropriate growth conditions imposed on the defining sequences to assure that the operator is self-adjoint, an appropriate bounded operator is chosen for the commutator equation. The structure of the resulting commutator, and its action on a strategic set of vectors related to the spectral decomposition of the given operator, will be discussed, and then used to obtain results on absolute continuity.

On a theme of Beurling

Ron Douglas

One of the most far reaching and influential results in functional analysis is Beurling's characterization of the invariant subspaces of the unilateral shift in terms of the function theory of Nevanlinna, Riesz and others. After considerable effort, over almost fifty years, we know that this result extends only partially to other functional Hilbert spaces such as the Bergmen space over the disk and very little seems to survive to spaces such as the Hardy space over the unit ball or polydisk in C^n.

In this talk we will review these issues and analyze some of the consequences of Beurling's theorem and their validity in other spaces focusing on relations between them. The emphasis will be on formulating precise questions for further study. We use the language of Hilbert modules in the context of the algebra of polynomials in several variables. Questions concerning results such as the lifting theorem and the Corona conjecture arise in this setting. Most examples considered will be the Hardy and Bergman spaces over the ball or polydisk.

Polynomially Weighted Bergman Spaces on the Ball

Ron Douglas

For any probability measure mu on the unit ball in C^n, one can attempt to define a Bergman space which is a subspace of the L^2 space for μ. However, there are several possible definitions for such a space. One would be all functions in $L^2(\mu)$ which are locally equal a. e. to a holomorphic function. A second would be the equivalence classes in $L^2(\mu)$ which contain a holomorphic function on the open ball. Another would be the closure in $L^2(\mu)$ of the polynomials or perhaps the functions holomorphic on a neighorhood of the closure of the ball.

In this talk I discuss a recent result by K. Wang and myself showing that all of these definitions coincide for measures of the form $d(\mu) = |p|^2 dm$, where m is Lebesgue measure on the ball and p is a polynomial. Moreover, in these cases, the Hilbert module defined by this weighted Bergman space is essentially normal; that is, all of the cross-commutators of the multiplication operators defined by the coordinate functions are compact (actually in a Schatten-von Neumann class). The proofs depend on techniques from harmonic analysis.

Noncommutative analogues of the Fejér-Riesz

Michael Dritschel

The classical Fejér-Riesz Theorem states that a nonnegative trigonometric polynomial can be factored as the absolute square of an analytic polynomial. Indeed, the factorization can be done with an outer polynomial. Various generalizations of this result have been considered. For example, Rosenblum showed that the result remained true for operator valued trigonometric polynomials. If one instead considers operator valued polynomials in several variables, one obtains factorization results for strictly positive polynomials, though outer factorizations become more problematic. In another direction, Scott McCullough proved a factorization result for so-called hereditary trigonometric polynomials in freely noncommuting variables (strict positivity not needed). In this talk we consider an analogue of (hereditary) trigonometric polynomials over inverse semigroups, and give a result which includes a strict form of McCullough's theorem as well as the multivariable version of Rosenblum's theorem.

Hardy spaces associated Schrödinger operators

J. Dziubański

Let $L = -\Delta + V$ be a Schrödinger operator on $\mathbb{R}^d$ with a nonnegative locally integrable potential V and let $K_t(x, y)$ be the integral kernels of the semigroup $\{T_t\}_{t>0}$ generated by $-L$. The Feynman-Kac formula implies that

$$0 \leq K_t(x, y) \leq (4\pi t)^{-d/2} \exp\big(- |x - y|^2/4t\big).$$

One possible definition of the Hardy space associated with L is by means of the maximal function

$$\mathcal{M}_L f(x) = \sup_{t>0} |T_t f(x)|.$$

We say that an $L^1(\mathbb{R}^d)$-function f belongs to H^1_L if $\mathcal{M}_L f \in L^1(\mathbb{R}^d)$. Then we set $\|f\|_{H^1_{L,\max}} = \|\mathcal{M}_L f\|_{L^1(\mathbb{R}^d)}$.

Hardy spaces associated with certain operators attracted attention of many authors.

In a very recent work S.Hofmann, G.Lu, D.Mitrea, M.Mitrea and L.Yan provide a general approach to the theory of H^1-spaces associated with semigroups satisfying the Davies-Gaffney estimates, and in particular Schrödinger semigroups, proving an abstract atomic decomposition of the elements of H^1_L.

The purpose of this talk is to present that the Hardy spaces for certain classes of Schrödinger operators admit other atomic decompositions and characterizations by the Riesz transforms $R_j = \partial_{x_j} L^{-1/2}$. The atoms that occur in these atomic decompositions are similar to those of the classical theory of real Hardy spaces, and, depending on the dimension d and behavior of the potential V, are either local or global with new cancellation conditions.

The talk is based on joint works with M. Preisner and J. Zienkiewicz.

On the asymptotics of the determinants of Toeplitz-Hankel matrices

T. Ehrhardt

We present results concerning the asymptotic behavior of the determinants of the matrices which are the Hadamard product of Toeplitz and Hankel matrices

$$M_{\mu,n}(a) = \Big(a_{j-k} m_{j+k}\Big)_{j,k=0}^{n-1}$$

as the matrix size n goes to infinity. Here a_k are the Fourier coefficient of a periodic function a, and m_k are the moments of a measure μ on the positive real axis, i.e.,

$$a_k = \frac{1}{2\pi} \int_0^{2\pi} a(e^{i\theta}) e^{-ik\theta}\, d\theta, \qquad m_k = \int_0^{\infty} x^k\, d\mu(x).$$

Depending on the conditions on the measure μ and under smoothness (and regularity) assumptions on a, two different kinds of asymptotic behavior is identified either

$$\det M_{\mu,n}(a) \sim G^n F, \quad \text{or} \quad \det M_{\mu,n}(a) \sim G^n n^{\Omega} F$$

as $n \to \infty$, with constants $G, F, \Omega \neq 0$. The result generalized the classical Szegö-Widom limit theorem. The matrices considered here appear in Random Matrix Theory when describing the asymptotics of linear statistics of the eigenvalues of certain (non-hermitian) random matrices. This talk is based on joint work with B Rider.

On some completion problems for partially specified matrices

Y. Eidelman

We consider different problems of completion of partially specified matrices to unitary matrices, to matrices with band inverses, to Green matrices and to mutually inverse matrices. A special attention is paid to the unitary completion problem. The latter results are used in the computation of the roots of polynomials. Connections with problems of eigenvalue computations for matrices with quasiseparable structure are analyzed. These connections allow to derive various fast algorithms.

The talk is based on a joint work with I. Gohberg, D. Bini and L. Gemignani.

A collection of results for composition operators on the half-plane

Sam Elliott

A composition operator is an operator on a space of functions induced by some pre-composition with a self-map of the domain. After a brief introduction to known results from the disc, we present a collection of results begun by Valentin Matache and then extended by the speaker jointly with Michael Jury and Andrew Wynn on composition operators on function spaces on the half-plane. While in general the disc case has been more widely studied, and operators behave better in this setting, there are a number of notable properties which have resisted general categorisation: one such question is that of a norm formula. Though all (analytic) composition operators on the disc have long been known to be bounded, at present no formula is known for the norm of a general composition operator on the Hardy spaces of the disc. We present such a formula for the half-plane, along with other related results which demonstrate the differences between the disc and half-plane cases.

The joint work presented here between the speaker and Michael Jury was begun at conferences in Sevilla and El Escorial in 2009. The speaker is very pleased to have the opportunity to return to Sevilla and present this work in Spain.

K-theoretical Algebraic Structures of Crossed Products Orbifolds Wreath Products

Carla Farsi

Joint work with Christopher Seaton.

In this talk I will present structure theorems for the wreath product ring of orbifold $C*$-algebras presented as crossed products of actions on manifolds by compact Lie groups. These wreath product ring structures are expressed in terms of K--thoretical orbifold inertial decompositions. In particular we show that this ring admits λ--ring and Hopf algebra structures both abstractly and directly. This generalizes results known for $C*$--algebras of orbifolds presented as crossed products of actions on manifolds by finite groups.

A Noncommutative Residue for Pseudo-Differential Operators on the Noncommutative Two Torus

F. Fathizadeh

We define a noncommutative residue on classical pseudo-differential operators on the noncommutative two torus and prove that up to a constant multiple, it is the unique trace on the algebra of classical pseudo-differential operators modulo infinitely smoothing operators. This is joint work with M. W. Wong.

Localization and Toeplitz Operators on Polyanalytic Fock Spaces

Nelson Faustino

We will explore the interplay between localization operators on the phase space and the structure of the Berezin symbol calculus in the context of polyanalytic Fock spaces, namely around the Coburn conjecture posed by L. A. Coburn in [2].

Recently the proof of this conjecture was obtained by Lo (cf. [4]) and Engli$\check{s}$ (cf. [3]) in the context of Segal-Bargmann-Fock spaces for nicer class of windows under the constraint that the σ belongs to $L^\infty(\mathbb{C})$.

On the sequel we will present an extension of this result to polyanalytic Fock spaces. While the generation is almost mimetic for the true polyanalytic Fock spaces, the Gabor analysis framework for vector-valued windows (cf. [1]) provides a meaningful extension of this conjecture to polyanalytic Fock spaces. Further extensions of this result to certain classes of Gelfand-Shilov spaces will be also discussed.

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A nonlocal operator C* algebra with a group of shifts having periodic points

Cláudio Fernandes

Representations on a Hilbert spaces are constructed for a nonlocal operator C* algebra $\mathcal{B}$ associated with an amenable discrete group of homeomorphisms with the same nonempty set of periodic points. Using the spectral measure related with a general isometric representation from $\mathcal{B}$ on a Hilbert space we generalize the reduction scheme of Litvinchuk, Gohberg and Krupnik for studying singular integral operators with finite cyclic groups of shifts. This generalization allow us to a symbol calculus for algebra $\mathcal{B}$ and Fredholm criteria for its elements. This talk is based on the joint work with M.A. Bastos and Yu. I. Karlovich.

Sampling theorems for transformations related to regular and irregular eigenvalue problems

G. Freiling

We derive sampling representations associated with boundary eigenvalue problems.
Our approach is connected with the analytical nature of the solutions of differential equations with a spectral parameter. The method requires neither basisness of eigenfunctions nor even their completeness.
As a kernel of the sampled transforms we use the Green's function multiplied by some entire function, which gives a rich variety of such kernels.
The sampling theorems are Lagrange- or Hermite-type interpolation theorems and generalize the classical sampling theorem of Whittaker-Kotel'nikov-Shannon.

Refinements of Young inequality and two reverse inequalities for refined Young inequality

Shigeru Furuichi

In this talk, we show refined Young inequalities for two positive operators. Our results refine the ordering relations among the arithmetic mean, the geometric mean and the harmonic mean for two positive operators. This is a sum-type refinement. We are also able to give a multiple-type refinement by the use of Specht's ratio. Finally, we give two different reverse inequalities for the sum-type refined Young inequality for two positive operators.

On a problem of Halmos: unitary equivalence of a matrix to its transpose

S.R. Garcia

Halmos asked whether every square complex matrix is unitarily equivalent to its transpose (UET). In this talk, we give a complete characterization of matrices which are UET. Surprisingly, the naïve conjecture that a matrix is UET if and only if it is unitarily equivalent to a complex symmetric (i.e., self-transpose) matrix is true in dimensions $n \leq 7$ but false for $n \geq 8$. In particular, unexpected building blocks begin to appear in dimensions 6 and 8. This is joint work with James E. Tener (UC Berkeley). Partially supported by NSF Grant DMS-1001614 (SRG) and NSF Graduate Research Fellowship (JET).

On computing the Newton correction for rank-structured eigenvalue problems

L. Gemignani

In this talk we discuss the use of functional iteration methods for solving general rank-structured eigenvalue problems. Efficient implementations of these methods can be obtained by using matrix methods exploiting the rank structure of the input matrix for the fast evaluation of the Newton correction of the associated characteristic polynomial.

A criterion for unitary similarity of upper triangular matrices in general position

Tatiana G. Gerasimova

Each square complex matrix is unitarily similar to an upper triangular matrix with diagonal entries in any prescribed order. Let $A = [a_{ij}]$ and $B = [b_{ij}]$ be upper triangular $n \times n$ matrices that

- are not similar to direct sums of matrices of smaller sizes, or
- are in general position and have the same main diagonal.

It is proved in [D. Farenick, V. Futorny, T.G. Gerasimova, V.V. Serheichuk, N. Shvai, Linear Algebra Appl. (2011), doi:10.1016/j.laa.2011.03.021] that A and B are unitarily similar if and only if

$$\|h(A_k)\| = \|h(B_k)\| \qquad \text{for all } h \in \mathbb{C}[x] \text{ and } k = 1, \dots, n,$$

where $A_k := [a_{ij}]_{i,j=1}^k$ and $B_k := [b_{ij}]_{i,j=1}^k$ are the principal $k \times k$ submatrices of A and B, and $\| \cdot \|$ is the Frobenius norm.

Consistent sampling of perturbed signals

J. Giribet

To sample a signal f means to obtain a sequence $\{f_n\}_{n\in\mathbb{Z}}$ of instantaneous values of a particular signal characteristic, this sequence are called the samples of f. The classical sampling scheme is based on the Whittaker-Kotelnikov-Shannon theorem. Given a signal $f \in \mathcal{PW}$ (the Paley-Wiener space), the Whittaker-Kotelnikov-Shannon theorem establishes that it is possible to reconstruct the signal f from its values at the integers $\{f(n)\}_{n\in\mathbb{Z}}$. When $f \notin \mathcal{PW}$, a common strategy is to apply a filter (certain bounded linear operator) to the signal f obtaining a new signal g. Then, the filtered signal g is sampled giving the sequence $\{g_n\}_{n\in\mathbb{N}}$. Although, the signal recovered by the samples $\{g_n\}$ will not generally coincide with the original signal f, approximates it. In fact, the recovered signal is the best approximation, i.e., the orthogonal projection, of the original signal f in $\mathcal{PW}$. A common way to represent the samples of a signal f, is by means of the inner product of f with vectors $\{v_n\}_n \in \mathbb{N}$ that span a closed subspace $\mathcal{S}$, called the *sampling subspace*. By the other hand, given the the samples $\{f_n\}_{n\in\mathbb{N}}$, the reconstructed signal $\hat{f}$ is given by $\hat{f} = \sum_{n\in\mathbb{N}} f_n w_n$, where $\{w_n\}_{n\in\mathbb{N}}$ span a closed subspace $\mathcal{R}$, called the *reconstruction subspace*. In the classical sampling scheme the reconstruction and the sampling subspaces are assumed to be the same. In signal processing applications, this not always the case, and then it is not always possible to recover the best approximation of the original signal. Thus, different sampling techniques must be used. In particular, the idea of consistent sampling, it means that the reconstructed signal $\hat{f}$ is not supposed to be the best approximation of the original signal, but f and $\hat{f}$ have the same samples. In an operator point of view, the idea of consistent sampling can be defined as: Given $H, F \in L(\mathcal{K}, \mathcal{H})$, the operator $X \in L(\mathcal{K})$ satisfies the *consistent sampling requirement for H and F* if $X \in \mathcal{CS}(F, H) := \{X \in L(\mathcal{K}) : FXH^* = (FXH^*)^2, N(FXH^*) = N(H^*)\}$.

In this work we study the a problem related with the consistent sampling of perturbed signals $f + \delta f$. More precisely, let μ be a Lebesgue-Stieltjes measure on $\mathbb{R}$ and let $\mathcal{H}$ be the Hilbert space $L^2(\mu)$. Suppose that $(\Omega, \mathcal{F}, P)$ is a probability space; if $z : \Omega \to \mathbb{R}$ is P-measurable then the *expectation* of z is $E(z) = \int_\Omega z(\omega)dP(\omega)$. Let $\delta f : \mathbb{R} \times \Omega \to \mathbb{R}$ be a $\mu \times P$-measurable function such that: 1) for almost every $t \in \mathbb{R}$, $E(\delta f(t, .)) = 0$, 2) for almost every $\omega \in \Omega$, $\delta f(., \omega) \in \mathcal{H}$, 3) $E(\|\delta f\|^2) = \int_\Omega \int_\mathbb{R} |\delta f(\omega, t)|^2 d\mu(t)dP(\omega) < \infty$.

We are interested in the following problem, related with signal processing applications:

Given a closed subspace $\mathcal{M}$ of $\mathcal{H}$, let $\mathcal{J} = \{X \in \mathcal{CS}(F, H) : E(FXH^*(f + \delta f)) = f$ for every $f \in \mathcal{M}\}$. Find $X_0 \in \mathcal{J}$ such that $E(\|FX_0H^*\delta f\|^2) \leq E(\|FX_0H^*\delta f\|^2)$ for every $X \in \mathcal{J}$.

The talk is based on a joint work with G. Corach.

The Burchnall-Chaundy C*-algebra, operator spaces over Hilbert modules, and the tau function.

J. Glazebrook

Commencing from the Burchnall-Chaundy (BC) algebra of pseudodifferential operators, we show that via an integral operation derived from the Sato correspondence, an associated C*-algebra is obtained which is a subalgebra of a Banach algebra of operators over a Hilbert-module. In view of the Sato-Segal-Wilson theory, the KP-flows are realized in terms of flows of multiplication operators on an infinite dimensional Grassmannian over this algebra.

We will describe how the Brown-Douglas-Fillmore theory enters into the picture, and show how elements of an extension group over the (BC) algebra can be related to families of tau functions. Further, we describe an operator-geometric setting for how the predeterminants of the latter can arise in the first place. This talk is based on joint work with M. Dupre' and E. Previato.

Transfer Functions for Markov Chains

R. Gohm

Noncommutative Markov chains are a generalization of the corresponding classical concept which appears in the mathematical description of open quantum systems. Their structure theory is very rich and leads naturally to applications of concepts from multi-variable operator theory such as dilations, row shifts, multi-analytic operators, stochastic multi-variable linear systems etc. We investigate the effectiveness of these tools for typical problems arising in this context. In particular we discuss the analogues of the transfer functions which play such an important role in classical system theory.

Tauberian operators

Manuel González Ortiz

Tauberian operators were introduced by Kalton and Wilansky to investigate some questions in summability theory. These operators have proved to be useful in the study of different topics: real interpolation theory [Beauzamy], factorization of operators [Davis, Figiel, Johnson and Pełczyński], equivalence between the Krein-Milman property and the Radon-Nikodým property [Schachermayer], weak Calkin algebras of operators [Astala and Tylli], approximation property of Banach spaces [Lima, Nygaard and Oja], James' characterization of reflexive Banach spaces [Neidinger and Rosenthal], construction of hereditarily indecomposable Banach spaces [Argyros and Felouzis], extension to operators of the principle of local reflexivity [Behrends], etc.

Here we will survey the properties and the main applications of tauberian operators and describe some related problems that remain open.

References.

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Spectral theory of large Wiener-Hopf operators with complex-symmetric kernels and rational symbol

Sergey Grudsky

This report is devoted to the asymptotic behavior of individual eigenvalues of truncated Wiener-Hopf integral operators over increasing intervals. The kernel of the operators is complex-symmetric and has a rational Fourier transform. Under additional hypotheses, the main result describes the location of the eigenvalues and provides their asymptotic expansions in terms of the reciprocal of the length of the truncation interval. Also determined is the structure of the eigenfunctions.

Dual Toeplitz Operators on the Sphere

H. Guediri

Let $\mathbb{B}_n$ be the unit ball of $\mathbb{C}^n$ and $\mathbb{S}_n$ be its boundary, (the unit sphere). Denote by $\mathcal{H}^2(\mathbb{S}_n)$ the Hardy space on the complex unit sphere. While on the circle, (i.e. for $n = 1$), we have $(\mathcal{H}^2)^\perp = \overline{z\mathcal{H}^2}$, the matter is much more involved in higher dimensions because

$$L^2(\mathbb{S}_n) \ominus \left\{\mathcal{H}^2(\mathbb{S}_n) + \overline{\mathcal{H}^2(\mathbb{S}_n)}\right\}$$

is sufficiently huge to cause remarkable differences from the one dimensional case. It suffices to observe that $L^2(\mathbb{S}_n)$ contains "radial-like" functions; and such phenomenon does not occur in the one-dimensional case.

For $f \in L^\infty(\mathbb{S}_n)$, define the dual Toeplitz operator $\mathcal{S}_f$ on $(\mathcal{H}^2(\mathbb{S}_n))^\perp$ to be a multiplication followed by a projection: $\mathcal{S}_f : u \in (\mathcal{H}^2(\mathbb{S}_n))^\perp \longrightarrow \mathcal{S}_f(u) := (I-\mathcal{P})(fu) \in (\mathcal{H}^2(\mathbb{S}_n))^\perp$, with $\mathcal{P}$ being the customary orthogonal Cauchy-Szegö projection from $L^2(\mathbb{S}_n)$ onto the Hardy space $\mathcal{H}^2(\mathbb{S}_n)$.

Due to the aforementioned characterization of the orthogonal complement of the Hardy space on the circle, dual Toeplitz operators on this space are anti-unitarily equivalent to Toeplitz operators. In higher dimensions, (for instance on the unit sphere), dual Toeplitz operators might behave quite differently and, therefore, seem to be a worth studying new class of Toeplitz-type operators. Our aim here is to introduce and start a systematic investigation of dual Toeplitz operators on the Hardy space of the unit sphere in $\mathbb{C}^n$.

In particular, we obtain a corresponding spectral inclusion theorem, which is known to force the underlying operator to behave elegantly. Brown-Halmos theorems, characterizing products and commutativity of dual Toeplitz operators, are also among our main assertions. Several consequences of these key assertions are also derived. Besides, a characterization of the class of quasinormal dual Toeplitz operators is also discussed.

Invertible weighted composition operators

Gajath Gunatillake

A weighted composition operator $C_{\psi,\varphi}$ takes an analytic map f on the open unit disc of the complex plane to the analytic map $\psi \cdot f \circ \varphi$ where φ is an analytic map of the open unit disc into itself and ψ is an analytic map on the open unit disc. This work studies the invertibility of such operators. The two maps ψ and φ are characterized when $C_{\psi,\varphi}$ acts on the Hardy space H^2. We also explore the spectra.

On Orlicz-Lorentz subspaces of bounded families and Approximation type operators

Manjul Gupta

For an arbitrary index set I , we introduce in this paper , the subspaces $l_{p,q,M}(I)$ and $l_{p,q,r}(I)$ of the space $l_\infty(I)$ of bounded families of scalars, where p,q,r are positive reals and M is an Orlicz function. We study their structural properties and characterize their elements . Besides we consider the product of the spaces $l_{p,q,r}(I)$ and the relationships amongst them for different positive indices p,q, and r.In the final section we establish representation of an operator of approximation type $l_{p,q,r}$ as an infinite series of finite rank operators . We also study the ideal structure of the class of all such operators and their product. These results generalize some of the earlier results proved for Lorentz spaces by A.Pietsch.
Joint work with Antara Bhar.

Muckenhoupt weights in variable exponent spaces

Peter Hasto

I discuss work with Lars Diening and David Cruz-Uribe. We define appropriate Muckenhoupt classes for variable exponent Lebesgue spaces; in other words, we characterize the set of weights ω for which the maximal operator is bounded on $L^{p(\cdot)}(\mathbb{R}^n, \omega)$. The exponent is assumed to satisfy the usual log-Hölder continuity condition.

On the numerical radius of the truncated adjoint Shift

Haykel Gaaya

For any n-by-n complex matrix T and any $1 \leqslant k \leqslant n$, let $\Lambda_k(T)$ the set of all $\lambda \in \mathbb{C}$ such that $PTP = \lambda P$ for some rank-k orthogonal projection P be its higher rank-k numerical range. I will show that if $\mathbb{S}$ is the n-dimensional shift on $\mathbb{C}^n$ then its rank-k numerical range is the circular disc centred in zero and with radius cos $\dfrac{k\pi}{n+1}$ if $1 < k \leqslant \left[\frac{n+1}{2}\right]$ and the empty set if $\left[\frac{n+1}{2}\right] < k \leqslant n$, where $[x]$ denote the integer part of x. This extends and rafines previous results of U. Haagerup, P. de la Harpe on the classical numerical range of the n-dimensional shift on$\mathbb{C}^n$. I will speak also about an interesting result for higher rank-k numerical range of nilpotent operator.

The polyanalytic Ginibre ensembles

Håkan Hedenmalm

The standard (complex) Ginibre ensemble can be viewed as a point process generated by the cut-off exponential function (studied, e.g., by Szego) as probability generating kernel. We study a variant on the theme which permits a certain degree of polyanalyticity. The degree of polyanalyticity may be interpreted as permitting a few higher Landau levels in the physical model. We find that in the limit as the degree tends to infinity and the degree of polyanalyticity also tends to infinity but much more slowly, if we blow up at a bulk point, we get convergence to the reproducing kernel of a Paley-Wiener space, with the unit disk as Fourier spectrum. At boundary points, the situation is much more complicated, but also more interesting. The outward rim of the spectral droplet appears to be associated with a relative of the Airy kernel. This reports on joint work with A. Haimi

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LMIs vs Convexity and Free Convexity

J. William Helton

Convexity is the most desired property of an optimization problem, since local minima (often easy to find numerically) are global. Within the subject of convex optimization the biggest advance in the last 20 years is semidefinite programing, which is the pursuit of linear matrix inequalities (LMIs). Many problems in many branches of science convert to LMIs, where in computational complexity and in linear systems engineering it has had a major impact.

Studying properties of LMIs presents an opportunity for operator theorists, since analyzing LMIs and finding their range of applicability involves a variety of operator techniques. Thanks to these there is getting to be a substantial theory of LMIs. It presents a range of theorems in free analysis which pertain to many classes of LMIs arising in linear systems.

Free biholmorphic maps

J. William Helton

Strictly singular operators with a compact power

Francisco L. Hernández

Compactness of the iterates of strictly singular (or Kato) bounded operators defined on Banach lattices E is analyzed. Conditions on the behavior of the disjoint sequences in E are given in order that every strictly singular operator T have square T^2 compact. A Banach lattice is disjointly homogeneous if every pair of disjoint normalized sequence have equivalent subsequences. Using this we present extensions of a result of V. Milman for L^p-spaces to the setting of rearrangement invariant function spaces. In particular we get (non-Hilbert) Orlicz function spaces on which every strictly singular operator is compact. On the other side we provide examples of rearrangement invariant function spaces of indices equal to p (for $1 \leq p < \infty$) with strictly singular non power-compact operators .

The talk is based on a joint work with J.Flores, E.Semenov and P.Tradacete.

Some modes of convergence related to homogenization

Anderts Holmbom

We discuss some concepts related to homogenization and G-convergence and the processes connected to the disagreement between the G-limit and the corresponding weak $L^2(\Omega)^{N\times N}$-limit

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Toeplitz operators on Fock-Sobolev spaces

Hong Rae Cho and Kehe Zhu

We consider the Fock-Sobolev space $F^{p,m}$ consisting of entire functions f such that $f^{(m)}$, the m-th order derivative of f, is in the Fock space F^p. We show that an entire function f is in $F^{p,m}$ if and only if the function $z^m f(z)$ is in F^p. We also characterize the Carleson measures for the spaces $F^{p,m}$, establish the boundedness of the weighted Fock projection and the Toeplitz operator on appropriate L^p spaces, identify the Banach dual of $F^{p,m}$, and compute the complex interpolation space between two $F^{p,m}$ spaces.

On wavelets related Toeplitz operators

Ondrej Hutník

Toeplitz operators acting on spaces of holomorphic functions and their analogs (under the name localization operators) arising in time-frequency (or, time-scale) analysis are nowadays intensively studied by many authors. In this talk we survey our results on the topic of Toeplitz operators based on the Calderón reproducing formula and acting on the spaces of wavelet transforms. In our study the admissible wavelets on the affine group are considered. We mention the representation of the space of wavelet transforms and its usage in investigating the basic properties of Calderón-Toeplitz operators with various symbols. We also show how certain classes of pseudodifferential operators naturally appear in this context.

On the duality of nontangential maximal functions and Carleson measures

Tuomas Hytönen

These results arose while studying the nature of certain solution spaces for boundary value problems in a recent work of Auscher and Rosén. Besides answering the original question, we found a rather complete description of the duality between several related spaces. This is joint work with A. Rosén (Linköping).

Compactness characterization of operators in the Toeplitz algebra of the Fock space

Joshua Isralowitz

For $1 < p < \infty$, let $\mathcal{T}_p$ be the Toeplitz algebra of the Bergman space $L^p_a(\mathbb{B}_n, dv)$, which is the closure of the algebra generated by Toeplitz operators T_f on $L^p_a(\mathbb{B}_n, dv)$ with bounded symbols f. In 2004, D. Suarez showed that a bounded operator A on $L^p_a(\mathbb{B}_n, dv)$ with $1 < p < \infty$ is compact if and only if $A \in \mathcal{T}_p$ and the Berezin transform $B(A)$ vanishes at the boundary $\partial\mathbb{B}_n$. In this talk, we will discuss extending this result to the standard weighted Fock space $F^p_\alpha(\mathbb{C}^n)$. This is joint work with W. Bauer.

On Fredholm and Volterra integral equations with a logarithmic kernel

P. Junghanns

We study the solvability of the Fredholm integral equation

$$\varphi(x) = \lambda \int_0^1 \varphi(\xi)\,\frac{\ln \frac{x}{\xi}}{x-\xi}\,d\xi + f(x)\,, \quad 0 < x < 1\,, \quad \lambda \in \mathbb{R}\,,$$

and the Volterra integral equation

$$\varphi(x) = \lambda \int_0^x \varphi(\xi)\,\frac{\ln \frac{x}{\xi}}{x-\xi}\,d\xi + f(x)\,, \quad 0 < x < 1\,, \quad \lambda \in \mathbb{R}\,,$$

using Wiener-Hopf methods (cf. [1, 2]) and a method recently developed in [3].

The talk is based on joint work with L. v. Wolfersdorf.

References

[1] R. Duduchava, *Integral Equations with Fixed Singularities*, BSB B. G. Teubner, Leipzig, 1979.

[2] L. G. Mikhaĭlov, *Integral Equations with Homogeneous Kernel of Degree -1* (Russian), Duschanbe, 1966.

[3] G. Vainikko, Cordal Volterra integral equations, *Numer. Funct. Anal. Optim.*, 30 (2009), 1145--1172.

State space formulas for rational solutions of corona type Bezout equations and Toeplitz operators

M.A. Kaashoek

This talk deals with rational solutions to the corona type Bezout equation $G(z)X(z) = I$, where G is a ``fat" stable rational matrix function. Toeplitz operators and state space methods from systems theory are used to derive state space formulas for the optimal H^2 solution and for maximum entropy H^∞ solutions. The formulas involve the stabilizing solutions of associate discrete algebraic Riccati equations of stochastic realization type. They show that these special solutions are rational and all have McMillan degree less than or equal to the McMillan degree of G.
The talk reviews recent joint work with A.E. Frazho, S. ter Horst and A.C.M. Ran.

A state space view on the Gohberg-Heinig inversion theorem for block Toeplitz matrices

M.A. Kaashoek

An explicit inversion formula will be presented for certain structured linear transformations that are closely related to finite block Toeplitz matrices. State space techniques from mathematical system theory will play an important role. The conditions of invertibility are illustrated by an example. The talk is based on joint work with F. van Schagen, and is an addition to the paper:

I. Gohberg, M.A. Kaashoek, F. van Schagen, On inversion of finite Toeplitz matrices with elements in an algebraic ring, *Lin. Alg. Appl.* **385** (2004), 381--389.

Noncommutative power series and noncommutative functions

Dmitry Kaliuzhnyi-Verbovetskyi

In various applications of formal power series their evaluations on square matrices (of any size or of size large enough) play an important role and allow to develop a noncommutative analog of analytic function theory. On the other hand, functions defined on square matrices of any size which respect direct sums and similarities and satisfy a local boundedness condition behave in many ways as analytic functions and have power series expansions --- a noncommutative analogue of Taylor series. We will discuss convergence questions for noncommutative power series and domains of analyticity for noncommutative functions.
The talk is based on a joint work with Victor Vinnikov.

Matrices with normal defect one

D. Kaliuzhnyi-Verbovetskyi

A $n \times n$ matrix A has normal defect one if it is not normal, however can be embedded as a north-western block into a normal matrix of size $(n+1) \times (n+1)$. The latter is called a minimal normal completion of A. A simple procedure is presented which allows one to check whether a given matrix has normal defect one, and if this is the case --- to construct all its minimal normal completions. A characterization of the generic case for each n under the assumption rank$(A^*A - AA^*) = 2$ (which is necessary for A to have normal defect one) is obtained. Both the complex and the real cases are considered. It is pointed out how these results can be used to solve the minimal commuting completion problem in the classes of pairs of $n \times n$ Hermitian (resp., symmetric, or symmetric/antisymmetric) matrices when the completed matrices are sought of size $(n+1) \times (n+1)$. The talk is based on my recent joint work with I. Spitkovsky and H. Woerdeman.

A charged particle in a homogeneous magnetic field accelerated by a time period Aharonov-Bohm flux

Tomáš Kalvoda

We consider a nonrelativistic quantum charged particle moving on a plane under the influence of a uniform magnetic field and driven by a periodically time-dependent Aharonov-Bohm flux. We observe an acceleration effect in the case when the Aharonov-Bohm flux depends on time as a sinusoidal function whose frequency is in resonance with the cyclotron frequency. In particular, the energy of the particle increases linearly for large times. An explicit formula for the acceleration rate is derived with the aid of the quantum averaging method, and then it is checked against a numerical solution with a very good agreement.
The talk is based on a joint work with P. Šťovíček.

Subnormal block Toeplitz operators with rational symbols

Dong-O Kang

In this talk, subnormality of block Toeplitz operators is considered. It can be easily verified that every analytic block Toeplitz operator is a subnormal operator. We show that every non-analytic subnormal block Toeplitz operator with a rational symbol with a certain restiction is normal. As a special case, we consider the (trigonometric) polynomial symbol case and derive a procedure to verify whether a given (trigonometric) polynomial symbol satisfies the condition(restriction) or not. In this process, we also verify that the condition imposed on the symbol is a mild restriction to avoid the obvious non-analytic non-normal case. This talk is based on the work with the above named mathematicians.
This is a joint work with Sung Hwang, In Hyun Kim and Woo Young Lee.

On a structure of the kernel of singular integral operators with endpoint singularities

Aleksandr Karelin, Anna Tarasenko

In the article [1,2] we obtained a direct relation between singular integral operators A with a model involution and a matrix characteristic singular integral operators: for an orientation-reversing shift it is a transform by two invertible operators $\mathcal{H}A\mathcal{E}$. We will refer to the formula as operator equality. Different applications of operator equalities to singular integral operators and to boundary value problems are considered.

Now, we consider a structure of the kernel of singular integral operator $B_\jmath$ in the space $L_2(\jmath),\ \jmath = (x_1, x_2);\ x_1, x_2 \in \mathbb{R}, \mathbb{R} = (-\infty, \div\infty)$:

$$B_\jmath = \tilde{a}I_\jmath + \tilde{c}S_\jmath + \tilde{d}U_{1,\jmath} \quad ,$$

where $S_\jmath$ is the Cauchy singular integral operator and

$$\left(U_{1,\jmath}\varphi\right)(x) = \frac{1}{\pi i}\int\limits_{x_1}^{x_2} \frac{(x_2 - x_1)\varphi(\tau)}{-2x\tau + (x_1 + x_2)(\tau + x) - 2x_1x_2}\, d\tau,$$

the coefficients $\tilde{a}, \tilde{c}, \tilde{d}$ are bounded measurable functions.

Results of [1,2] are applied to studying $B_\jmath$.

[1] A. A. Karelin, On a relation between singular integral operators with a Carleman linear-fractional shift and matrix characteristic operators without s hift, Boletin Soc. Mat. Mexicana Vol. 7 No. 12 (2001), pp. 235--246.

[2] A. Karelin, Aplications of operator equalities to singular integral operators and to Riemann boundary value problems, Math. Nachr. Vol. 280 No. 9-10 (2007), pp. 1108--1117.

On singular integral operators with semi-almost periodic coefficients on variable Lebesgue spaces

A. Karlovich

Let a be a semi-almost periodic matrix function with the almost periodic representatives a_l and a_r at $-\infty$ and $+\infty$, respectively. Suppose $p : \mathbb{R} \to (1, \infty)$ is a slowly oscillating exponent such that the Cauchy singular integral operator S is bounded on the variable Lebesgue space $L^{p(\cdot)}(\mathbb{R})$. We prove that if the operator $aP + Q$ with $P = (I + S)/2$ and $Q = (I - S)/2$ is Fredholm on the variable Lebesgue space $L^{p(\cdot)}_N(\mathbb{R})$, then the operators $a_lP + Q$ and $a_rP + Q$ are invertible on standard Lebesgue spaces $L^{q_l}_N(\mathbb{R})$ and $L^{q_r}_N(\mathbb{R})$ for some exponents q_l and q_r between the lower and the upper limits of p at $-\infty$ and $+\infty$, respectively. This is a joint work with Ilya Spitkovsky.

Convolution type operators with oscillating symbols on a union of intervals

Yu. Karlovich

We establish Fredholm criteria for convolution type operators with oscillating matrix symbols on weighted Lebesgue spaces on a union of intervals with weights in a subclass of Muckenhoupt weights. The operator symbols are continuous on the real line and admitting mixed (slowly oscillating and semi-almost periodic) discontinuities at $\pm\infty$. The study makes use of

the Allan-Douglas local principle, limit operators techniques, boundedness and compactness results for pseudodifferential operators with non-regular symbols and the Fredholm theory for Wiener-Hopf operators with semi-almost periodic matrix symbols.

The talk is based on a joint work with J. Loreto-Hernández.

The Riemann boundary value problem for analytic matrix on non-rectifiable arc

B. Kats

Let Γ be a Jordan arc on the complex plane with endpoints a_1 and a_2, $\Gamma^\circ := \Gamma \setminus \{a_1, a_2\}$. We assume that the arc is non-rectifiable.

We consider the matrix Riemann--Hilbert boundary value problem on the arc Γ, i.e., the problem on determination of holomorphic in $\mathbb{C} \setminus \Gamma$ matrix

$$Y(z) = \begin{pmatrix} Y_{11}(z) & Y_{12}(z) \\ Y_{21}(z) & Y_{22}(z) \end{pmatrix}$$

satisfying equality

$$Y^+(t) = Y^-(t)G(t), t \in \Gamma^\circ, \tag{12}$$

and certain restrictions on its behavior near the points a_1, a_2 and ∞. Here $G(t)$ stands for a matrix defined on Γ. In particular, we study the Focas--Its--Kitaev version of the problem (12), i.e. the case

$$M(t) = \begin{pmatrix} 1 & w(t) \\ 0 & 1 \end{pmatrix}, Y(z) \sim \begin{pmatrix} z^n & 0 \\ 0 & z^{-n} \end{pmatrix}, z \to \infty.$$

We prove solvability of the problem for sufficiently large n. The first entry of Y in this case is a generalized complex orthogonal polynomial of degree n on non-rectifiable arc Γ.

The talk is based on a joint work with D. Katz.

Contributions to the theory of C*-correspondences with applications to multivariable dynamics

E. Katsoulis

Motivated by the theory of tensor algebras and multivariable C^*-dynamics, we revisit two fundamental techniques in the theory of C^*-correspondences, the ``addition of a tail" to a non-injective $C^*$-correspondence m and the dilation of an injective $C^*$-correspondence to an essential Hilbert bimodule. We provide a very broad scheme for ``adding a tail" to a non-injective C^*-correspondence; our scheme includes the ``tail" of Muhly and Tomforde as a special case. We illustrate the diversity and necessity of our tails with several examples from the theory of multivariable C^*-dynamics. We also exhibit a transparent picture for the dilation of an injective C^*-correspondence to an essential Hilbert bimodule. As an application of our constructs, we prove two results in the theory of multivariable dynamics that extend results of Davidson and Roydor and others. We also discuss the impact of our results on the description of the C^*-envelope of a tensor algebra as the Cuntz-Pimsner algebra of the associated C^*-correspondence. The talk is based on a joint work with E. Kakariadis.

Factorization of operators in Krein spaces and linear-fractional relations of operator balls

V. Khatskevich

For plus-operators in a Krein space, we consider a linear fractional relation defined on a subset of the closed unit operator ball. The classes of operators with the empty domain of definition for such a relation are described. The sufficient (and necessary, in some meaning) conditions for the chain rule to be valid are given. In particular, we consider the special case of linear-fractional transformations. In the case of Hilbert spaces H_1 and H_2, each linear-fractional transformation of the closed unit ball K of the space $L(H_1, H_2)$ is of the form

$$F_T(K) = (T_{21} + T_{22}K)(T_11 + T_{12}K)^{-1}$$

and is generated by the plus-operator T. We consider applications of our results to the well-known Krein-Phillips problem of invariant subspaces of special type for sets of plus-operators acting in Krein spaces.
This is a joint work with V. Senderov.

Tensor-structured computation of the Fock operator in quantum chemistry

Venera Khoromskaia

The classical Hartree-Fock equation is one of the basic *ab initio* models is electronic structure calculations. The numerical solution of the Hartree-Fock equation, a nonlinear eigenvalue problem with the Fock operator containing 3D and 6D convolution integrals with the Newton potential, is a challenging task. Due to the nonlinear dependence of the Fock operator on the solution of the equation, it can be solved only iteratively, by the self-consistent field iterations. Traditionally, its solution is based on a rigorous analytical precomputation of the arising convolution type integrals in $\mathbb{R}^3$ which requires the naturally separable basis.
We discuss the novel tensor-structured methods for the numerical solution of the Hartree-Fock equation, which can be used as well in other models in quantum chemistry. These methods include efficient algorithms for the separable representation of the discretized functions and integral operators in $\mathbb{R}^3$ using the canonical, Tucker and mixed tensor formats and the corresponding fast tensor operations. The core of our ``grey-box" solver is the rank-structured computation of the nonlinear Hartree and the (nonlocal) exchange parts of the Fock operator in $\mathbb{R}^3$, discretized on a sequence of $n \times n \times n$ Cartesian grids. The arising 3D and 6D convolutions are replaced by 1D algebraic operations implemented with $O(n \log n)$ complexity. Note that in terms of usual estimation by volume size $N_{vol} = n^3$, the tensor-structured operations are of sublinear complexity, $O(N_{vol}^{1/3})$.
The robust multigrid canonical-to-Tucker rank reduction algorithm with the controllable accuracy enables usage of fine Cartesian grids up to $n^3 \approx 10^{12}$. This yields high resolution of the involved computational quantities and allows arbitrary location of atoms in a molecule as in the conventional mesh-free analytical-based solution of the Hartree-Fock equation. Numerical results for several moderate size molecules demonstrate efficiency of the tensor-structured methods in electronic structure calculations.

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Spectral formulae for elementary operators

D. Kitson

A mapping $\mathcal{E} : \mathcal{A} \to \mathcal{A}$ on an algebra $\mathcal{A}$ is called an elementary operator if it is expressible in the form $x \mapsto \sum_{j=1}^{n} a_j x b_j$ for some elements a_j and b_j contained in $\mathcal{A}$ (or more generally in the multiplier algebra of $\mathcal{A}$). We will present formulae for computing the Browder essential spectrum of an elementary operator acting on $\mathcal{B}(H)$ (the bounded operators on a Hilbert space) or on any norm ideal of $\mathcal{B}(H)$.

The Convex Positivstellensatz for noncommutative polynomials

I. Klep

Given polynomials p and q, it is natural to ask: does one dominate the other? That is,

$$\text{does} \quad q(x) \geq 0 \quad \text{imply} \quad p(x) \geq 0\,? \tag{Q}$$

In this talk we focus on *noncommutative* polynomials p, q and substitute matrices for the variables x_j. In case the positivity domain $\mathcal{D} = \{X \mid q(X) \succeq 0\}$ is *convex*, the domination question (Q) has an elegant answer. First of all, $\mathcal{D}$ then has an LMI representation, i.e., $\mathcal{D} = \{X \mid L(X) \succeq 0\}$ for a linear pencil L. Then we obtain the following ``perfect" *Positivstellensatz*: p is *positive semidefinite* on the noncommutative LMI domain $\mathcal{D}$ if and only if it has a weighted sum of squares representation with optimal degree bounds:

$$p(x) = s(x)^T s(x) + \sum_j f_j(x)^T\, L(x)\, f_j(x), \tag{B}$$

where $s(x), f_j(x)$ are vectors of polynomials of degree no greater than deg$(p)/2$.

A main ingredient of the proof is an analysis of extensions of *Hankel matrices*.

(The talk is based on joint work with J.W. Helton and S. McCullough.)

Rational inner functions in the Schur-Agler class

G. Knese

The Schur-Agler class is a subclass of the bounded analytic functions on the polydisk with close ties to operator theory. We shall describe our recent investigations into the properties of rational inner functions in this class. Non-minimality of transfer function realization, necessary and sufficient conditions for membership (in special cases), and low degree examples are among the topics we will discuss.

Boundedness criteria of Fourier operators in weighted grand Lebesgue spaces

Vakhtang Kokilashvili

The goal of our talk is to present the results dealing with mapping properties of linear and nonlinear harmonic analysis operators in new function spaces. In particular, we consider the weighted grand Lebesgue space $L^{p),\theta}$ $(1 < p < \infty, \ \ \theta > 0)$ defined by the norm

$$\|f\|_{L_w^{p),\theta}} := \sup_{0<\varepsilon\leq p-1} \left(\varepsilon^\theta \int_T |f(t)|^{p-\varepsilon} w(t) dt \right)^{\frac{1}{p-\varepsilon}} < \infty.$$

These spaces were introduced by T. Iwaniec and C. Sbordone in unweighted case. The weighted versions of these spaces for the first time were studied by A. Fiorenza, B. Gupta and P. Jain to establish the boundedness of the Hardy--Littlewood maximal operator. It is known that weighted grand Lebesgue spaces are nonreflexive and nonrearrangement invariant spaces.

We intend to discuss the following topics:

1) boundedness criteria for singular integrals of various type: conjugate functions of several variables, Cauchy singular integrals, commutators;

2) necessary and sufficient conditions governing the boundedness of the Fourier operators $\sup\limits_n |\sigma_n^\alpha(f,x)|$, $\alpha \geq 0$, $\sup\limits_{0<r<1} |u_r(f,x)|$, Fourier multipliers and Littlewood--Paley functions;

3) two--weight $L_w^p \mapsto L_v^{q)}$ boundedness for fractional maximal and potential operators;

Further, by the symbol $\mathcal{L}_w^{p),\theta}$ is denoted the subset of those functions f from $L^{p),\theta}$ for which

$$\lim_{\varepsilon\to 0} \left(\varepsilon^\theta \int_T |f(t)|^{p-\varepsilon} w(t) dt \right)^{\frac{1}{p-\varepsilon}} = 0.$$

The norm convergence and summability criteria for single and multiple Fourier trigonometric series of functions $f \in \mathcal{L}_w^{p),\theta}$ will be presented.

Toeplitz products with Pluriharmonic symbols on the Hardy space over the ball

Hyungwoon Koo

On the Hardy space over the unit ball in $\mathbf{C}^n$, we consider operators which have the form of a finite sum of products of several Toeplitz operators. We study characterizing problems of when such an operator is compact or of finite rank. Some of our results show higher dimensional phenomena.

The Inverse of a Two-level Positive Definite Toeplitz Operator Matrix

Selcuk Koyuncu

Joint work with Hugo J.Woerdeman.

The Gohberg-Semencul formula allows one to express the entries of the inverse of a Toeplitz matrix using only a few entries (the first row and the first column) of the matrix, under some nonsingularity condition. In this talk we will provide a two variable generalization of the Gohberg-Semencul formula in the case of a positive definite two-level Toeplitz matrix with a symbol of the form $\frac{1}{|p|^2}$ where p is a stable polynomial of two variables. We also consider the case of operator valued two-level Toeplitz matrices. In addition, we propose an approximation of the inverse of a multilevel Toeplitz matrix with a positive symbol, and use it as the initial value for a Hotelling iteration to compute the inverse. Numerical results are included.

Fixed Point Theorems for Block Operator Matrix and an Application to Problem Arising in Growing Cell Populations

B. Krichen

In this paper we study fixed point results for operator matrices with nonlinear entries. These results are applied to prove the existence of solutions for mixed problems arising in growing cell populations.
The talk is based on a joint work with A. Jeribi and A. Ben Amar.

An analogue of the spectral mapping theorem for condition spectrum

G. Krishna Kumar

For $0 < \epsilon < 1$, the ϵ-condition spectrum of an element a in a complex unital Banach algebra A is defined as,

$$\sigma_\epsilon(a) = \left\{ \lambda \in \mathbb{C} : \lambda - a \text{ is not invertible or } \|\lambda - a\| \|(\lambda - a)^{-1}\| \geq \frac{1}{\epsilon} \right\}.$$

This is a generalization of the idea of spectrum introduced in [1]. In this paper we prove a mapping theorem for condition spectrum, extending an earlier result in [1]. Let f be an analytic function in an open set D containing $\sigma_\epsilon(a)$. We study the relationship between the sets $\sigma_\epsilon(f(a))$ and $f(\sigma_\epsilon(a))$. In general these two sets are different. We define functions $\phi(\epsilon), \psi(\epsilon)$(that take small values for small values of ϵ) and prove that $f(\sigma_\epsilon(a)) \subseteq \sigma_{\phi(\epsilon)}(f(a))$ and $\sigma_\epsilon(f(a)) \subseteq f(\sigma_{\psi(\epsilon)}(a))$. The classical Spectral Mapping Theorem is shown as a special case of this result. We give estimate for these functions in some special cases and finally illustrate the results by numerical computations.

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The talk is based on a joint work with S. H. Kulkarni.

Total positivity: a cone-theoretic approach

O. Kushel

This talk is devoted to the generalization of the theory of total positivity. We say that a linear operator $A : \mathbb{R}^n \to \mathbb{R}^n$ is generalized totally positive (GTP), if its jth exterior power $\wedge^j A$ preserves a proper cone $K_j \subset \wedge^j \mathbb{R}^n$ for every $j = 1, \ldots, n$. We also define generalized strictly totally positive (GSTP) operators. We prove that the spectrum of a GSTP operator is positive and simple, moreover, its eigenvectors are localized in special sets. The existence of invariant cones of finite ranks is shown under some additional conditions. Basic properties of GTP and GSTP operators are analyzed. Some new insights and alternative proofs of the well-known results of Gantmacher and Krein describing the properties of TP and STP matrices are presented.

Similarity of Operators in the Bergman Space Setting

Hyun Kwon

We describe the contractions that are similar to M_z^*, the adjoint of the operator of multiplication by z, on the Bergman Space A^2. This talk is based on the work being carried out with R. G. Douglas.

On Toeplitz operators on the harmonic Bergman space

M. Loaiza

Toeplitz operators with continuous symbols acting on the harmonic Bergman space of the unit disk have a nice behavior. For example the commutator and semicommutator of two Toeplitz operators with continuous symbols is compact. In this talk we analyze the C*-algebra generated by Toeplitz operators with piecewise continuous symbols on the harmonic Bergman space.

The talk is based on a joint work with M. Lozano.

Regularity and topology of minimizers of average distance functional in dynamic evolutional problem.

X.Y. Lu

Minimizing movement theory was introduced in 1993 by De Giorgi to study evolutional processes. We will focus on the geometrical and analytical properties of the sets during an evolution, and our main goal is to estimate whether and when they will change topology.

Given a locally convex domain Ω in R^2 with finite Hausdorff measure boundary, a Hausdorff one-dimensional subset $\Sigma \subset \Omega$ (the ``fracture"), and 2 functionals: an ``energy" F, which stands for the functional that is minimized during the process (energy, entropy, etc); and a ``dissipation" $D$, i.e. the ``cost" to pass from one configuration to another, we consider the following minimizing movement problem:

$$\begin{cases} \Sigma_0 & \text{(initial datum)} \\ \Sigma_n \in \text{argmin}_{\Sigma_{n-1} \subset \Sigma'} F(\Sigma') + D(\Sigma_{n-1}, \Sigma') \end{cases} .$$

Letting the incremental length go to 0 we have the continouos (rate-independent) case, in which the evolution is continouos.

Then, with direct estimates on the "branching time" (time t in the evolution at which the minimizing set Σ_t changes topology) upperbounds, we will show that even very similar initial conditions can lead to totally different behaviors, and we will show that significant difference exists even as the dissipation exponent is above or below 3/2, as for the first case the evolution countinues for any time, but for the second case the configuration may reach a stable equilibrium from which every further evolution is not permitted.

Moreover, we show that under some regularity assumptions on the initial data, the rate-independent evolution keeps regular, i.e. the tangent vector varies with regularity.

Pseudo-Taylor expansions and the Carathéodory-Fejér problem

Z. Lykova

We give a new solvability criterion for the *boundary Carathéodory-Fejér problem*: given a point $x \in \mathbb{R}$ and, a finite set of target values $a^0, a^1, \ldots, a^n \in \mathbb{R}$, to construct a function f in the Pick class such that the limit of $f^{(k)}(z)/k!$ as $z \to x$ nontangentially in the upper half plane is a^k for $k = 0, 1, \ldots, n$. The criterion is in terms of positivity of an associated Hankel matrix. The proof is based on a reduction method due to Julia and Nevanlinna. The talk is based on a joint work with J. Agler and N. J. Young.

Products of orthogonal projections and polar decompositions

A. Maestripieri

We characterize the sets $\mathfrak{X}$ of all products PQ, and $\mathfrak{Y}$ of all products PQP, where P, Q run over all orthogonal projections and we solve the problems $\arg\min\{\|P - Q\| : (P, Q) \in \mathcal{Z}\}$, for $\mathcal{Z} = \mathfrak{X}$ or $\mathfrak{Y}$. We also determine the polar decompositions and Moore-Penrose pseudoinverses of elements of $\mathfrak{X}$.

The talk is based on a joint work with G. Corach.

α- Left Derivable mapping at Zero on CDSL algebras

Asia Majeed

We show that a linear mapping $\delta : \mathcal{A} \to M$ is an α -left derivable mapping at zero if $\delta(ST) = \alpha(T)\delta(S) + \alpha(S)\delta(T)$ for any S, T $\in \mathcal{A}$ with $ST = 0$.Where $\mathcal{A}$ is a unital algebra. As a corollary, we prove that α -left derivable mapping at zero on CDCSL algebras is zero under certain conditions. Where α is a surjective homomorphism.

Scalar Riemann-Hilbert problems on a Riemann surface and applications

M. T. Malheiro

We study the solvability of scalar Riemann-Hilbert problems relative to a contour on a torus Σ which are equivalent to some matrix Riemann-Hilbert problems in the complex plane. With this purpose a new concept of meromorphic Σ-factorization is introduced and discussed. The results are applied to solve matrix Riemann-Hilbert problems and study some properties of a class of Toeplitz operators with 2×2 matrix symbols.

The talk is based on a joint work with M. C. Câmara.

Invertibility and Fredholm property of a linear operator in a sum of its invariant subspaces

A. Markus

Let A be a linear bounded operator in a Banach space X which is a sum of two its invariant subspaces X_1 and X_2. We consider several questions on connections between properties of operator A and of its restrictions to X_1 and X_2. If the sum of X_1 and X_2 is direct, then the answers are always simple and well-known. Therefore we consider the case when this sum is not direct (i. e. $X_1 \cap X_2 \neq \{0\}$). At first we discuss the connection between the invertibility of A and of operators $A|X_1$, $A|X_2$, $A|X_1 \cap X_2$. As a corrolary we obtain some relations between the spectra of these operators.
In the second part of the talk we consider relations between the Fredholm property and the indices of all mentioned operators. This part is based on a joint work with H. Schulze.

On the dimension of the kernel of singular integral operators with non-Carleman shift

R. Marreiros

We consider the operator $T = I - cUP_+$: $L_2^n(\mathbb{R}) \to L_2^n(\mathbb{R})$, on the real line, where I is the identity operator, $c \in C^{n\times n}(\mathring{\mathbb{R}})$ is a continuous matrix function on $\mathring{\mathbb{R}} = \mathbb{R} \cup \{\infty\}$, the one point compactification of $\mathbb{R}$, $(U\varphi)(t) = \varphi(t+\mu)$, $\mu \in \mathbb{R}$, is the shift operator, and $P_\pm = \frac{1}{2}(I \pm S)$ are the complementary projection operators, with $(S\varphi)(t) = (\pi i)^{-1} \int_{\mathbb{R}} \varphi(\tau)(\tau - t)^{-1} d\tau$ the operator of singular integration with Cauchy kernel. It is supposed that all the eigenvalues of the matrix $c(t)$ at ∞, simultaneously belong either to the interior of the unit circle $\mathbb{T}$ or to its exterior. Under these conditions, estimates for the dimension of the kernel of the operator T are obtained. We obtain analogous estimates, under similar conditions, for an operator with polynomial coefficient relative to the shift operator.

This is a joint work with V. Kravchenko.

Unitary perturbations of compressed shifts and sampling theory

R.T.W Martin

Sampling theory can be described as the study of reproducing kernel Hilbert spaces $\mathcal{H}$ which have a special sampling property: functions belonging to these spaces are perfectly reconstructible from their values taken on certain discrete sets of points. This happens, for example, if $\mathcal{H}$ has a total orthogonal set of point evaluation vectors. Here, if $\mathcal{H}$ consists of functions on a set $X \subset \mathbb{C}$, a vector $\delta_x \in \mathcal{H}$ is called the point evaluation vector at $x \in X$ if $\langle f, \delta_x \rangle = f(x)$ for all $f \in \mathcal{H}$. Sampling theory is an active area of research in both pure mathematics and communication engineering, and has applications to signal processing. In this talk we will discuss a connection between vector-valued sampling theory, and model subspaces K^2_Θ of the Hardy space $H^2_n(\mathbb{U}) \subset \oplus_{i=1}^n L^2(\mathbb{R})$ of n-component vector functions on the upper half-plane $\mathbb{U}$.

In particular, given a purely contractive matrix-valued analytic function Θ on $\mathbb{U}$, a $\mathcal{U}(n)$-parameter family of unitary perturbations of the restriction of the backwards shift to the de Branges-Rovnyak space K^2_Θ will be presented. Here $\mathcal{U}(n)$ denotes the group of unitary $n \times n$ matrices, and the shift on $H^2_n(\mathbb{U})$ is the operator of multiplication by $\mu(z) := \frac{z-i}{z+i}$. These perturbations are higher dimensional analogues of the unitary perturbations introduced by D.N. Clark in the case where Θ is a scalar-valued ($n = 1$) inner function, and by E. Fricain in the case where Θ is scalar-valued and an extreme point of the closed unit ball of H^∞. By following results of Clark and Fricain in the scalar case, a necessary and sufficient condition on Θ for K^2_Θ to contain a total orthogonal set of point evaluation vectors (and hence have the sampling property) is provided. These investigations further lead to a matrix-valued disintegration theorem for the Aleksandrov-Clark measures associated with the matrix-valued contractive analytic function Θ.

Numerical Solutions for Conformal Maps and Explicit Riemann-Hilbert Problems

F. Martin

Calculating conformal maps on uniform meshes suffers from the *crowding* phenomenon which occurs if we map the unit disc onto an elongated region. To overcome this problem non-uniform meshes are considered and a method for evaluating the Hilbert transform on these meshes is indicated. The Hilbert transform is not only important for computing conformal maps but also in signal theory and computational analysis. Furthermore we introduce a modification of Wegmann's method and prove its convergence. The conformal map of the unit disc onto an ellipse serves as an illustration. At the end explicit Riemann-Hilbert problems are considered. We present an iteration for solving these problems using the Hilbert transform described above. As a special case linear Riemann-Hilbert problems are studied.

The talk is based on a joint work with Prof. E. Wegert and is supported by the DFG grant We 1704/8-2.

Norm-attaining composition operators on the Bloch spaces

M. J. Martín

Let ϕ be a non-constant analytic function in the unit disk $\mathbb{D}$ with $\phi(\mathbb{D}) \subset \mathbb{D}$. For every holomorphic function f on $\mathbb{D}$, the *composition operator* C_ϕ yields the function $C_\phi(f) = f\circ\phi$. The map ϕ is called the symbol of the linear operator C_ϕ.

In this talk we are interested in the characterization of the *norm-attaining* composition operators on the Bloch space $\mathcal{B}$ and on the little Bloch space $\mathcal{B}_0$ modulo constant functions. Recall that a bounded linear operator T on a Banach space X attains its norm on X if there exists a function $f \in X$ with norm 1 such that $\|T\| = \|\, T\, f\|$. Such a function f is called an *extremal function for the norm of* T.

We prove that every composition operator C_ϕ on the Bloch space (modulo constant functions) attains its norm and characterize the norm-attaining composition operators on $\mathcal{B}_0$ (modulo constant functions) in terms of a condition related to the hyperbolic derivative of the symbol ϕ. We also identify all the extremal functions for $\|C_\phi\|$ in both spaces.

Frames for Krein spaces

F. Martínez Pería

Given a Hilbert space $\mathcal{H}$, a family of vectors $\{f_i\}_{i\in I}$ in $\mathcal{H}$ is a frame for $\mathcal{H}$ if there exist constants $0 < A \leq B$ such that

$$A\,\|f\|^2 \leq \sum_{i\in I} |\langle f, f_i\rangle|^2 \leq B\,\|f\|^2.$$

An obvious example of a frame for a Hilbert space is a Riesz basis. Notice that every frame for $\mathcal{H}$ is a generating system for $\mathcal{H}$, but frames become interesting when they are linearly dependent sets. Indeed, if $\{f_i\}_{i\in I}$ is a (linearly dependent) frame for $\mathcal{H}$ and $\{\langle f, f_i\rangle\}_{i\in I}$ are the frame coefficients of a vector $f \in \mathcal{H}$, it is possible to reconstruct f faithfully, even if some of the coefficients are missing. In fact, the redundancy of data obtained after analyzing a vector with a frame, turns frames into a useful tool in engineering applications such as signal processing.

On the other hand, given a Krein space $\mathcal{K}$ with fundamental symmetry J, the notion of J-orthonormalized system (and basis) is linked with the existence of a (maximal) dual pair. Recall that a pair of subspaces (L_+, L_-) in $\mathcal{K}$ is a dual pair if L_+ is J-nonnegative, L_- is J-nonpositive and they are J-orthogonal.

Fixed a Krein space $\mathcal{K}$ with fundamental symmetry J, the purpose of this talk is to study a particular class of frames, hereafter mentioned as J-frames. To each J-frame for $\mathcal{K}$ there is associated a pair of uniformly J-definite maximal subspaces, but they are not necessarily J-orthogonal. Some characterizations are discussed and some basic frame theory problems are extrapolated to J-frames.

The talk is based on a joint work with J. Giribet, A. Maestripieri and P. Massey.

Products of m-isometries

Antonio Martinón

Joint work with Teresa Bermúdez and Juan Agustín.

A operator T on a Banach space X is called an (m,p)-isometry if it satisfies the equality $\sum_{k=0}^{m}\binom{m}{k}(-1)^{m-k}\|T^k x\|^p = 0$, for all $x \in X$. In this paper we prove that if T is an (n,p)-isometry, S is an (m,p)-isometry and they commute, then TS is an $(m+n-1,p)$-isometry. This result is applied to the elementary operators of length 1 acting on the Hilbert-Schmidt operators and we prove a conjecture enunciated by Botelho and Jamison.

On C*-algebras from interval maps

N. Martins

Given a unimodal interval map, we construct partial isometries acting on Hilbert spaces associated to the orbit of each point. Then we prove that such partial isometries give rise to representations of a C*-algebra associated to the subshift encoding the kneading sequence of the critical point.

The talk is based on a joint work with C. Correia Ramos and P. Pinto.

Singular and fractional integral operators on new function spaces

K. Matsuoka

We show boundedness of singular integral operators and fractional integral operators on function spaces B^σ, which have been introduced recently to unify λ-central Morrey spaces, λ-central mean oscillation spaces and the usual Morrey-Campanato spaces. Using our function spaces, we can unify the boundedness of operators on several classical function spaces.

The talk is based on a joint work with Y. Komori-Furuya, E. Nakai and Y. Sawano.

A new family of Hardy spaces on manifolds of negative curvature

G. Mauceri

Let M be a complete connected noncompact Riemannian manifold with Ricci curvature bounded from below, positive injectivity radius and spectral gap. We introduce a sequence $X_0(M), X_1(M), \ldots$ of new Hardy spaces on M and show that these spaces may be used to obtain endpoint estimates for various singular integral operators associated to the Laplace--Beltrami operator: spectral multipliers, Riesz transforms. The talk is based on joint work with S. Meda and M. Vallarino.

Avram-Parter and Szegő theorems: convex test functions and counterexamples

E. Maximenko

The asymptotic distribution of singular and proper values of Toeplitz matrices is studied as the orden of the matrix tends to infinity. Developing ideas of S. Serra and P. Tilli we generalize Avram-Parter and Szegő limit formulas to the case of convex test functions. Moreover we construct counterexamples with real integrable symbols and increasing but non-convex test functions when these formulas fail. The talk is based on a joint work with A. Böttcher and S. Grudsky.

Elliptic boundary problems with rough coefficients

Svitlana Mayboroda

We are going to discuss some recent progress and surprising counterexamples on solvability of boundary problems for second order elliptic equations with complex bounded measurable coefficients.

A note on the reduced Iwahori-Hecke C^*-algebra of $GL(n)$

Sergio Mendes

According to the Baum-Connes conjecture for $G = GL(n)$, there is a canonical isomorphism

$$\mu : K_*^G(\underline{E}G) \to K_* C_r^* G$$

where $\underline{E}G$ is a universal example for the action of G. Quite specific, $\underline{E}G$ is the affine Bruhat-Tits building for G, also denoted βG. Let $C_r^*(G//I)$ denote the reduced Iwahori-Hecke C^*-algebra of $GL(n)$ and let Σ be a single apartment in βG. Then, $\Sigma = \underline{E}W$ is a model for the universal example $\underline{E}W$ of the the affine Weyl group W of G. Localized to the apartment Σ, the Baum-Connes conjecture modulo torsion is an isomorphism

$$K_j(C_r^*(G//I) \otimes_{\mathbb{Z}} \mathbb{C} \cong K_j^W(\Sigma) \otimes_{\mathbb{Z}} \mathbb{C}.$$

Let E/F be a finite Galois extension of local nonarchimedean fields. Base change lifts a representation of $GL(n, F)$ to a representation of $GL(n, E)$. This creates a map from the tempered dual of $GL(n, F)$ to the tempered dual of $GL(n, E)$. We use the Baum-Connes conjecture to relate functoriality of buildings with base change. We illustrate this relation with the reduced Iwahori-Hecke C^*-algebra of $GL(n, E)$ and $GL(n, F)$.

Weak compactness of Volterra type integral operators

S. Miihkinen

Let $\mathbb{D}$ be the open unit disk in the complex plane $\mathbb{C}$ and $g\colon \mathbb{D} \to \mathbb{C}$ an analytic function. We consider an integration operator T_g defined by

$$T_g f(z) = \int_0^z f(\zeta) g'(\zeta) d\zeta, \ z \in \mathbb{D},$$

for functions f analytic in $\mathbb{D}$. This generalized Volterra operator was introduced by Ch. Pommerenke to study exponentials of BMOA functions.

We present a result stating that T_g is weakly compact on Hardy space $H^1(\mathbb{D})$ precisely when it is compact. This can be shown by deriving estimates for the essential and weak essential norms of T_g.

The talk is based on a joint work with J. Laitila and P. Nieminen.

Criteria for Existence of Riesz Bases consisting of Root Functions of Hill and 1D periodic Dirac Operators

Boris Mityagin

The series of necessary and sufficient conditions (both geometric and analytic ones) for the systems of root functions of Hill and Dirac operators with periodic and antiperiodic boundary conditions to contain Riesz bases in a Hilbert space L^2 or bases in Banach rearrangement invariant function spaces is given. We prove equiconvergence theorems for Hill operators with singular H^{-1}-potentials as well. This is a joint work with Professor Plamen Djakov, Sabanci University.

Transformations on self-adjoint operators preserving commutativity or a measure of commutativity

Lajos Molnár

Let H be a complex Hilbert space with dim $H \geq 3$ and denote by $B_s(H)$ the space of self-adjoint bounded linear operators on H. We describe the structure of all (non-linear) bijective transformations $\phi : B_s(H) \to B_s(H)$ which satisfy

$$\phi(A)\phi(B) = \phi(B)\phi(A) \Longleftrightarrow AB = BA, \quad A, B \in B_s(H)$$

or satisfy

$$\|\phi(A)\phi(B) - \phi(B)\phi(A)\| = \|AB - BA\|, \quad A, B \in B_s(H).$$

The talk is based on joint works with P. Šemrl and W. Timmermann.

Hamiltonian perturbation of imaginary eigenvalues and the passivation problem

J. Moro

Given a Hamiltonian matrix with purely imaginary eigenvalues, we consider the problem of finding Hamiltonian perturbation matrices of minimal size which expel all eigenvalues from the imaginary axis. Such a perturbation analysis is motivated by the so-called *passivation problem* in Control Theory: given a non-passive control system (i.e., a system which generates energy by itself), the problem consists in finding a reasonably small perturbation which restores passivity to the system. The key to rephrase this problem in the Hamiltonian context is to connect the transfer function, evaluated on the imaginary axis, with the imaginary eigenvalues of a certain auxiliary Hamiltonian matrix.

Hamiltonian matrix structure imposes strong restrictions on the behavior of eigenvalues, e.g., a simple imaginary eigenvalue cannot leave the imaginary axis unless it coalesces first with another eigenvalue. One of the key ingredients to understand such constraints is the *sign characteristic* of purely imaginary eigenvalues, a crucial invariant in the structured perturbation analysis. Taking this into account, a structured spectral perturbation theory is presented which, when appropriately combined with structure-preserving eigenvalue algorithms, leads to numerical algorithms for the passivation problem

The material in this talk corresponds to joint work with Rafik Alam, Shreemayee Bora, Michael Karow and Volker Mehrmann

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An Embedding for Hardy Spaces on Riemannian Manifolds

A. J. Morris

A considerable amount of recent work has been devoted to constructing Hardy spaces that are suitably adapted to certain differential operators. The intuition for this approach is given by the classical Hardy space $H^1(\mathbb{R}^n)$ and its relationship with the Laplacian $\Delta = \sum_{j=1}^n \partial^2/\partial x_j^2$. The new Hardy spaces are often defined as an abstract completion, and in some cases it is unknown whether the abstract space can be realised as a function space. We will consider the Hardy spaces H_D^p defined by Auscher, McIntosh and Russ for the Hodge--Dirac operator D on a Riemannian manifold, and show how they can be realised as a subspace of L^p for all $p \in [1, 2]$. The proof relies on the fact that D is an elliptic, self-adjoint, differential operator with finite propagation speed. This is joint work with P. Auscher and A. McIntosh.

Ensemble Kalman filter in reservoir engineering

Geir Nævdal

During the last decade the ensemble Kalman filter (EnKF) has been considered as one of the most promising methods to update the models for fluid flow in oil reservoirs to take into account available production data, and it is currently an active research area. The ensemble Kalman filter is a Monte Carlo approach to the Kalman filter that is suitable for large scale non-linear problems. In this talk we will describe shortly the problem under consideration and point to some mathematical challenges for this methodology.

Relative entropy preserving maps and isometries on density operators

G. Nagy

In this talk, different kinds of preservers on sets of density operators on a finite-dimensional complex Hilbert space, H are discussed. The first part of the presentation is devoted to maps on sets of density operators which leave certain measures of relative entropy invariant. These entropic quantities are (1) the Belavkin-Staszewski relative entropy, (2) the Tsallis relative entropy, (3) the quadratic relative entropy and (4) the Jensen-Shannon divergence. We characterize those transformations on the set of all density operators on H which preserve one of these relative entropies. Moreover, the structure is determined of the surjective transformations on the set of the invertible density operators on H which leave one of the quantities (2),(3) invariant. In the second part, we consider those metrics on the space of density operators on H which come from the von Neumann-Schatten p-norms. We describe the form of the corresponding isometries. The talk is based on a joint work with Lajos Molnár.

Essential norms of weighted composition operators and Aleksandrov measures

Pekka Nieminen

We derive exact formulas for the essential and weak essential norms of some weighted analytic composition operators acting on H^2 and other function spaces in the unit disc, extending and improving earlier results due to Sarason and to Kriete and Moorhouse. These formulas involve the Aleksandrov-Clark measures associated to the symbol of the operator. The results are based on a variant of a general method due to Weis of constructing best compact and weakly compact approximants for linear operators on L^1 spaces.

The Spectrum of Fourier-Muckenhoupt Multipliers

Nikolai Nikolski

A Fourier multiplier on the circle group $\mathbb{T}$ is a bounded operator T on a function space X such that $Tx_j = \lambda_j x_j$, where $x_j = e^{ijx}$, $j \in \mathbb{Z}$, λ_j being complex numbers. The question is, for which spaces X the following Spectral Localization Property (SLP) holds: $\sigma(T) = clos\{\lambda_j(T) : j \in \mathbb{Z}\}$ for every X-multiplier T. Considering the case $X = L^2(\mathbb{T}, w)$ with a Muckenhoupt weight $w \in (A_2)$, we give examples of w with and without SLP. Some observations on more general settings are also presented.
A part of this report concerns a joint project with I.Verbitsky (Univ. of Missouri-Columbia).

Self similarity in PDEs, new approach

Benhamidouche Nouredine

In this talk, we present a new approach to find a general self similar exact solutions to some nonlinear PDEs. The approach which we will present one will be called "the traveling profiles method" (TPM). Consider the following equation :

$$\frac{\partial u}{\partial t} = A_x u \tag{1.1}$$

wher $A_x u$ is a linear or nonlinear differential operator.

Our approach is to find a solution in the genarl form

$$u(x,t) = c(t)\,\psi\left(\frac{x - b(t)}{a(t)}\right), \text{ with } a, b, c \in R \tag{1.2}$$

where ψ is in L^2 , that one will call "the based-profile", and the parameteres $c(t)$, $a(t)$, $b(t)$ are real valued functions of t .

The coefficients $c(t)$, $a(t)$, $b(t)$ are determined by the solution of minimizition problem :

$$\min_{\dot{a},\dot{b},\dot{c}} \int_{-\infty}^{+\infty} \left| \frac{\partial u}{\partial t} - A_x u \right|^2 dx, \tag{1.3}$$

and the " based-profile" is determined as solution of a differential equation.

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Bergman space systems

A. Olofsson

Following Halmos, by a wandering subspace of the unweighted Bergman space A_2 of sqare area integrable analytic functions in the unit disc $\mathbb{D}$ is meant a closed subspace $\mathcal{E}$ of A_2 such that $\mathcal{E} \perp S_2^k(\mathcal{E})$ in A_2 for $k \geq 1$, where $S_2 : f \mapsto zf$ is the Bergman shift operator. There are by now well-known results by Aleman, Richter, Sundberg, Shimorin, Hedenmalm, and others, pointing at the need to understand better the structure of wandering subspaces in the Bergman spaces. In this talk we shall discuss some recent work to the extent that wandering subspaces are naturally parameterized by a class of operator-valued analytic functions, called operator-valued Bergman inner functions, which in the A_2-case can be realized as functions of the form

$$W(z) = D + zC\Big((I - zA)^{-1} + (I - zA)^{-2}\Big)B, \quad z \in \mathbb{D},$$

where (A, B, C, D) is a tuple of operators satisfying some canonical conditions. Formulas of this type make a clear connection to mathematical systems theory and suggest us to build a systems theory for weighted Bergman space norms. In this context the operator-valued Bergman inner functions are realized as transfer functions for a related class of discrete time linear systems.

Operators arising from generalized multiresolution analyses

Judith A. Packer

Generalized multiresolution analyses (GMRAs) corresponding to a discrete abelian translation group Γ and a dilation operator δ defined on abstract Hilbert spaces can be described by their multiplicity functions m defined on $\widehat{\Gamma}$ and matrix-valued filter functions H defined on appropriately chosen set related to m and $\widehat{\Gamma}$. We discuss the isometry S_H associated to a specified filter system H, a construction with its origins in the work of Bratteli and Jorgensen, and give necessary and sufficient conditions for this isometry to be pure. A construction that produces an abstract GMRA from any functions m and H meeting required conditions is described. An equivalence relation is defined on different filter systems H associated to the same multiplicity function m, and all equivalence classes of GMRA's associated to a specified m are described by cohomological conditions.
This work is joint with L. Baggett, V. Furst, and K. Merrill.

Dimension reduction in some problems involving self-adjoint extensions

Konstantin Pankrashkin

We would like to discuss some quantum-mechanical problems in which self-adjoint extensions appear naturally (we mostly concentrate on Aharonov-Bohm and quantum graph operators). A reduction to discrete operators allows one to calculate explicitly some spectral and scattering characterstics of these systems.

The talk is based on the papers

- J. Kellendonk, K. Pankrashkin, S. Richard: Levinson's theorem and higher degree traces for Aharonov-Bohm operators. J. Math. Phys. 52 (2011) 052102.
- K. Pankrashkin, S. Richard: Spectral and scattering theory for the Aharonov-Bohm operators. Rev. Math. Phys. 23 (2011) 53--81.
- J. Br□ning, V. Geyler, K. Pankrashkin Spectra of self-adjoint extensions and applications to solvable Schrödinger operators. Rev. Math. Phys. 20 (2008) 1--70.

Hankel operators on weighted Bergman spaces

J. Pau

We study boundedness, compactness and membership in Schatten-Von Newman ideals of Hankel operators on weighted Bergman spaces with rapidly decreasing weights.

Trace formulae for perturbations of class $\boldsymbol{S_m}$

V.V. Peller

The talk is based on joint work with A.B. Aleksandrov. We obtain general trace formulae in the case of perturbation of self-adjoint operators by self-adjoint operators of class $\boldsymbol{S}_m$, where m is a positive integer. In a recent paper by Potapov, Skripka, and Sukochev a trace formula for operator Taylor polynomials was obtained. This formula includes the Lifshits--Krein trace formula in the case $m = 1$ and the Koplienko trace formula in the case $m = 2$. We establish most general trace formulae in the case of perturbation of Schatten--von Neumann class $\boldsymbol{S}_m$. We also improve the trace formula obtained in by Potapov, Skripka, and Sukochev for operator Taylor polynomials and prove it for arbitrary functions in the Besov space $B^m_{\infty 1}(\mathbb{R})$.

We consider several other special cases of our general trace formulae. In particular, we establish a trace formula for mth order operator differences.

Structured matrices with bidiagonal decompositions

J.M. Peña

Bidiagonal factorizations have played a crucial role for some structured classes of matrices such us nonsingular totally nonnegative matrices (matrices with all their minors nonnegative) or strictly sign regular matrices. Important theoretical properties of these matrices can be deduced from this factorization. Moreover, the bidiagonal factorization can be used to perform accurately many computations with these matrices. There are other classes of matrices admitting a bidiagonal factorization. Here we also present some recent advances on the computation with matrices with special bidiagonal factorizations.

Optimal constant for Bergman projection of bounded functions

A. Perälä

We obtain the optimal constant for the Bergman projection of essentially bounded functions onto the Bloch space. Similarly, the optimal constant for the related question about the little Bloch space is calculated. We also discuss some related problems.

Analytic Solutions of Integral Equations with *Mathematica* software

Ana C. Conceição and José C. Pereira

We constructed an algorithm, [AEq] (described in [1]), for obtaining the analytic solutions of some classes of integral equations related with the Cauchy type singular integrals (see [2]). Recently, we implemented on a computer the [AEq] algorithm with *Mathematica*. The *Mathematica* software is a state-of-the-art computer algebra system capable of performing complex numeric and symbolic computations. In this talk we explore the usage of such system in the implementation and optimization of analytic methods for obtaining the solutions of the integral equations. We present some examples to illustrate how the automation of the solving process potentiates the exploration of the $\{Problem, Solution\}$ space in an effective manner.
The talk is based on a joint work with Viktor G. Kravchenko

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Quadratic and subexponential decay estimates for commutators of singular integral operators

Carlos Pérez

Commutators of singular integral operators with BMO functions were introduced in the seventies by Coifman-Rochberg and Weiss. These are very interesting operators for many reasons and their study became a classical topic in modern harmonic analysis. One reason of this interest is due to the fact that they are more singular than the usual singular integral operators. This idea can be expressed in many ways. In this lecture we plan to give three more reasons showing this worst behavior. One of them is related to a sharp L2 weighted estimate with respect A2 weights but the novelty is that the bound in term of the A2 constant of the weight is quadratic and no better while in the case of singular integrals is simply linear. The second reason is due to the fact that there is an appropriate local sub-exponential decay which in the case of singular integrals is of exponential type instead. The third reason is related to the controlled of the commutators by iterations of the maximal function with a sharp new A_∞ constant. Half of the lecture is part of a joint work with D. Chung and C. Pereyra and the second half with C. Ortiz and E. Rela.

Quasi-monotone weight functions and their characteristics and applications

L.-E. Persson

A weight function $w(x)$ on $(0,l)$ or (l,∞), is said to be quasi-monotone if $w(x)\,x^{-a_0} \leq C_0 w(y)\,y^{a_0}$ for some $a_0 \in R$, $C_0 \geq 1$, and $\times \leq y$ or $y < x$. In this talk we present both well-known and new results concerning quasi-monotone functions. In particular, some new results concerning the close connection to index numbers and generalized Bari-Stechkin classes are proved and applied. Moreover, some new regularization results are proved and several applications are pointed out, e.g. in interpolation theory, Fourier analysis, Hardy-type inequalities, singular operators and homogenization theory.

The talk is based on a joint work with N. Samko and P. Wall.

A class of time-shift invariant linear evolutionary equations and some applications

Rainer Picard

A time-shift invariant class of linear evolutionary operator equations is introduced, which covers a number of problems such as initial boundary value problems with delay, integro-differential equations and PDAE in a unified set-up. Well-posedness and causality of the solution operator is established and to illustrate the utility of the approach some applications are dicussed.

Pro-C*-algebra structures from profinite groups

P.R. Pinto

We investigate when a C*-algebra A can be given different structures of pro-C*-algebras in the sense of Voiculescu, besides the trivial one where the seminorms are all equal to the given norm of A. We produce such nontrivial structures for the full group *-algebra C$^*(G)$ with G being a residually finite group or the free group $\mathbb{F}_n$ in n generators. The talk is based on a joint work with R. El Harti and N.C. Phillips.

Spectral order and its extensions

A. Płaneta

Spectral order was defined by M. P. Olson in 1971 for bounded selfadjoint operators. The motivation to consider a new order was the fact that the set of all selfadjoint bounded operators with usual order given by quadratic forms is not lattice ordered. As shown by Kadison, the set $\mathcal{S}$ of all bounded selfadjoint operators on a complex Hilbert space $\mathcal{H}$ is an anti-lattice, which means that for $A, B \in \mathcal{S}$, a greatest lower bound for A and B exists with respect to the usual ordering ``$\leqslant$" in $\mathcal{S}$ if and only if $A$ and $B$ are comparable (cf. [1]). A little bit earlier, Sherman proved that if the set of all selfadjoint elements of a $C^*$-algebra $\mathcal{A}$ of bounded linear operators on $\mathcal{H}$ is lattice ordered by ``$\leqslant$", then $\mathcal{A}$ is commutative (cf. [3]). Olson showed by himself that the set of all selfadjoint elements of a von Neumann algebra of bounded linear operators on $\mathcal{H}$ is a conditionally complete lattice with respect to the spectral order (cf. [2]). The aim of this talk is to present the spectral order in the case of unbounded selfadjoint operators and n-tuples of commuting unbounded selfadjoint operators.

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The talk is based on a joint work with J. Stochel.

Invariant subspaces of composition operators in Dirichlet space

M. Ponce-Escudero

In this talk we provide a description of the lattice of invariant subspaces for a certain class of composition operators acting on Dirichlet space $\mathcal{D}$. Specifically, we study C_φ where φ is a linear fractional transformation conjugate to a translation on the upper half-plane (known as parabolic non-automorphisms). Denoting by $\mathcal{D}_0$ the orthogonal complement of the constants, we can write $\mathcal{D} = [1] \oplus \mathcal{D}_0$. We show that the lattice of invariant subspaces of C_φ is

$$Lat_{\mathcal{D}} C_\varphi = \left\{ [1] \oplus \left\{ f \in \mathcal{D}_0 \colon \mathcal{F}\left[f\left(\frac{z-i}{z+i} \right) \right] = 0 \text{ on } A \right\} \colon A \subseteq \mathbb{R}_+ \text{measurable.} \right\}$$

Here $\mathcal{F}$ denotes the Fourier transform.

This characterization differs from that of the the invariant subspaces of C_φ acting on the classical Hardy space $\mathcal{H}^2$. In the former space, all invariant subspaces are spanned by its eigenfunctions. Indeed, any eigenfunction in $\mathcal{H}^2$ is of the form $e_t(z) = \exp(t(z+1)/(z-1))$. In $\mathcal{H}^2$ we know that

$$Lat_{\mathcal{H}^2} C_\varphi = \{\text{span}\{e_t \colon t \in F\} \colon F \text{ is closed } \}$$

These eigenfunctions do not belong to $\mathcal{D}$, resulting in a quite different characterization of invariant subspaces for C_φ. The talk is based on a joint work with A. Montes-Rodríguez.

Composition operators on noncommutative Hardy spaces

Gelu Popescu

We initiate the study of composition operators on the noncommutative Hardy space $H^2_{\mathbf{ball}}$. Several classical results about composition operators (boundedness, norm estimates, spectral properties, compactness, similarity) have free analogues in our noncommutative multivariable setting. The most prominent feature of this talk is the interaction between the noncommutative analytic function theory in the unit ball of $B(\mathcal{H})^n$, the operator algebras generated by the left creation operators on the full Fock space with n generators, and the classical complex function theory in the unit ball of $\mathbb{C}^n$.

Joint similarity to operators in noncommutative varieties

Gelu Popescu

We present several results on the joint similarity to n-tuples of operators in noncommutative varieties $\mathcal{V}_\mathcal{P} \subset B(\mathcal{H})^n$, associated with noncommutative polynomials $\mathcal{P}$ in n indeterminates, where $B(\mathcal{H})$ is the algebra of all bounded linear operators on a Hilbert space $\mathcal{H}$. Several classical results concerning the similarity to contractions (or special contractions such as parts of shifts, isometries, unitaries) have analogues in our noncommutative multivariable setting.

Phillips-Raeburn spectral triples

Denis Potapov

The Phillips-Raeburn spectral triple associated with a finite von Neumann algebra $\mathcal{M}$ equipped with a weakly continuous action α is the triple $(\mathcal{A}, \mathcal{H}, D)$, where $\mathcal{A}$ is the algebra of all infinitely differentiable elements with respect to the action α represented in the crossed-product von Neumann algebra $\mathcal{M} \rtimes_\alpha \mathbb{R}x$ acting on a Hilbert space $\mathcal{H}$; and D is the differentiation operator on $\mathcal{H}$. The main result shows that the spectral triple $(\mathcal{A}, \mathcal{H}, D)$ is QC-*smooth*.

Crystal frameworks, matrix-valued functions and rigidity operators

S.C. Power

An introduction and survey is given of some recent work on the dynamics of *crystal frameworks*, that is, of translationally periodic discrete bond-node structures in $\mathbb{R}^d$, for $d = 2, 3, \dots$. We discuss the rigidity matrix, a fundamental object from finite bar-joint framework theory, rigidity operators, matrix-function representations and low energy phonons. These phonons in material crystals such as quartz and zeolites, are known as rigid unit modes, or RUMs, and are associated with the relative motions of rigid units, such as SiO_4 tetrahedra in the tetrahedral polyhedral bond-node model for quartz. This functional analysis perspective in material science seems to be new. We also introduce semi-infinite crystal frameworks and their multi-variable Toeplitz operators.

Variable exponent Campanato spaces

H. Rafeiro

We study variable exponent Campanato spaces $\mathcal{L}^{p(\cdot),\lambda(\cdot)}(X)$ on spaces of homogeneous type. We show the equivalence, up to norms, with variable exponent Morrey spaces $L^{p(\cdot),\lambda(\cdot)}(X)$ ($\lambda_+ < 1$) and with variable exponent Hölder spaces $H^{\alpha(\cdot)}(X)$ ($\lambda_- > 1$).

The talk is based on a joint work with S. Samko.

On classical operators of real analysis having vanishing mean oscillation functions.

M.A. Ragusa

The purpose of the author is to study regularity results of solutions of some partial differential equations having the coefficients of the higher order derivatives in the following class of vanishing mean oscillation. At first let us set the definition of bounded mean oscillation function that appear at first in the note by John and Nirenberg (*Commun. Pure Appl. Math.*, 1961).

A function $f \in L^1_{loc}(R^n)$ belongs to $BMO(R^n)$ if the seminorm

$$\|f\|_* \equiv \sup_{B(x,\rho)} \frac{1}{|B(x,\rho)|} \int\limits_{B(x,\rho)} |f(y) - f_{x,\rho}| dy < \infty.$$

where $B(x, \rho)$ ranges in the class of the balls of R^n centered in $x \in R^n$ having radius ρ and $f_{x,\rho}$ is the mean integral of the function f associated to $B(x, \rho)$. If we are not interested in specifying which the center is, we only consider f_ρ. Let us state the definition of the subspace of vanishing mean oscillation functions, given at first by Sarason (*Trans. Amer. Math. Soc.*, 1975). Let $f \in BMO(R^n)$ and

$$\eta(f, R) = \sup_{\rho \leq R} \frac{1}{|B_\rho|} \int\limits_{B_\rho} |f(y) - f_\rho| dy$$

where $B\rho$ ranges over the class of the balls of R^n of radius ρ. A function $f \in VMO(R^n)$ if

$$\lim_{R \to 0} \eta(f, R) = 0.$$

Regularity properties are based on establish representation formulas for the highest order derivatives of the solutions directly for the case of variable coefficients in terms of singular integrals and commutator whose norm can be made small because of the coefficients are vanishing mean oscillation functions.

The Maximal Function and Hilbert Transform on Weighted Spaces: Theorems and Examples

M. C. Reguera

For $1 < p < \infty$, we find examples of weights, in the one weight setting as well as the two weight setting, for which the Hardy-Littlewood Maximal Operator is bounded in the corresponding weighted L^p spaces, but the Hilbert transform is not. This disproves a conjecture by Muckenhoupt and Wheeden that intended to characterize boundedness of the Hilbert transform in the two weight setting in terms of the boundedness of the maximal operator.
The talk is based on joint work with J. Scurry.

Furstenberg sets and Hausdorff measures

Ezequiel Rela

For a given $\alpha \in (0,1]$, we will say that a set $E \subset \mathbb{R}^2$ belongs to the F_α class (Furstenberg sets of type α) if, for any direction $e \in \mathbb{S}$, there exists a unit line segment ℓ_e in the direction of e such that

$$\dim_H(\ell_e \cap E) \geq \alpha. \tag{13}$$

If we define $\gamma(\alpha) = \inf\{\dim_H(E) : E \in F_\alpha\}$, then the Furstenberg problem is to determine $\gamma(\alpha)$. So far, the best known results are:

$$\max\{2\alpha; \frac{1}{2} + \alpha\} \leq \gamma(\alpha) \leq \frac{1}{2} + \frac{3}{2}\alpha. \tag{14}$$

Motivated by our interest in the extreme case $\alpha = 0$, we consider the more general class of Furstenberg sets $F_{\mathfrak{h}}$, replacing the hypothesis (13) by

$$\mathcal{H}^{\mathfrak{h}}(\ell_e \cap E) > 0, \tag{15}$$

where $\mathcal{H}^{\mathfrak{h}}$ stands for the Hausdorff measure associated to the gauge function $\mathfrak{h}$. We prove that the lower bounds in (14) can be generalized in the sense that the appropriate dimension function for any set in the class $F_{\mathfrak{h}}$ must be dimensionally not much smaller than $\mathfrak{h}^2$ or $\sqrt{\cdot}\mathfrak{h}$. We also generalize the construction yielding the upper bound. It follows that it is possible to extend inequalities (14) to the limit case $\alpha = 0$ for a class of zero dimensional gauge functions.

With the same techniques we study a related problem. If the directions are taken in a subset $L \subset \mathbb{S}$ such that $\dim_H(L) \geq \beta > 0$, then for any Furstenberg set E of type α associated to the set L of directions we have that

$$\dim_H(E) \geq \max\left\{2\alpha + \beta - 1; \frac{\beta}{2} + \alpha\right\}.$$

This is a joint work with Ursula Molter.

A_1 conjecture and Bellman function

Alexander Reznikov

Joint work with Fedor Nazarov, Alexander Volberg and Vasily Vasyunin.
By the A_1 conjecture people understand the question: can the weak $L^1(wdx)$ norm of a Calderon-Zygmund operator be bounded by a constant, depending only on the operator, times the so called A_1 characteristic of the weight w? Since the estimate

$$\|T\|_{L^1(wdx)\to L^{1,\infty}(wdx)} \leqslant C(T) \cdot [w]_1 \cdot (1 + \log([w]_1))$$

was known, the question is: can one erase the logarithmic term?

We give a negative answer to this question using the Bellman Function. We also notice that this result is stronger then the negative answer to the famous Muckenhoupt-Wheeden conjecture, which was given earlier in [1] and [2].

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Some aspects of positive completions problems

Leiba Rodman

Dedicated to the memory of M. Bakonyi

In it's most basic and well studied form, the positive completion (or extension) problem may be stated as follows: Given a cone C that consists of all Hermitian matrices having zeros in fixed specified entries, and given a Hermitian matrix A, find if there exists a positive definite or positive semidefinite matrix in the set $A + C$, and if yes, describe all such matrices. The matrices may be real or complex. Study of positive completion problems on their numerous guises and generalizations was a central theme in M. Bakonyi's research work. In the talk, several aspects of the positive completion problem will by highlighted, including positive matrices over C^*-algebras and connections with graph theory. Open problems will be stated.

Perturbation theory for classes of structured matrices

Leiba Rodman

The lecture will focus on behavior of eigenvalues, root subspaces, etc. of a matrix with structure under small perturbations of the matrix. The perturbations may be small in norm or small in rank, and are subject to the same structure restrictions as the original matrix. Various structures will be considered, with emphasis on Hamiltonian matrices. Large part of the material is joint work with C. Mehl, V. Mehrmann, and A.C.M. Ran.

On The Solutions of a Singular Integral Equation with Shift and Continuous Coefficients

E. M. Rojas

Existence and uniqueness of solutions, as well as their explicit representations, are obtained for singular integral equations with weighted Carleman shift and continuous coefficients which cannot be reduced to binomial boundary value problems.

The talk is based on a joint work with L. P. Castro.

Matrix representations of inverses of Toeplitz and Toeplitz-plus-Hankel matrices

K. Rost

Inverses of Toeplitz matrices can be represented as Bezoutians of two polynomials. A matrix representation of such a Bezoutian was constructed for the first time by Gohberg/Semenul in 1972. To generalize this result we design matrix representations for Toeplitz-plus-Hankel Bezoutians, in particular for split Bezoutians B. These are higly structured matrices since all of their rows and columns are symmetric or skewsymmetric vectors. Moreover, recursion formulas for the entries of B are presented, and B is represented as a linear combination of very simple (even sparse) split Bezoutians. These results are important, for example, for constructing matrix representations of inverses of centrosymmetric or centro-skewsymmetric Toeplitz-plus-Hankel matrices.

IFractional powers of operators and Poincaré inequalities

Emmanuel Russ

Let μ be a probability measure on $\mathbb{R}^n$. Under suitable assumptions which ensure that μ satisfies a Poincaré inequality in L^2, we prove a fractional version of this inequality, involving non local quantities. We also deal with the case of Lie groups and Riemannian manifolds. Finally, we investigate some properties of Bessel spaces associated with suitable second order elliptic operators.

Applications of the theory of reproducing kernels to convolutions and integral transforms

Saburou Saitoh

We introduce a very general concept of convolutions by means of the theory of reproducing kernels which turns out to be useful for several concrete examples and applications.

In particular, we will give concrete applications as follows:

In Fourier analysis, the convolution operator is fundamental; as new examples, we introduce naturally further new convolutions of three types and we shall derive the convolution inequalities for the new convolutions, the results may be derived naturally from the theory of reproducing kernels.

For arbitrary non-identically zero functions f, we will introduce some natural fractional functions f_1 having f as denominators and we shall consider their representations f_1 by appropriate numerator functions within a reproducing kernel Hilbert spaces framework. That is, in the present work we would like to introduce very general fractional functions (e.g., having the possibility of admitting zeros in their denominators) by means of the theory of reproducing kernels.

For some general integral equations with the mixed Toeplitz and Hankel kernels represented by convolutions, some analytical representations of the solutions will be given by using the theory of reproducing kernels and the Moore-Penrose generalized inverses.

As a powerful method, I would like to refer to the fact that the Tikhonov regularization method combined with the theory of reproducing kernels is the most general and powerful numerical method within the framework of the above mentioned cases.

The talk is based on a joint work with L. P. Castro.

A new family of topological rings with applications in linear systems

Guy Salomon

Recently, and using the theory of stochastic distributions, the theory of linear stochastic systems was reduced to the theory of linear systems on a certain commutative ring (which is also a nuclear space).
Motivated by this example we define a wide family of nuclear spaces and give a characterization to the existence of a continuous multiplication (in an appropriate manner) on these spaces, thus obtaining commutative topological rings.

We characterize invertible elements of these rings, and present applications to linear system theory.
This is joint work with Daniel Alpay.

Weighted estimates of the Cauchy singular integral operator in generalized Morrey spaces

N. Samko

We study the weighted boundedness of Hardy and singular operators in generalized Morrey spaces $L^{p,\omega}(\mathbb{R}^n, \varphi)$. The conditions for the boundedness are obtained in terms of numerical inequalities relating the Matuszewska-Orlich indices of the weight function φ with the parameter p and index numbers of the characteristic ω of the space. Conditions of the boundedness in terms of such numerical inequalities are important in applications, in particular in the theory of singular integral equations and related boundary value problems for analytic functions. We find conditions for the singular integral operator $\mathfrak{A}$ to be Fredholm in the spaces under consideration.
The talk is based on a joint work with D. Lukkassen, A. Meidell and L-E. Persson

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On potentials in generalized Hölder spaces over uniform domains

S. Samko

In this talk, based on a joint paper with Lars Diening, we show that Riesz-type potential operators of order α over uniform domains Ω in $\mathbb{R}^n$ map the subspace $H_0^\lambda(\Omega)$ of functions in Hölder space $H^\lambda(\Omega)$ vanishing on $\partial\Omega$, into the space $H^{\lambda+\alpha}(\Omega)$, if $\lambda+\alpha \leq 1$. This is proved in a more general setting of generalized Hölder spaces with a given dominant of continuity modulus. Statements of such a kind are known for instance for the whole space $\mathbb{R}^n$ or more generally for metric measure spaces with cancellation property. In the case of domains in $\mathbb{R}^n$ when the cancellation property fails, our proofs are based on a special treatment of potential of a constant function.

Solution of some generalized Riemann boundary value problem with shift in the real line

J. C. Sanchez Rodriguez

In the real line we consider singular integral operators with a linear Carleman shift and complex conjugation, acting in $\widetilde{L}_2(\mathbb{R})$, the space of all Lebesgue measurable complex value functions on $\mathbb{R}$ with $p = 2$ power. We show that the original singular integral operator with shift and conjugation is, after extension, equivalent to a singular integral operator without shift and with a 4×4 square matrix coefficients. By exploiting the properties of the factorization of the symbol of this last operator, it is possible to describe the solution of a generalized Riemann boundary value problem with a Carleman shift.

Eigenfunctions of generalized Hilbert matrices

A. Sarafoleanu

We study Hilbert-type infinite matrices of the form $H_\lambda = \left(\frac{1}{m+n+\lambda}\right)_{m,n\geq 0}$ where $\lambda \in \mathbb{C} \setminus \mathbb{Z}$. These matrices extend to bounded operators on H^2 or on the Korenblum classes $A^{-\tau}$. We investigate eigenfunctions of these operators using their commutant with certain differential operators. The results we obtain match those proved earlier by M. Rosenblum for the case $\lambda \in \mathbb{R}$.
The talk is based on a joint work with A. Aleman and A. Montes Rodríguez.

Addittive maps preserving further spectrales radii

M. Sarih

Let X be a complex Banach space and let $\mathcal{L}(X)$ be the algebras of all bounded linear operators on X. We characterise additive continious maps from $\mathcal{L}(X)$ onto itself wich preserve further spectrales radii such us the inner local radius and the local spectral radius at a nonzero fixed vector.
The talk is based on a joint work with M. Bendaoud.

On Representation Theorems for Indefinite Quadratic Forms

Stephan Schmitz

Let $\mathfrak{H}$ be a complex Hilbert space with the inner product $\langle \cdot, \cdot \rangle$. We will be dealing with the class of sesquilinear forms given by

$$\mathfrak{b}[x, y] = \langle A^{1/2}x, HA^{1/2}y \rangle, \ x, y \in \mathrm{Dom}[\mathfrak{b}] = \mathrm{Dom}(A^{1/2}).$$

If A is a *positive definite* self-adjoint operator in $\mathfrak{H}$ and H is a bounded, not necessarily positive, self-adjoint operator in $\mathfrak{H}$, then the following results hold true (see [1] and the references quoted therein) :

(i) The first representation theorem: If H has a bounded inverse, then there is a unique self-adjoint boundedly invertible operator B with Dom$(B) \subset$ Dom$[\mathfrak{b}]$ associated with the form $\mathfrak{b}$, that is,

$$\mathfrak{b}[x,y] = \langle x, By\rangle \text{ for all } x \in \text{Dom}[\mathfrak{b}],\ y \in \text{Dom}(B) \subset \text{Dom}[\mathfrak{b}].$$

(ii) The second representation theorem: If, in addition, the domains of $|B|^{1/2}$ and $A^{1/2}$ agree, that is,

$$\text{Dom}(|B|^{1/2}) = \text{Dom}(A^{1/2}),$$

then the form $\mathfrak{b}$ is represented by B in the sense that

$$\mathfrak{b}[x,y] = \langle |B|^{1/2}x, \text{sign}(B)|B|^{1/2}y\rangle \text{ for all } x, y \in \text{Dom}[\mathfrak{b}] = \text{Dom}(|B|^{1/2}),$$

with sign(B) the sign of the operator B.

Our aim is to extend the first and second reprsentation theorems to the case of semi-definite operators A. In contrast to the previous case the operator associated with the form in general does not have a spectral gap around zero.
The talk is based on joint work with V. Kostrykin.

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Estimates for Dirichlet polynomials

Kristian Seip

We will discuss some estimates on coefficient sums $\sum_{n=1}^{N} |a_n|$ and similar quantities in terms of L^p norms of Dirichlet polynomials $\sum_{n=1}^{N} a_n n^{-s}$. Topics that will enter this presentation, are the Bohnenblust-Hille inequality for homogeneous polynomials and the problem of weak factorization of the Hardy space H^1 and Hankel forms on the infinite-dimensional polydisc.

Boundary velocity equivalence and different flow schemes in inverse boundary value problem of aerohydrodynamics

E. Semenko

The plane flow past a connected contour by potential stationary streaming of an ideal incompressible fluid is considered. The classes of boundary velocities corresponding to one streamlined contour at a different oncoming flow parameters are described. The solution of inverse problem of hydrodynamics, i.e. the determination of contour by given boundary velocity, is resulted for any flow schemes including when flow hasn't stagnation points on contour. The question of boundary velocity equivalence, i.e. of correspondence of them to one streamlined contour, is considered.

Strictly singular operators and inclusions

E. M. Semenov

A linear operator A between two Banach spaces E and F is called strictly singular (SS in short) if A fails to be an isomorphism on any infinite dimensional subspace of E. An operator A from E to F is called super strictly singular (SSS in short) if the sequence of Bernstein widths $b_n(A)$ tends to 0 when $n \to \infty$, where

$$b_n(A) = \sup_{Q \subset E, \dim Q = n} \inf_{x \in Q, \|x\|_E = 1} \|Ax\|_F.$$

It is clear that $K \subset SSS \subset SS$, where K denotes the class of compact operators. In general these operator ideals are different. However any SS operator in a l_p-space ($1 \leqslant p < \infty$) is compact. This was proved by I. Gohberg, A. Markus and I. Feldman (for $p = 2$ it was done before by J. Calkin).
It is well known that there exist non-compact strictly singular operators in $L_p[0,1]$ ($1 \leqslant p \leqslant \infty, p \neq 2$).

Theorem 1. *Let $1 < q < r < \infty$. If an operator A is bounded in $L_q[0,1]$ and $L_r[0,1]$ and $A \in SS(L_p)$ for some $p \in [q,r]$, then $A \in K(L_p)$ for all $p \in (q,r)$.*

This theorem fails for operators on L_p spaces of infinite measure. Denote $V_p = SS \setminus K(L_p)$.

Theorem 2. *Let $1 < q < r < \infty$. The set $V_q \bigcap V_r$ is not empty iff $q < 2 < r$.*

A Banach space E of measurable functions on $[0,1]$ is said to be rearrangement invariant (r. i. in short) if E is a Banach lattice and the norms of equimeasurable functions are equal. A classical Grothendick theorem states that the inclusion $L_\infty \subset L_p$ is SS for any $p < \infty$.

Theorem 3. *If E is an r. i. space and $E \neq L_\infty$, then the inclusion $L_\infty \subset E$ is SSS and*

$$\varphi_E\left(1/n\right) \leqslant b_n(I(L_\infty, E)) \leqslant \varphi_E^{1/2}\left(1/n\right), \qquad n = 1, 2, \ldots,$$

where $\varphi_E(t)$ is the norm of the characteristic function of $[0,t]$ in E.

Harmonic spheres conjecture

Armen Sergeev

Harmonic spheres are given by the smooth maps of the Riemann sphere into Riemannian manifolds which are the extremals of the energy functional defined by Dirichlet integral. They satisfy nonlinear elliptic equations, generalizing Laplace--Beltrami equation. If the targeting Riemannian manifold is Kähler then holomorphic and anti-holomorphic spheres deliver local minima of the energy functional, however, this functional usually have also non-minimal extremals.
On the other hand, Yang--Mills fields are the extremals of a functional given by the Yang--Mills action. Local minima of this functional are called instantons and anti-instantons. It was believed that they exhaust all critical points of Yang--Mills action on the 4-dimensional Euclidean space $\mathbb{R}^4$, until examples of non-minimal Yang--Mills fields were constructed.
There is an evident formal similarity between Yang--Mills fields and harmonic maps and it became clear after the Atiyah's paper of 1984 that there is a deep reason for such a similarity. In our talk we formulate a harmonic spheres conjecture which asserts that there is a direct correspondence between the moduli space of Yang--Mills G-fields on $\mathbb{R}^4$ and the space of based harmonic spheres in the loop space ΩG where G is a compact Lie group. The talk will be devoted to the discussion of this conjecture and idea of its proof.

Inverse scattering problem with fixed energy for nonlinear Schrödinger equation

V.S. Serov

We are dealing with the generalized nonlinear Schrödinger equation

$$i\frac{\partial}{\partial t}E(x,t) = -\Delta E(x,t) + h(x,|E|)E(x,t),$$

where function h satisfies some special conditions. Considering harmonic time-dependence $E(x,t) = u(x)e^{-i\omega t}$ with frequency $\omega > 0$ we obtain time-independent equation

$$-\Delta u(x) + h(x,|u|)u(x) = \omega u(x).$$

Our main example of such type of equations is the following equation with *absorption*

$$-\Delta u + \alpha_0 u + \frac{\alpha_1 |u|^2}{1 + r|u|^2}u = k^2 u,$$

where the coefficients α_0 and α_1 are (in general) complex-valued functions with compact support, parameter r is positive and $k^2 > 0$ is fixed. This equation describes the *saturation model* in nonlinear optics. We study inverse scattering problem with fixed energy. Our main goal is to prove the uniqueness result of three dimensional inverse fixed energy problem for these nonlinear equations.

The Riemann--Hilbert boundary value problem and univalent mappings of polygonal domains with infinite sets of vertices.

P. Shabalin

In this report we generalize the Schwarz-Christoffel formula for a conformal mapping of a half-plane onto a polygon for the case when the number of vertices of a certain polygon is infinite. We assume that the inner angles at unknown vertices and the image of the vertices under the conformal mapping on the real line are given. Under various restrictions on values of the angles and on the sequences of points of the real line that are preimages of the vertices the formulae for such a mapping is obtained. The construction of mapping is based on the solution of the Riemann-Hilbert boundary value problem for the case when coefficients in the boundary condition have two infinite sequences of discontinuity points of the first kind and a finite or infinite index. We obtain the univalence criteria for constructed mappings.
The talk is based on a joint work with R. Salimov.

Singular degenerate problems occurring in atmospheric dispersion of pollutants

Aida Shahmurova

The main objective of the present paper is to discuss singular perturbation BVPs for degenerate linear differential-operator equation

$$Lu = -\varepsilon u^{[2]}(x) + Au(x) + \varepsilon^{\frac{1}{2}} A_1(x) u^{[1]}(x) + A_2(x) u(x) = f,$$

and BVPs for nonlinear degenerate equation

$$-u^{[2]}(x) + A\left(x, u, u^{[1]}\right) u(x) = f\left(x, u, u^{[1]}\right),$$

on $(0,1)$, where

$$D_x^{[i]} u = u^{[i]}(x) = \left[x^{\gamma} \frac{d}{dx}\right]^i u(x), \ \gamma > 1$$

and ε is a small parameter. In applications maximal regularity properties of Cauchy problem for the degenerate parabolic equation with small parameter

$$u_t - \varepsilon D_x^{[2]} u(t,x) + Au(t,x) + \varepsilon^{\frac{1}{2}} A_1(x)\, u^{[1]}(t,x)$$

$$+A_2(x)\, u(t,x) = f(y,x)\,, t \in R_+, x \in (0,1)\,,$$

is established, where A, A_1, A_2 are linear operators in a Banach space E. Several conditions for the uniform separability and the resolvent estimates for the linear problem are given in abstract L_p-spaces. Moreover, the existence and uniqueness of maximal regular solution of nonlinear problem are obtained.
The above singular perturbation problems occur in different situation of fluid mechanics environmental engineering et.s.

Separable differential operators in Banach spaces and applications

Veli Shakhmurov

Main aim of the present talk, is to show the uniform separability properties of differential operator O_t generated by multipoint boundary value problems for the following differential equation with small parameter

$$(-1)^m\, tu^{(2m)}(x) + Au(x) + \sum_{j=0}^{2m-1} t^{\frac{j}{2m}} A_k(x)\, u^{(j)}(x) + \lambda u = f(x)\,, \qquad (16)$$

where A and A_k are linear operators in Banach space E, λ is a complex spectral and t is a small parameters.
Under certain conditions we prove that the problem (1) is separable in $L_p(0,1;E)$ and the uniform coercive estimates for the resolvent of the O_t is true, i.e.

$$\sum_{i=0}^{2m} |\lambda|^{1-\frac{i}{2m}}\, t^{\frac{i}{2m}} \left\| \frac{d^i}{dt^i} \left[O_t{+}\lambda\right]^{-1} \right\|_{B(L_p(0,1;E))} + \left\| A\left[O_t + \lambda\right]^{-1} \right\|_{B(L_p(0,1;E))} \leq C.$$

It implies that the operator O_t is positive in $L_p(0,1;E)$ and generates an analytic semigroup. By using this properties we show that the mixed problem for the following parabolic equation:

$$\frac{\partial u}{\partial y} + (-1)^m\, t \frac{\partial^{2m} u}{\partial x^{2m}} + Au(x,y) = f(y,x)\,,\ x \in (0,1)\,,\ y \in (0,\infty)$$

has a unique strange solution in $L_p\left((0,1)\times(0,\infty);E\right)$ and the following uniform estimate holds:

$$\left\|\frac{\partial u}{\partial y}\right\|_{L_p}+\left\|t\frac{\partial^{2m}u}{\partial x^{2m}}\right\|_{L_p}+\|Au\|_{L_p}\leq C\,\|f\|_{L_p}.$$

Classes of Vessels of Three or More Commuting Operators

E. Shamovich

Operator vessels were defined by M. Livsic to facilitate the study of spectral theory of commuting non-selfadjoint operators. In particular the case of operators with finite dimensional imaginary parts lead to interesting algebro-geometric questions.
The connection goes through an overdetermined, time invariant system associated to each vessel. Since the system is overdetermined it admits a set of input compatibility PDEs. Then passing to frequency domain one gets a system of algebraic equations. Those algebraic equations define an affine scheme, the discriminant scheme. The case of two operators was well studied by J. Ball and V. Vinnikov. In this case the discriminant scheme is always purely one-dimensional. The discriminant scheme comes with a coherent sheaf on it. Sections of this sheaf provide us with solutions to the original system of input compatibility PDEs. However, certain maximality assumption is necessary to ensure that the sheaf is not too small..
On the other hand if we have three or more operators, the discriminant scheme need no longer be one dimensional. An example of a discriminant scheme consisting of isolated points was already introduced by Livsic.
In this lecture we will show necessary and sufficient condition for a vessel of commuting operators to posses a one-dimensional discriminant scheme. We will then proceed to show that the discriminant scheme in this case admits no isolated points. We will also show examples of the structure of the determinantal scheme in the case of three operators. Those examples will demonstrate the embedded point phenomena absent from the two operator case. Finally we will speak of still open questions regarding maximality of the kernel sheaf, the vessel's joint characteristic function and desingularizations of the determinantal scheme. The talk is based on a joint work with V. Vinnikov

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State feedback for overdetermined 2D systems: Pole placement for bundle maps over an algebraic curve

L. Shaul

We discuss the pole placement problem for overdetermined multidimensional systems. We begin by introducing the notion of a Livsic-Kravitsky commutative operator vessel, showing that its transfer function is a meromorphic bundle map between two vector bundles over an algebraic curve. Next, we discuss state feedback for these kind of systems. Finally, we present a function-theoretic obstruction for solving a pole placement problem in this setting. The talk is based on a joint work with V. Vinnikov.

Canonical Factorization of a System of Operator Equations

N. Shayanfar

Let be given a complicated system of equations, where the matrix defining the system is constant. We show that the system can be reduced to an equivalent one with higher order operators having only one variable.
The solvability of the system is described in the case when the matrix polynomial corresponding to the system admits a Smith factorization. When posed as operator factorization, some new features of the transformation become apparent. The key to our new approach is the selection of the operator as a variable of the matrix polynomial referred above, since canonical factorization of this matrix polynomial will be described in terms of the variable.
Our main result is an explicit construction of a matrix polynomial equation for the system of operator equations. Results of the reduction on the system of integral equations are given as an application.

The talk is based on a joint work with M. Hadizadeh.

On the spectral characteristics of the Sturm--Liouville problem with Cantor type measure

I. A. Sheipak

We consider the boundary problem

$$\begin{gathered} -y'' - \lambda \rho y = 0, \\ y(0) + \gamma_0 y'(0) = y(1) + \gamma_1 y'(1) = 0, \end{gathered} \tag{1}$$

where the weight ρ is a self-similar measure.

If ρ has non-arithmetic type of self-similarity then the counting function of the eigenvalues for problem (1) has the following asymptotics

$$N(\lambda) = \lambda^D(s + o(1)).$$

In this case s is a constant.
If ρ has arithmetic type of self-similarity then the counting function of the eigenvalues for problem (1) has the following asymptotics

$$N(\lambda) = \lambda^D(s(\ln\lambda) + o(1)). \tag{2}$$

In this case $s(\ln\lambda)$ is some periodic function and possibly it can be a constant too.
The positive constant D can be uniquely evaluated from the self-similarity parameters of the measure ρ. For singular non-discrete ρ inequality $0 < D < \dfrac{1}{2}$ holds.
The fact of the non-degeneracy of the periodic function $s(\ln\lambda)$ (when it is not a constant) is important for many applications but has not been studied yet. The characteristics of function s are not known also. They are the main purpose of our investigation.
Let us describe the class of measures we are working in. Let $\varkappa \geqslant 2$ be a natural number. For arbitrary number $a \in (0, \frac{1}{\varkappa})$ we define $b = \frac{1-a\varkappa}{\varkappa-1}$, $\alpha_{2k} = k(a+b)$, $\alpha_{2k+1} = \alpha_{2k} + a$, $k = 0, 1, \ldots, \varkappa - 1$. There is a unique continuous function P such that it is constant on the intervals $(\alpha_{2k+1}, \alpha_{2k+2})$ and on the intervals $(\alpha_{2k}, \alpha_{2k+1})$ the relationship $P(ax + \alpha_{2k}) = \frac{1}{\varkappa}P(x) + \frac{k}{\varkappa}$ holds, where $x \in (0, 1)$. The function P is *a Cantor type function* and weight $\rho = P'$ (in the sense of the distributions) is *a Cantor type measure*.
Theorem 1. If ρ is a Cantor type measure then the eigenvalues λ_n of the problem (1) with Neumann boundary conditions have the property of spectral periodicity:

$$\lambda_{\varkappa n} = \frac{\varkappa}{a}\lambda_n, \quad n \in \mathbb{N}.$$

Theorem 2. If ρ is a Cantor type measure then the counting function of eigenvalues of problem (1) with any self-adjoint boundary conditions has asymptotics (2) where $s(t) = e^{-Dt}\sigma(t)$, $\sigma(t)$ is a singular continuous function, $D = \ln\varkappa \left(\ln\frac{\varkappa}{a}\right)^{-1}$.
Consequently, in this case function s is not a constant.
The talk is based on the joint work with A.A.Vladimirov. The work is supported by the Russian Fund for Basic Research (grant No. 10-01-00423).

A Complete Unitary Similarity Invariant for Unicellular Matrices

N. Shvai

We consider the *unitary similarity problem*: under what necessary and sufficient conditions are two $n \times n$ complex matrices unitarily similar? A classical solution is Specht's theorem: two $n \times n$ complex matrices A and B are unitarily similar if and only if trace $\omega(A, A^*) =$ trace $\omega(B, B^*)$ for every word ω in two noncommuting variables x and y. We can and will consider only upper triangular matrices since each matrix is unitarily similar to upper triangular. A square matrix is *indecomposable for similarity* if it is similar to a Jordan block.
The following theorem was proved in [1] for the operator norm $\|M\| = \max_{|v|=1} |Mv|$ and in [2] for the Frobenius norm.
Theorem. Let $A = [a_{ij}]$ and $B = [b_{ij}]$ be two upper triangular complex $n \times n$ matrices that are indecomposable for similarity. Then A and B are unitarily similar if and only if

$$\|f(A_i)\| = \|f(B_i)\| \qquad \text{for all } f \in \mathbb{C}[x],\ i = 1, \dots n,$$

in which $A_k := [a_{ij}]_{i,j=1}^k$ and $B_k := [b_{ij}]_{i,j=1}^k$ are the leading principal $k \times k$ submatrices of A and B, and $\| \cdot \|$ is the operator or Frobenius norm.

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Calderón-Zygmund operators related to Jacobi expansions

Peter Sjögren

This is joint work with A. Nowak. We study several fundamental operators in harmonic analysis related to Jacobi expansions, including Riesz transforms, imaginary powers of the Jacobi operator, the Jacobi-Poisson semigroup maximal operator and square functions. We show that these are (vector-valued) Calderón-Zygmund operators in the sense of the associated space of homogeneous type, and hence their mapping properties follow from the general theory. Our proofs rely on an explicit formula for the Jacobi-Poisson kernel, which we derive from a product formula for Jacobi polynomials.

On Stable Numerical Algorithms for Irregular Operator Equations

Alexandra Smirnova

A lot of important mathematical models give rise to either irregular operator equations or to operator equations whose regularity is extremely difficult to investigate, for instance numerous nonlinear inverse problems in PDEs. The regularity condition generalizes the requirement on the derivative to be different from zero in a neighborhood of the root. This requirement is used for the convergence analysis of Newton's scheme in a one dimensional case. Without the regularity condition, Newton's iterations are not necessarily well-defined. The lack of regularity is a major obstacle when it comes to applicability of not only the Newton method, but all *classical* iterative methods, gradient-type methods for example, although often these methods are formally executable for irregular problems as well. Thus, one has to construct special *regularized* numerical procedures for solving operator equations without the regularity condition in Banach and Hilbert spaces.

In the modern theory of *irregular (ill-posed, unstable)* problems, numerous regularized computational methods are known. These methods are being constantly perfected and augmented with new algorithms. Applied inverse problems are the main sources of this development. We will present a general framework of regularization theory for unstable operator equations and illustrate its performance on some key imaging models.

Bounded Toeplitz and Hankel products on the weighted Bergman spaces

P. Sobolewski

We study the boundedness of the products of Toeplitz operators $T_f T_{\bar{g}}$, Hankel operators $H_f H_g^*$ and the mixed Toeplitz and Hankel products $H_g T_{\bar{f}}$ on the weighted Bargman space A^p_α of the unit ball where $1 < p < \infty,\ \alpha > -1$. The talk is based on a joint work with M. Michalska.

On the Eigenvalue Problem for a Certain Class of Jacobi Matrices

F. Štampach

A function $\mathfrak{F}$ with simple and nice algebraic properties is defined on a subset of the space of complex sequences. Some special functions are expressible in terms of $\mathfrak{F}$, first of all the Bessel functions of the first kind. A compact formula in terms of the function $\mathfrak{F}$ is given for the characteristic function of a finite dimensional symmetric Jacobi matrix. Further, we focus on a particular class of semi-infinite symmetric Jacobi matrices. With the aid of $\mathfrak{F}$, a complex function of one complex variable is constructed having the property that the spectrum of the Jacobi matrix under investigation coincides with the set of all real zeros of the function. Furthermore, a vector-valued function on the complex plain is defined such that its values on spectral points of the Jacobi matrix are equal to corresponding eigenvectors. A simple formula for the ℓ^2-norm of the eigenvectors is given. Finally, a relation between $\mathfrak{F}$ and the Green function of a Jacobi operator, especially the Weyl m-function, is found. Results are demonstrated on several examples. The talk is based on a joint work with P. Šťovíček.

Fractional operators with semigroups in PDEs and Harmonic Analysis

P. R. Stinga

In the last 5 years there has been an increasing interest in the study of nonlinear problems involving fractional powers of second order partial differential equations, like the fractional Laplacian. We can say that a such development was initiated by the works of L. Caffarelli and L. Silvestre on the obstacle problem for the fractional Laplacian. In PDE problems the pointwise formulae for the fractional operators are crucial, as well as localization techniques to handle nonlocal operators, and also Schauder and Harnack's estimates.

On the other hand, fractional operators are classical objects in Harmonic Analysis and Functional Analysis. The fractional Laplacian appears, for instance, in the attempt to define fractional derivatives. In this context the fractional Laplacian can be defined using spectral methods.

In this talk I will present an unified approach to consider fractional powers of differential operators that avoids the Fourier transform. The main tool will be the heat-diffusion semigroup generated by the operator. In this way many PDE properties valid for the fractional Laplacian can be derived in a general and clear way. We will prove a local characterization for general fractional operators via an extension problem in the spirit of Caffarelli-Silvestre. As a consequence we will derive a Harnack's inequality for the fractional harmonic oscillator.

The approach also gives us the explicit pointwise formulae for the operators, without taking inverse Fourier transform. In the case of the harmonic oscillator, the formula leads to the natural definition of Hölder spaces. Then adapted Schauder estimates are obtained.

Finally, we are able to characterize Hölder spaces in the Schrödinger context using semigroups that will permit us to prove boundedness results of fractional operators in a very simple way.

Multipliers and their inverses

D. T. Stoeva

During the last years, Gabor multipliers have been used in many areas, for example in seismic data analysis and psychoacoustics. Therefore, from practical point of view, it is of interest to investigate the possibilities for invertibility of multipliers in order to use their inverses. Theoretically, multipliers of general sequences are also of interest.
The present talk deals with multipliers when one of the sequences is a frame. We give sufficient conditions for the invertibility of multipliers, determine their inverses by series and give bounds for the n-term error approximation. We discuss the sharpness of the bounds of variables, used in the sufficient conditions. Further, some numerical results on the topic are presented.
The talk is based on a joint work with P. Balazs.

Variational principles for eigenvalues in gaps

M. Strauss

We characterise eigenvalues which lie in gaps in the essential spectrum of operator valued functions. The approach can then be used to characterise eigenvalues for various classes of operator, including self-adjoint operators and block operator matrices which are self-adjoint in a Hilbert or Krein space. The talk is based on a joint work with M. Langer.

On some vector differential operators of infinite order

Tetyana Stulova

Let E be a complex Banach space and $\varphi(z) = \sum\limits_{n=0}^{\infty} C_n z^n$ be a formal power series that the coefficients are bounded linear operators on E.
Consider a problem on applicability of the following differential operator of infinite order

$$\varphi\left(\frac{d}{dz}\right) g = \sum_{n=0}^{\infty} C_n g^{(n)}$$

to the space $\mathcal{H}(\mathbb{C}, E)$ of all entire E-valued functions.

Theorem 1. *Let $\varphi(z)$ be an entire operator-function of exponential type, i.e. $\|\varphi(z)\| \leq \gamma e^{\beta|z|}$ for some $\gamma > 0$, $\beta > 0$ and all $z \in \mathbb{C}$. If $g \in \mathcal{H}(\mathbb{C}, E)$, then the series $\sum\limits_{n=0}^{\infty} C_n g^{(n)}(z)$ converges uniformly in every disk and thus $\varphi\left(\frac{d}{dz}\right)$ is continuous linear operator in the space $\mathcal{H}(\mathbb{C}, E)$ on which the topology of uniform convergence on compacts is considered. In addition, not more than exponential growth of $\varphi(z)$ is the necessary condition: if $\varphi(z) = \sum\limits_{n=0}^{\infty} C_n z^n$ is such power series, that the series $\sum\limits_{n=0}^{\infty} C_n g^{(n)}(0)$ converges for all $g \in \mathcal{H}(\mathbb{C}, E)$, then $\varphi(z)$ is an entire function of exponential type.*

Let V be an arbitrary complex vector space and $V\left[\left[\zeta, \frac{1}{\zeta}\right]\right]$ be the space of all formal Laurent series with coefficient of V. For $f(\zeta) = \sum\limits_{n=0}^{\infty} b_n \zeta^n \in V\left[\left[\zeta, \frac{1}{\zeta}\right]\right]$ we set

$$\oint f(\zeta) d\zeta = 2\pi i \, b_{-1} \tag{17}$$

Theorem 2. *Let φ and g satisfy conditions of Theorem 1, $g(z) = \sum\limits_{n=0}^{\infty} a_n z^n$ and*

$$G(\zeta) = \sum_{n=0}^{\infty} \frac{n! a_n}{\zeta^{n+1}}$$

be the formal Laplace-Borel' transformation of power series $g(z)$. Then the product $e^{z\zeta}\varphi(\zeta) G(\zeta)$ is well defined as an element of $E[[z]]\left[\left[\zeta, \frac{1}{\zeta}\right]\right]$ (i.e. in this case the space of formal power series by z with coefficient in E place the role of the space V) and

$$\left(\varphi\left(\frac{d}{dz}\right) g\right)(z) = \frac{1}{2\pi i} \oint e^{z\zeta} \varphi(\zeta) G(\zeta) \, d\zeta,$$

where the integral is considered in the sense of (17).

Corollary 3. *Let $T : E \to E$ be a quasinilpotent operator, that the Fredholm resolvent $(1 - zT)^{-1}$ has exponential type, and $g(z)$ is an arbitrary E-valued entire function. Then the differential equation*

$$Tw' + g(z) = w, \tag{18}$$

has unique entire solution $w(z) = \sum\limits_{n=0}^{\infty} T^n g^{(n)}(z)$ and this solution can be presented in the following integral form

$$w(z) = \frac{1}{2\pi i} \oint e^{z\zeta} (1 - \zeta T)^{-1} G(\zeta) \, d\zeta,$$

where the integral is considered in the sense of (17).

Joint work with Sergey Gefter.

On linearizator of strong dumped pencils

L.Sukhocheva

Let $\widehat{\mathcal{H}} = \mathcal{H}^+ \oplus \mathcal{H}^-$ be Krein space and A be arbitrary selfajoint operator. We construct Hilbert space $\mathcal{H}$ and strong dumped pencil $L(\lambda) = \lambda^2 I + \lambda B + C$ such that the operator A is a linearizator of $L(\lambda)$.
Definition. The pencil $L(\lambda) = \lambda^2 I + \lambda B + C$ is called strong damped if

$$(Bf, f)^2 - 4(f, f)(Cf, f) > 0, \ f \in \mathcal{H}, \ f \neq 0.$$

The main result is the following
Theorem. Let A be uniformly positive in a Krein space $\widehat{\mathcal{H}}$. Then $A - \lambda I$ be linearizator of strong damped pencil if and only if $\alpha > \max\{\lambda : \lambda \in \sigma(A)\}$.

Optimal solutions to matrix-valued Nehari problems and related limit theorems

S. ter Horst

In a 1990 paper Helton and Young showed that under certain conditions the optimal solution of the Nehari problem corresponding to a finite rank Hankel operator with scalar entries can be efficiently approximated by certain functions defined in terms of finite dimensional restrictions of the Hankel operator. In this paper it is shown that these approximants appear as optimal solutions to restricted Nehari problems. The latter problems can be solved using relaxed commutant lifting theory. This observation is used to extent the Helton and Young approximation result to a matrix-valued setting. As in the Helton and Young paper the rate of convergence depends on the choice of the initial space in the approximation scheme.

Nonnegative continuous functions with rational Laplace transform

Margarita Tetlalmatzi-Montiel

29/05/2011

In this talk we address the following problem: *Suppose* $\Phi(s) = p(s)/q(s)$*, where* p *and* q *are relative prime polynomials with* $\deg p < \deg q$*, is continuous for* $s \geq 0$*. Find necessary and sufficient conditions on* p *and* q *so that* $\Phi(s)$ *is the Laplace transform of a nonnegative continuous function on* $[0, \infty]$. The solution of this problem is of interest in the study of the so called matrix exponential distributions in probability theory. Using elementary methods we present a complete solution when $\deg q \leq 3$ and significant progress when $\deg q = 4$; then we compare our results with those of M. Fackrell, et al., who solve the problem in terms of certain convex sets when $\deg q \leq 3$. The talk is based on work done with M. Bladt.

Numerical range of $C_0(N)$ contractions

D. Timotin

A conjecture of Halmos proved by Choi and Li states that the closure of the numerical range of a contraction on a Hilbert space is the intersection of the closure of the numerical ranges of all its unitary dilations. We show that for $C_0(N)$ contractions one can restrict the intersection to a smaller family of dilations. This generalizes a finite dimensional result of Gau and Wu. This presentation is based on joint work with Chafiq Benhida and Pamela Gorkin.

Operator approximation for optimal data compression

A. Torokhti

Let $X = (\Omega, \Sigma, \mu)$ be a probability space, $K_X = \{\mathbf{x}(t, \cdot) \in L^2(\Omega, \mathbb{R}^m) \mid t \in T\}$ and $K_Y = \{\mathbf{y}(t, \cdot) \in L^2(\Omega, \mathbb{R}^n) \mid t \in T\}$ where $T := [a,\ b] \subseteq \mathbb{R}$. Let $\mathbf{x}(t, \cdot)$ and $\mathbf{y}(t, \cdot)$ be a reference signal and an observed signal, respectively.

Most of the literature on the subject of optimal data compression discusses the properties of a data compression transform for an *individual* random signal $\mathbf{y}(t, \cdot)$. Clearly, in the case of arbitrarily large signal sets, K_X and K_Y, such an approach is not suitable since it implies an associated arbitrarily large set of transforms.

In this paper, for compression of data set K_Y, a *single* transform $\mathcal{F} : K_Y \to K_X$ is proposed based on a combination of ideas of piece-wise interpolation and the rank-constrained approximation. Transform $\mathcal{F}$ is presented in the form of a sum with p sub-transforms given, for $j = 1, \ldots, p$, by $\mathcal{F}_j[\mathbf{y}(t, \cdot)] = \mathbf{a}_j + \mathcal{G}_j[\mathbf{y}(t, \cdot)]$. Here, $\mathbf{a}_j \in L^2(\Omega, \mathbb{R}^m)$ and $\mathcal{G}_j$ is an operator represented by matrix $G_j \in \mathbb{R}^{m \times n}$ so that $[\mathcal{G}_j(\mathbf{y})](t, \omega) = G_j[\mathbf{y}(t, \omega)]$, for $\omega \in \Omega$. For $j = 1, \ldots, p$, $\mathbf{a}_j$ and $\mathcal{G}_j$ are determined from interpolation-like conditions on sets K_X and K_Y, and from rank-constrained problems. This device provides the transform flexibility to incorporate variation of observed data and leads to performance improvement compared to the known methods. The work is based on an extension of results in [1--5].

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Riordan arrays via the classical umbral calculus

Maria M. Torres

An exponential Riordan array is an infinite lower triangular matrix described by two formal exponential series. Riordan arrays were introduced as a generalization of the well known Pascal triangle and have been studied in connection with combinatorial identities and walk problems. In this talk, I will speak about a symbolic treatment of the exponential Riordan group based on the renewed approach to umbral calculus initiated by G.-C. Rota and B. D. Taylor and recently developed by E. Di Nardo and D. Senato among other authors. The talk is based on a joint work with J. Agapito, A. Mestre and P. Petrullo.

Operator ideals and domination

P. Tradacete

In the framework of Banach lattices a central topic is the study of relations between order and Banach space structure.
Given an operator ideal $\mathcal{I}$ we are interested in the following problems:

- **Domination problem:** *Let* $0 \leq S \leq T : E \to F$*. Under which conditions on the Banach lattices* E *and* F *does* $T \in \mathcal{I}$ *imply* $S \in \mathcal{I}$*?*

- **Power problem:** *Let* $0 \leq S \leq T : E \to E$*. Does there exist* $n \in \mathbb{N}$ *such that if* $T \in \mathcal{I}$ *then* $R^n \in \mathcal{I}$*?*

For instance, a classical result due to P. Dodds and D. Fremlin asserts that for the ideal of compact operators $\mathcal{K}$, the domination problem has a positive answer when E^* and F are order continuous and for the power problem $n = 3$ works (both being optimal). We will survey some old and new results for other operator ideals.

Singular integrals made easy

Sergei Treil

In the theory of singular integral operators significant effort is often required to rigorously define such an operator. This is due to the fact that the kernels of such operators are not locally integrable on the diagonal $s = t$, so the integral formally defining the operator T or its bilinear form $\langle Tf, g\rangle$ is not well defined (the integrand in not in L^1) even for ``nice" f and g.

However, as it turned out, the situation with interpretation of singular integral operators is much simpler, than it seems; to investigate the boundedness one only needs to study an elementary and well defined *restricted* bilinear form. While our result is not a replacement for the hard analysis still necessary to investigate the boundedness of a singular integral operator, it simplifies the setup significantly.

The main idea is embarrassingly simple, and we should be ashamed that we did not arrive to it much earlier. In our defense we can only say that this idea was overlooked by the harmonic analysts before us.

The talk is based on a joint work with C. Liaw.

Singular indefinite Sturm-Liouville problems

Carsten Trunk

We consider singular Sturm-Liouville operators with an indefinite weight, i.e. operators of the form

$$A = \operatorname{sgn}(\cdot)\left(-\frac{d^2}{dx^2} + V\right) \tag{19}$$

on $\mathbb{R}$. It is assumed that V is a real-valued, locally integrable potential such that the limits $\lim_{x\to\pm\infty} V(x)$ exist and are finite.

Closely related to the operator A in (19) is the *definite* Sturm-Liouville operator

$$B = -\frac{d^2}{dx^2} + V,$$

which is selfadjoint and semi-bounded in the Hilbert space $L^2(\mathbb{R})$.

Our aim is to describe the spectrum of the indefinite Sturm-Liouville operator A. We will discuss four cases which are organized according to the location of the spectrum $\sigma(B)$ of the definite Sturm-Liouville operator B and the essential spectrum $\sigma_{ess}(B)$ of B:

1. $\sigma(B) \subset (0, \infty)$. This case is called *left-definite*.

2. $\sigma(B) \cap (-\infty, 0)$ consists of finitely many points and $\sigma_{ess}(B) \subset [0, \infty)$.

3. $\sigma(B) \cap (-\infty, 0)$ consists of infinitely many points and $\sigma_{ess}(B) \subset [0, \infty)$.

4. No further restrictions on the spectrum/essential spectrum of B.

Hurwitz rational functions and stable matrices

M. Tyaglov

Let R be the following rational function

$$R(z) = \frac{p(z)}{q(z)} = t_0 z^{r-m} + t_1 z^{r-m-1} + t_2 z^{r-m-2} + \dots,$$

where $h(z)$ and $g(z)$ are *real* polynomials:

$$\begin{aligned} p(z) &= b_0 z^r + b_1 z^{r-1} + \dots + b_{r-1} z + b_r, \qquad b_0 > 0,\ b_j \in \mathbb{R}, \\ q(z) &= c_0 z^m + c_1 z^{m-1} + \dots + c_{m-1} z + c_m, \qquad b_0 > 0,\ b_j \in \mathbb{R}. \end{aligned}$$

and $r, m \in \mathbb{N} \bigcup \{0\}$ $(r + m > 0)$.

The function R is called *Hurwitz* if the polynomials $p(z)$ and $q(-z)$ are Hurwitz stable (or one of them is Hurwitz stable while the second one is a constant.

Theorem 1. *The function R is Hurwitz if and only if all the leading principal minors of the infinite Hurwitz matrix*

$$\mathcal{H}(R) = \begin{pmatrix} t_1 & t_3 & t_5 & t_7 & t_9 & \dots \\ t_0 & t_2 & t_4 & t_6 & t_8 & \dots \\ 0 & t_1 & t_3 & t_5 & t_7 & \dots \\ 0 & t_0 & t_2 & t_4 & t_6 & \dots \\ 0 & 0 & t_1 & t_3 & t_5 & \dots \\ 0 & 0 & t_0 & t_2 & t_4 & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots \end{pmatrix}$$

are positive up to order $n = \deg p + \deg q$.

A criterion of Hurwitzness of the function R in terms of the coefficients of the polynomials p and q and *a new type of matrices* are presented. We also discuss some application of Hurwitz rational functions to stable matrices, that is, matrices whose eigenvalues have negative real part.
The talk is based on a joint work with Yu. Barkovsky.

Composition operators on BMOA and related spaces

H-O. Tylli

Let $BMOA$ be the Banach space of analytic functions $f\colon D \to \mathbb{C}$ whose boundary values have bounded mean oscillation on the unit circle, and $VMOA$ the closed subspace consisting of the functions having vanishing mean oscillation. Let C_φ be the composition operator $f \mapsto f \circ \varphi$, where φ is an analytic self-map of the unit disc D.
I will describe several characterizations of the (weak) compactness of C_φ on $BMOA$ and $VMOA$, including one which simplifies the conditions on the Nevanlinna counting function of φ due to W. Smith (Proc. AMS, 1999).
This talk is based on joint work with Jussi Laitila (Essex), Pekka J. Nieminen (Helsinki) and Eero Saksman (Helsinki), to appear in Complex Anal. Oper. Theory.

Orthogonal functions and matrix computations

M. Van Barel

Orthogonal polynomials on the real line satisfy a three term recurrence relation. This relation can be written in matrix notation by using a tridiagonal matrix. Similarly, orthogonal polynomials on the unit circle satisfy a Szegő recurrence relation that corresponds to an (almost) unitary Hessenberg matrix. It turns out that orthogonal rational functions with prescribed poles satisfy a recurrence relation that corresponds to diagonal plus semiseparable matrices. This leads to efficient algorithms for computing the recurrence parameters for these orthogonal rational functions by solving corresponding linear algebra problems. In this talk we will study several of these connections between orthogonal functions and matrix computations and give some numerical examples illustrating the numerical behaviour of these algorithms.
Let $(e^{tA})_{t\geq 0}$ be a C_0-contraction semigroup on a 2-smooth Banach space E, let $(W_t)_{t\geq 0}$ be a cylindrical Brownian motion in a Hilbert space H, and let $(g_t)_{t\geq 0}$ be a progressively measurable process with values in the space $\gamma(H,E)$ of all γ-radonifying operators from H to E. We prove that for all $0 < p < \infty$ there exists a constant C, depending only on p and E, such that for all $T \geq 0$ we have

$$\mathbb{E} \sup_{0\leq t\leq T} \Big\| \int_0^t e^{(t-s)A} g_s \, dW_s \Big\|^p \leq C\mathbb{E}\Big(\int_0^T \|g_t\|^2_{\gamma(H,E)} \, dt \Big)^{\frac{p}{2}}.$$

For $p \geq 2$ the proof is based on the observation that $\psi(x) = \|x\|^p$ is Fréchet differentiable and its derivative satisfies the Lipschitz estimate

$$\|\psi'(x) - \psi'(y)\| \leq C(\|x\| + \|y\|)^{p-2}\|x - y\|.$$

The extension to $0 < p < 2$ proceeds via Lenglart's inequality. This is joint work with Jiahui Zhu.

A maximal inequality for stochastic convolutions

Jan van Neerven

A Commutator Approach to Absolute Continuity

J. Dombrowski

Commutator equations provide a useful technique for studying the absolute continuity of spectral measures associated with both bounded and unbounded self-adjoint operators. This technique, which has its roots in the work of C. R. Putnam, will be used to study the spectral properties of certain subclasses of unbounded self-adjoint Jacobi matrix operators. A Jacobi matrix operator can be represented by a tridiagonal matrix with diagonal sequence $\{b_n\}$ and subdiagonal sequence $\{a_n\}$, acting on a dense subset of $\ell^{2\cdot}$. Given such an unbounded Jacobi matrix operator, with appropriate growth conditions imposed on the defining sequences to assure that the operator is self-adjoint, an appropriate bounded operator is chosen for the commutator equation. The structure of the resulting commutator, and its action on a strategic set of vectors related to the spectral decomposition of the given operator, will be discussed, and then used to obtain results on absolute continuity.

Two-dimensional singular integral operators via poly-Bergman spaces

N. Vasilevski

We discuss a direct and very transparent connection between the poly-Bergman type spaces on the upper half-plane Π and the action of the following two-dimensional singular integral operators

$$(S_\Pi\varphi)(z) = -\frac{1}{\pi}\int_\Pi \frac{\varphi(\zeta)\,dv(\zeta)}{(\zeta - z)^2} \quad \text{and} \quad (S_\Pi^*\varphi)(z) = -\frac{1}{\pi}\int_\Pi \frac{\varphi(\zeta)\,dv(\zeta)}{(\overline{\zeta} - \overline{z})^2},$$

where dv is the standard Lebesgue plane measure.
We show that both S_Π and S_Π^* admit a simple (functional) model: *each of them is unitary equivalent to the direct sum of two unilateral shifts, forward and backward, both taken with infinite multiplicity*.
This fact leads to an easy access to the majority of properties of these operators.

Maximal regularity for stochastic evolution equations

M.C. Veraar

In this talk we present a recently developed theory of maximal regularity for stochastic evolution equation. The theory is based on beautiful interplays between operator theory (functional calculus), geometry of Banach spaces (UMD), harmonic analysis (square functions, maximal functions), and stochastic analysis. Before our work, positive results were only known in very special cases such as L^2-maximal regularity in Hilbert spaces. The talk is based on joint work with Jan van Neerven and Lutz Weis.

The spectral theory for periodic delay equations revisited

Sjoerd Verduyn Lunel

In this talk we apply some recent abstract results and present necessary and sufficient conditions for completeness of the span of eigenvectors of Hilbert-Schmidt operators of order one. In particular, we will present detailed results if the operator under study is a finite rank perturbation of a Volterra operator. In this case, one can define a characteristic matrix (in the sense of Kaashoek and Verduyn Lunel, TAMS 1992). This allows a detailed spectral analysis of the operator and an explicit computation of the resolvent operator so that the necessary and sufficient conditions for completeness can be verified. As an application, we can study the spectral properties of the monodromy operator of a periodic functional differential equations in a fairly general setting.

Noncommutative functions and their power series expansions

V. Vinnikov

A *noncommutative (nc) space* $\mathcal{V}_{\text{nc}}$ over a vector space $\mathcal{V}$ is the disjoint union of the spaces of $n \times n$ matrices over $\mathcal{V}$ for all n; if $\mathcal{V}$ is an operator space, $\mathcal{V}_{\text{nc}}$ carries a natural topology, that we call the uniformly-open topology, induced by the sequence of matrix norms. A subset of $\mathcal{V}_{\text{nc}}$ is called a *nc set* if it is closed under direct sums. A function f from a nc set in $\mathcal{V}_{\text{nc}}$ to $\mathcal{W}_{\text{nc}}$, for vector spaces $\mathcal{V}$ and $\mathcal{W}$, is called a *nc function* if it maps $n \times n$ matrices to $n \times n$ matrices for all n and satisfies certain compatibility conditions as we vary the matrix size n --- namely, if it respects direct sums and similarities, or equivalently, intertwinings. Polynomials, rational functions, and power series in d noncommuting inderterminates are examples of nc

functions on (nc subsets of) the noncommutative space over $\mathbb{C}^d$. Other natural examples arise in (operator-valued) free probability.
It follows from the compatibility conditions (the respect of direct sums and similarities) that nc functions have very strong regularity properties: if a nc function is locally bounded (in the uniformly-open topology) then it is necessary continuous (in that topology) and analytic in every matrix size n (as a function from an open subset of $\mathcal{V}^{n\times n}$ to $\mathcal{W}^{n\times n}$). It also admits a nc power series expansion. If $\dim \mathcal{V} = d < \infty$ and the center is a scalar point, these are simply power series in d noncommuting indeterminates. The main emphasis of the talk will be to discuss the structure of the power series in the general situation and their relation to some standard constructions in operator space theory such as ``faux" products and completely bounded multilinear mappings.
This is a joint work with Dmitry Kaliuzhnyi-Verbovetskyi.

Realization and interpolation for noncommutative functions

V. Vinnikov

I will discuss concepts, conjectures, and preliminary results generalizing the classical results on contractive analytic functions (or analytic functions with positive imaginary part) on the unit disc (or the upper halfplane) to the setting of noncommutative functions, that is functions defined on appropriate matrices of all sizes over a given vector space $\mathcal{V}$, which satisfy certain compatibility conditions as we vary the size of matrices --- they respect direct sums and similarities. The space $\mathcal{V}$ here is an operator space (or an operator system), so that we can define the noncommutative unit disc (or the noncommutative upper halfplane). In case $\mathcal{V}$ is finite dimensional, we recover essentially the results of Ball--Groenewald--Malakorn on the d-variable noncommutative Schur class whose elements are viewed as formal power series in d noncommuting indeterminates.
This is a joint journey with Joe Ball, Serban Belinschi, and Mihai Popa.

Spectral Analysis of Integrodifferential Equations in Hilbert space

V. Vlasov

We study integrodifferential equations with unbounded operator coefficients in a Hilbert space H.

$$\frac{d^2u(t)}{dt^2} + K(0)A^2u(t) + \int_0^t K'(t-s)A^2u(s)ds = f(t), \quad t \in \mathbb{R}_+, \tag{20}$$

$$u(+0) = \varphi_0, \quad u^{(1)}(+0) = \varphi_1, \tag{21}$$

where A is a self-adjoint positive operator with compact inverse acting on H, kernel $K(t)$ is a scalar convex downwards decreasing function which belongs to the space $W_1^1(\mathbb{R}_+)$.
The equation (20) is an abstract form of Gurtin-Pipkin integrodifferential equation, which describes heat propagation in media with memory and sound propagation in viscoelastic media; it also arises in homogenization problems in porous media (Darcy law).
We obtain the results on correct solvability of the problem (20), (21) in weighted Sobolev spaces on a positive semiaxis $\mathbb{R}_+$. Additionally assuming that kernel $K(t)$ has the form

$$K(t) = \sum_{j=1}^{\infty} \frac{c_j}{\gamma_j} e^{-\gamma_j t}, \tag{22}$$

where $c_j > 0$, $\gamma_{j+1} > \gamma_j > 0$, $j \in \mathbb{N}$, $\gamma_j \to +\infty$ $(j \to +\infty)$ We provide the spectral analysis of the operator-function $L(\lambda)$ which is a symbol of the equation (20).
On the base of distribution for eigenvalues of the operator-function $L(\lambda)$ we obtain the representations of the solutions of integrodifferential equations mentioned above. These representations are the series of exponentals corresponding to eigenvalues of the operator-function $L(\lambda)$.
The talk is based on a joint work with N. Rautian.

Zero products of Toeplitz operators

D. Vukotić

In their seminal paper in 1963/64, Brown and Halmos proved what is now a classical fact: the product (*i.e.*, composition) of two Toeplitz operators on the Hardy space H^2 is the zero operator if and only if one of the factors is zero. Subsequently, Halmos had asked whether this continues to hold for products of an arbitrary number of Toeplitz operators. There has been relatively little progress on this problem until recently and the conjecture has only been proved for up to 6 operators. In a joint paper with A. Aleman, we settled the question for n operators in the affirmative by proving the result in a more general context of vector-valued Hardy spaces (with arbitrary exponents larger than one). The vector-valued approach was a key ingredient, together with a lemma due to Douglas and the concept of nearly invariant subspaces as studied by Hitt and Sarason. In this talk, we will give a brief overview of the main ideas of our proof.

Weighted composition operators between conformally invariant spaces

D. Vukotić

In this talk we review several new results characterizing the bounded and compact weighted composition operators between certain conformally invariant spaces of analytic functions in the disk, especially those from the space of bounded analytic functions into the disk algebra or analytic Besov spaces.

Spectral properties of an indefinite 1-dimensional p-Laplace operator

B.A. Watson

For a weighted Sturm-Liouville-type problem of the form

$$-\Delta_p y = (p-1)(\lambda r - q)\text{sgn}y|y|^{p-1}, \quad \text{on } (0,1)$$

with Sturmian-type boundary conditions (Δ_p being the p-Laplacian), we examine the structure, asymptotics and parametric dependence of the eigenvalues, together with properties of the eigenfunctions such as oscillation and interlacing of zeros. We discuss definitions and consequences of left and right (semi-) definiteness, and also the fully indefinite case.
This talk is based on joint work with P.A. Binding and P.J. Browne.

The truncated matrix valued multivariable K-moment problem

Hugo J. Woerdeman

The matrix-valued truncated K-moment problem on $\mathbb{R}^d$ requires necessary and sufficient conditions for a multisequence of Hermitian matrices $\{S_\gamma\}_{\gamma\in\Gamma}$, where Γ is a finite subset of $\mathbb{N}_0^d$, to be the corresponding moments of a positive matrix-valued Borel measure σ and also the support of σ must lie in some given non-empty set $K \subseteq \mathbb{R}^d$, i.e.

$$S_\gamma = \int_{\mathbb{R}^d} \xi^\gamma d\sigma(\xi), \quad \gamma \in \Gamma, \tag{23}$$

and

$$\text{supp } \sigma \subseteq K. \tag{24}$$

In this joint work with David Kimsey we obtain necessary and sufficient conditions for the existence of a finitely atomic measure which satisfies (23) and (24). In particular, our result can handle the case when the indexing set that corresponds to the powers of total degree at most $2n+1$. We will also discuss a similar result in the complex setting.

The uniqueness of solutions to A. Horn's problem

Hugo J. Woerdeman

Given n-tuples of real numbers λ, μ, and ν, A. Horn's problem asks for Hermitian matrices A and B so that $\sigma(A) = \lambda, \sigma(B) = \mu$, and $\sigma(A + B) = \nu$, where $\sigma(\cdot)$ denotes the spectrum. In this paper we are exploring when the solution matrices A and B are unique up to unitary similarity. In the case that λ, μ, and ν lie in $(\alpha\mathbb{Q} + \beta)^n$ for some real α and β we prove that uniqueness occurs exactly when there exists a unique Littlewood-Richardson function associated with λ, μ, and ν. In particular, if λ, μ, and ν are partitions, we prove that the uniqueness of Littlewood-Richardson function can be replaced by corresponding Littlewood-Richardson coefficient $C^{\nu}_{\lambda,\mu}$ equals 1. Also, we conjecture that the uniqueness of Littlewood-Richardson function with λ, μ, and ν and the uniqueness of the solution up to unitary similarity of Horn's problem occur simultaneously for any real n-tuples λ, μ, and ν. Aside from the above case, we provide another piece of evidence for this conjecture, namely for the case when A or B is of rank one. This work is joint work with Lei Cao.

Hilbert space operators with and without Fölner sequences and matrices far from normal ones

Dmitry Yakubovich

Let $T \in \mathcal{L}(\mathcal{H})$ be a bounded linear operator acting on a complex separable Hilbert space $\mathcal{H}$. An increasing sequence of non-zero finite rank orthogonal projections $\{P_n\}_{n\in\mathbb{N}}$ strongly converging to $I_{\mathcal{H}}$ is called a Følner sequence for T, if

$$\lim_n \frac{\|TP_n - P_nT\|_p}{\|P_n\|_p} = 0, \qquad p \in \{1, 2\}$$

(this condition holds for $p = 1$ iff it holds for $p = 2$). In this work, we address the following question: *which classes of operators T have a Følner sequence?* We prove, applying Brown--Douglas--Fillmore, that all essentially normal operators do. We also exhibit examples of an operators that have no Følner sequence. These examples are based on certain properties of the Cuntz algebras $\mathcal{O}_n$, $n \geq 2$.

In the second part of this work, we give an asymptotical statement about finite matrices, which is equivalent to the fact that not all operators have Følner sequences. In a sense, this statement is about the existence of a sequence of complex $n \times n$ matrices M_n, which are far from normal ones. Here we use a reduction of a square matrix to a certain ``wedge" form by means of a unitary similarity transformation.

The talk is based on a joint work with Fernando Lledó. The work was supported by grants MTM2009-12740-C03-01 and MTM2008-06621-C02-01/MTM of the Ministry of Science and Innovation of Spain and FEDER.

Operators of Fractional Cauchy Transform Spaces

El-Bachir Yallaoui

For $\alpha > 0$ and $z \in \mathbf{D}$, we define the space of fractional Cauchy transform spaces F_α to be the family of all functions $f(z)$ such that $f(z) = \int_{\mathbf{T}} K_x^\alpha(z)d\mu(x)$ where the Cauchy kernel $K_x(z)$ is given by $K_x(z) = \dfrac{1}{1-\overline{x}z}$**T** is the unit circle, **M** represents the set of all complex--valued Borel measures on **T** and μ varies over all measures in **M**. The class F_α is a Banach space with respect to the norm $\|f\|_{F_\alpha} = \inf \|\mu\|_{\mathbf{M}}$. We will present some results on composition (weighted) operators and multiplication operators for values of $\alpha \geq 1$.

Uncertainty relations for metric adjusted skew information and correlation measure

K. Yanagi

Let $M_{n,sa}(\mathbb{C})$ be the set of all $n \times n$ complex self-adjoint matrices and let $M_{n,+,1}(\mathbb{C})$ be the set of strictly positive density matrices. Let $\mathcal{F}_{op}$ be the class of functions which are symmetric, normalized operator monotone. For $f \in \mathcal{F}_{op}$ satisfying $f(0) \neq 0$ we set $\tilde{f}(x) = 1/2\{(x+1)-(x-1)^2 f(0)/f(x)\},\ x > 0$. We define the class of monotone metrices (also said quantum Fisher informations) by the following formula: $\langle A, B\rangle_{\rho,f} = Tr(A \cdot m_f(L_\rho, R_\rho)^{-1}(B))$, where $L_\rho(A) = \rho A, R_\rho(A) = A\rho$, $m_f(A,B) = A^{1/2}f(A^{-1/2}BA^{-1/2})A^{1/2}$ $(A, B \in M_{n,sa}(\mathbb{C}))$. For $A, B \in M_{n,sa}$ and $\rho \in M_{n,+,1}$ we define the following quantities:

$$Corr_\rho^f(A,B) = \frac{f(0)}{2}\langle i[\rho, A], i[\rho, B]\rangle_{\rho,f}, \quad I_\rho^f(A) = Corr_\rho^f(A,A),$$

$$U_\rho^f(A) = \sqrt{V_\rho(A)^2 - (V_\rho(A) - I_\rho^f(A))^2}.$$

The quantity $I_\rho^f(A)$ is known as metric adjusted skew information and $Corr_\rho^f(A,B)$ is also known as metric adjusted correlation measure. In this talk, we give the following theorem.
Theorem For $f \in \mathcal{F}_{op}$ satisfying $f(0) \neq 0$, if

$$\frac{x+1}{2} + \tilde{f}(x) \geq 2f(x),$$

then it holds

$$U_\rho^f(A) \cdot U_\rho^f(B) \geq f(0)|Tr(\rho[A,B])|^2,$$

$$U_\rho^f(A) \cdot U_\rho^f(B) \geq 4f(0)|Corr_\rho^f(A,B)|^2,$$

where $A, B \in M_{n,sa}(\mathbb{C})$ and $\rho \in M_{n,+,1}(\mathbb{C})$.
The talk is based on a joint work with S. Furuichi.

Uniqueness of Preduals for some Analytic Function Spaces

Onur Yavuz

A well known result of Ando says that $H^\infty(\mathbb{D})$ has a unique predual. Let A be a finitely connected domain in the plane. We show that $H^\infty(A)$ has a unique isometric predual. We also prove a couple of theorems about the structure of the unique predual.
This is a joint work with Mohan Ravichandran.

The structured spectrum of an operator

Nicholas Young

In studying the spectral theory of an operator T on a Hilbert space H one may formally replace the field of complex numbers by a unital C^*-subalgebra of $\mathcal{L}(H)$. Thus, if E is such a C^*-subalgebra, we may define the spectrum of T relative to E by

$$\sigma_E(T) = \{\Lambda \in E : T - \Lambda \text{ is not boundedly invertible in } \mathcal{L}(H)\} \subset E.$$

This notion (with very special E) has arisen independently in two branches of function theory, one stemming from H^∞ control and the other from the generalization of Loewner's Theorem (about matrix monotone functions) to several variables. We shall consider the issues that arise, illustrated by two theorems, one from each of these two areas.

Parametrizing Structure Preserving Transformations of Matrix Polynomials

I. Zaballa