

# Simplifying canard theory with piecewise-linear systems

## Applications to neuronal dynamics

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Canards are special solutions of dynamical systems with multiple timescales. They are special for at least two reasons: first, they follow repelling dynamical objects (slow manifolds) for long time intervals; second, they are associated with exponentially narrow parameter variations typically called *explosions*. Since they were discovered in the van der Pol oscillator at the end of the 1970s, they have been the subject of intense mathematical research, giving rise to fine analytical studies (e.g. using matched asymptotics or geometric desingularisation techniques). They are also very challenging numerically as they arise in stiff systems. Canard dynamics has become increasingly popular over the years also because these objects naturally appear in mathematical models that are relevant to a plethora of application areas, including neuroscience, chemistry, plasma physics and population dynamics, to name a few.

In this talk, I will give an overview of recent results aiming to revisit canard theory from the perspective of slow-fast piecewise-linear (PWL) dynamical systems. This framework is well known to retain all salient features of nonlinear systems while being particularly amenable to analysis, including precise quantitative estimates. Taking several examples, I will attempt to show how the PWL approach helps to simplify canard theory without losing any of its rich dynamics. I will present application of these results to neuron models,

where the PWL approach is relevant since neurons are modelled in first approximation as circuits and PWL systems are notoriously very efficient to capture circuit dynamics. Starting with planar systems, where canards organise the transition between rest and spiking states, I will then consider three-dimensional cases where complex oscillatory behaviours mixing subthreshold oscillations and spikes (referred to as mixed-mode oscillations or MMOs) can also be understood by means of canard solutions and captured in a minimal PWL setting.

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