

# Complexity in discrete-time population models: other bifurcation diagrams are possible

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It is well-known that simple deterministic models governed by one-dimensional maps can display chaotic behavior. Pioneering work in this direction has been made based on discrete-time population models with overcompensatory growth, where increasing the growth rate leads to a period-doubling bifurcation route to chaos which is represented by usual bifurcation diagrams [8].

In many population models, it is more interesting the response of population abundance to changes in other parameters, such as harvesting effort in exploited populations or culling intensity in the control of plagues. Managers can control these parameters at some extent, searching for desirable outcomes (for example, a maximum sustainable yield in exploited populations, or preventing the risk of extinction in endangered species). It has been observed that an increasing mortality rate may give rise to new phenomena, sometimes counterintuitive, such as sudden collapses [5, 10], stability switches [2, 6], and the hydra effect (a population increasing in response to an increase in its per-capita mortality rate) [1, 4, 6].

In this talk, we review these phenomena in simple population models subject to different harvest strategies, and we highlight the importance of several often underestimated issues that are crucial for management, such as census timing [4], intervention time [3, 9], and carry-over effects [7, 9].

**Keywords:** population dynamics, bifurcation, stability, overcompensation

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## References

- [1] P. A. Abrams. When does greater mortality increase population size? The long story and diverse mechanisms underlying the hydra effect. *Ecol. Lett.*, 12:462–474, 2009.
- [2] C. N. Anderson *et al.* Why fishing magnifies fluctuations in fish abundance. *Nature*, 452:835–839, 2008.
- [3] B. Cid, F. M. Hilker, and E. Liz. Harvest timing and its population dynamic consequences in a discrete single-species model. *Math. Biosci.*, 248:78–87, 2014.
- [4] F. M. Hilker, and E. Liz. Harvesting, census timing and “hidden” hydra effects. *Ecol. Complex.*, 14:95–107, 2013.
- [5] E. Liz. Complex dynamics of survival and extinction in simple population models with harvesting. *Theor. Ecol.*, 3:209–221, 2010.
- [6] E. Liz, and A. Ruiz-Herrera. The hydra effect, bubbles, and chaos in a simple discrete population model with constant effort harvesting. *J. Math. Biol.*, 65:997–1016, 2012.
- [7] E. Liz, and A. Ruiz-Herrera. The impact of carry-over effects in population dynamics and management. *Preprint*, 2016.
- [8] R. M. May. Simple mathematical models with very complicated dynamics. *Nature*, 261:459–467, 1976.
- [9] I. I. Ratikainen *et al.* When density dependence is not instantaneous: theoretical developments and management implications. *Ecol. Lett.*, 11:184–198, 2008.
- [10] S. Sinha, and S. Parthasarathy. Unusual dynamics of extinction in a simple ecological model. *Proc. Natl. Acad. Sci. USA*, 93:1504–1508, 1996.