We study a cancer model given by a three-dimensional system of ordinary differential equations, depending on eight parameters, which describe the interaction among healthy cells, tumor cells and effector cells of immune system. The model was originally proposed in [1], where it was shown that the system exhibit a chaotic attractor, for certain fixed values of the parameters. Some topological properties of this attractor was studied in [2]. In this work we study how such a chaotic attractor is formed. More precisely, by varying one of the parameters of the system, we prove that a supercritical Hopf bifurcation occurs, leading to the creation of a stable limit cycle. By studying the continuation of this limit cycle, increasing the parameter value from the Hopf point, we numerically found the occurrence of a cascade of period-doubling bifurcations which leads to the formation of the chaotic attractor shown to exist in [1]. Furthermore, analyzing this dynamical behavior from the biological point of view we show that, after the occurrence of chaotic dynamics, both the tumor cells and the immune system cells vanish and only the healthy cells survive. So in this model and for the set of parameters considered, the total cure of the disease is possible, after a kind of battle between the immune system and cancer cells, represented by the period-doubling bifurcations and the chaotic dynamics. Indeed, the parameter varied in the performed bifurcation analysis represents exactly the interaction between the cancer cells and the immune system cells. The results obtained are presented in detail in [3]. Related results have recently been presented in [4].

References


