# First Joint Meeting between the RSME and the AMS

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# Abstracts

## Session 02

## Algebraic Geometry

Organizers:

Félix Delgado (Universidad de Valladolid)
Andrey Todorov (University of California at Santa Cruz)

# Index of Abstracts

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## *Invariants of Curves and Surfaces from Abel to Jung and Severi*

**S. S. Abhyankar** (Purdue University)

Soon after the invention of calculus by Newton and Leibnitz in the seventeenth century, the problem of finding the arc length of an ellipse gave rise to integrals of square roots of cubic polynomials. Although these could not be integrated in terms of known functions, in the eighteenth century Euler and Legendre created an elaborate theory about them. In the early part of the nineteenth century, first Abel and then Jacobi solved the mystery by inverting these and other integrals of algebraic functions. This led them to consider the genus of the corresponding algebraic curve as a birational invariant. In the latter part of that century Riemann, Weierstrass and Dedekind constructed a detailed theory of algebraic functions of one variable around this invariant. Soon afterwards, at the juncture of the nineteenth and twentieth centuries, a series of analogous invariants of surfaces were introduced by many pioneers such as Salmon in Ireland, Noether, Zeuthen and Jung in Germany and Segre, Castelnuovo, Enriques and Severi in Italy. I shall sketch this classical theory and its application to unsolved problems such as the Jacobian problem.

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## *Geometry of the plane Cremona maps*

**M. Alberich Carramiñana** (Universidad Politécnica de Cataluña)

Given a plane Cremona map $\Phi : \mathbb{P}^2_1 \dashrightarrow \mathbb{P}^2_2$, the two-dimensional linear system in $\mathbb{P}^2_1$ (homaloidal net) $\mathcal{H} = \Phi^*(|\mathcal{L}|)$ which is the pullback of the net of lines $|\mathcal{L}|$ of $\mathbb{P}^2_2$ determines $\Phi$ modulus projectivity. To $\mathcal{H}$ we associate the weighted cluster $(K, \mu)$, where $K$ are the base points (proper or infinitely near) of $\mathcal{H}$ and $\mu$ assigns to each $p \in K$ the multiplicity $\mu_p$ at $p$ of generic curves in $\mathcal{H}$. If all the points of $K$ are proper, the map $\Phi$ is called simple. If $\Phi$ and $\Phi^{-1}$ are both simple, they are called bisimple. We will show some problems concerning plane Cremona maps from the viewpoint of the geometry of the singularities of their base points. This will include the classic problems, which restricted to the case of bisimple maps; arithmetical versus geometrical questions; singularities of the inverse map; distinguished elements of the homaloidal net; composition and Noether's factorization theorem.

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## *Rationality of moduli spaces of stable vector bundles*

**L. Costa** (Universidad de Barcelona)

According to the general philosophy which claims that moduli spaces $M_X(r; c_1, c_2)$ of rank $r$ stable vector bundles on an algebraic variety $X$ inherits a lot of geometrical properties of $X$, the moduli spaces $M_{\mathbf{P}^2}(r; c_1, c_2)$ are expected to be rational. In this talk, we will consider this question and in particular, using elementary transformations and the notion of stably rationality, we will prove that the moduli space $M_{\mathbf{P}^2}(r; c_1, c_2)$ is rational in a huge number of cases.

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## *Integrable systems, matrix models, and open GW invariants*

**R. Donagi** (University of Pennsylvania)

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### *On the geometric Langlands correspondence*

**E. Frenkel** (University of California, Berkeley)

Let $X$ be a smooth, complete, geometrically connected curve defined either over a finite field of complex field. The geometric Langlands conjecture states that to each irreducible rank $n$ local system $E$ on $X$ one can attach a perverse sheaf on the moduli stack of rank $n$ bundles on $X$ (irreducible on each connected component), which is a Hecke eigensheaf with respect to $E$. In a joint work D.Gaitsgory and K.Vilonen and the speaker the geometric Langlands conjecture was derived from a certain vanishing conjecture. The vanishing conjecture was in turn also proved by the same authors when $X$ is defined over a finite field and, by a different method, by D.Gaitsgory when $X$ is defined both over a finite field and over the complex field. This gives a proof of the geometric Langlands conjecture for $GL(n)$ in both cases.

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### *Motivic Igusa zeta function and monodromy conjecture*

**I. Luengo Velasco** (Universidad Complutense)

In the 70's Igusa used $p$-adic integration to associate to a polynomial $f \in Z[x_1, \dot{;} x_n]$ a zeta function $Z_f(s)$, which encoded arithmetic properties of $f$, like number of solution of congruences. He conjectured that each pole of $Z_f(s)$, gives an eigenvalue of the complex monodromy of a Milnor fibration of $f$ (**monodromy conjecture**). Denef and Loeser give a motivic version of $Z_f(s)$, by using motivic integration in spaces of arcs.

In this talk we will review recent progress on the monodromy conjecture for Igusa's zeta function and and its motivic version, including our own results in work with E. Artal-Bartolo, Pierrette Cassou-Nogués and A. Melle-Hernández.

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### *Tetrahedral Curves*

**J. Migliore** (University of Notre Dame)

This is joint work with Uwe Nagel. Let $\lambda_{i,j}$ be the line in $\mathbb{P}^3$ defined by $x_i = x_j = 0$, with $0 \le i < j \le 3$. There are six such lines. For any $k$, the $k$-th infinitesimal neighborhood of $\lambda_{i,j}$ is the subscheme of $\mathbb{P}^3$ defined by the $(k+1)$-st power of the ideal $(x_i, x_j)$. This subscheme is always arithmetically Cohen-Macaulay (ACM). However, the union of two or more such subschemes is not necessarily ACM. Its ideal is an unmixed monomial ideal of height two. Schwartau described when certain such curves are ACM, namely he considered only curves supported on a certain four of the six lines. We consider the more general set of curves, supported on all six lines, and study the possible deficiency modules of such a curve. In particular we see when such a curve can be ACM. The first trick is the realization that there is a particular reduction possible that preserves the deficiency module. We then describe the curves that are minimal with respect to this reduction: we provide a simple algorithm (involving only integers) which computes the minimal curve in the class of a given tedrahedral curve. We also give the minimal free resolutions of these ideals. They turn out to be minimal also in the sense of Lazarsfeld and Rao.

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## *Dimension of families of determinantal schemes*

**R. M. Miró-Roig** (Universidad de Barcelona)

(Joint work with Jan Kleppe)

Given integers $a_0, a_1, ..., a_{t+c-2}$ and $b_1, ..., b_t$ we denote by $W(\underline{b}; \underline{a}) \subset\ Hi^p(\mathbf{P}^{n+c})$ (resp. $W_s(\underline{b}; \underline{a})$) the locus of good (resp. standard) determinantal schemes $X \subset \mathbf{P}^{n+c}$ of codimension $c$ defined by the maximal minors of a $t \times (t+c-1)$ matrix $(f_{ij})^{i=1,...,t}_{j=0,...,t+c-2}$ where $f_{ij} \in k[x_0, ..., x_{n+c}]$ is a homogeneous polynomial of degree $a_j - b_i$.

In the talk I will address the following three problems :

(1) To determine the dimension of $W(\underline{b}; \underline{a})$ and $W_s(\underline{b}; \underline{a})$ in terms of $a_j$ and $b_i$;

(2) To determine whether the closure of $W(\underline{b}; \underline{a})$ is an irreducible component of $Hi^p(\mathbf{P}^{n+c})$; and

(3) To determine when $Hi^p(\mathbf{P}^{n+c})$ is generically smooth along $W(\underline{b}; \underline{a})$.

Concerning question (1) I will give an upper bound for the dimension of $W(\underline{b}; \underline{a})$ and $W_s(\underline{b}; \underline{a})$ which works for all integers $a_i$ and $b_j$. We conjecture that this bound is sharp. The conjecture is proved for $2 \leq c \leq 5$, and for $c \geq 6$ under some restriction on $a_i$ and $b_j$. For questions (2) and (3) we have an affirmative answer for $2 \leq c \leq 4$ and $n \geq 2$, and for $c \geq 5$ under certain numerical assumptions.

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## *Polyhedrality of the cone of curves of a rational surface*

**F. J. Monserrat Delpalillo** (Universidad Jaume I)

We give a sufficient condition for the polyhedrality of the cone of curves associated to a smooth projective rational surface $Z$. This condition depends on the proximity relations obtained from the birational map which gives $Z$ from a relatively minimal surface.

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## *Equations of Hurwitz schemes in the infinite Grassmannian*

**J. M. Muñoz Porras** (Universidad de Salamanca)

The main result proved in the paper is the computation of the explicit equations defining the Hurwitz schemes of coverings with punctures as subschemes of the Sato infinite Grassmannian. As an application, we characterize the existence of certain linear series on a smooth curve in terms of soliton equations.

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## *On sections with isolated zeroes of twisted vector bundles*

**A. Campillo** (Universidad de Valladolid)

**J. Olivares Vazquez*** (CIMAT, México)

Let $E \longrightarrow M$ be a holomorphic rank $n$ vector bundle over a compact Kähler manifold of dimension $n$, having a positive (or ample) line bundle $L \longrightarrow M$ and consider a global section $s$, with isolated singularities, of the twisted bundle $E \otimes L^{\otimes r}$, where $r$ is an integer.

We prove that if $r$ is large enough, then $s$ is uniquely determined, up to a global endomorphism of the bundle $E$, by its subscheme of singular points (which we call the singular subscheme of $s$). If in particular $E$ is simple, then $s$ is uniquely determined, up to a scalar factor, by its singular subscheme.

We recall that the last statement holds in case $s$ is a holomorphic foliation by curves, with isolated singularities, on a projective manifold $M$ with stable tangent bundle, so it holds in particular if $M$ is a compact irreducible Hermitian symmetric space or a Calabi-Yau manifold.

If $L \longrightarrow \mathbf{P}^n$ is the hyperplane bundle, we show that it holds for every $r \geq 1$.

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## *Addition Formulae for Non-Abelian Theta Functions*

**E. Gómez González** (Universidad de Salamanca)

**F. J. Plaza Martín*** (Universidad de Salamanca)

This paper generalizes for non-abelian theta functions a number of formulae valid for theta functions of Jacobian varieties. The addition formula, the relation with the Szëgo kernel and with the multicomponent KP hierarchy and the behavior under cyclic coverings are given.