
# First Joint Meeting between the RSME and the AMS

*Sevilla, June 18–21, 2003*

# Abstracts

## Session 03

## Algebraic Topology

**Organizers:**

Alejandro Adem (University of Wiscoinsin)
Jaume Aguadé (Universitat Autónoma de Barcelona)
Eric M. Friedlander (Northwestern University)

# Index of Abstracts

**(In this index, in case of multiple authors, only the speaker is shown)**

## *Fixity and Group Actions*

**Alejandro Adem Díaz de León*** (University of Wisconsin)

**James F. Davis** (Indiana University)

**Ozgun Unlu** (University of Wisconsin)

We use methods from representation theory, homotopy theory and propagation to construct free group actions on products of spheres.

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## *p-local finite groups*

**Carles Broto Blanco** (Universitat Autónoma de Barcelona)

This talk will be based on joint work with Ran Levi and Bob Oliver on $p$-local finite groups.

The $p$-local structure of a finite group determines and it is determined by the $p$-completed classifying space. This suggested the definition of $p$-local finite group, an algebraic object that only contains $p$-local information and admits a classifying space. This new class of classifying spaces generalize the class of $p$-completed classifying spaces of finite groups and share with them many of the same properties.

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## *Stable localizations preserve certain ring spectra*

**Carles Casacuberta Vergés*** (Universitat de Barcelona)

**Javier Gutiérrez** (Universitat de Barcelona)

We consider homotopy idempotent functors in any suitable symmetric monoidal category of spectra. In earlier work we showed that such functors preserve homotopy ring spectra and homotopy module spectra, under suitable connectivity conditions (which are unnecessary if the given idempotent functor commutes with the suspension operator). In this talk we improve this result by showing that strict ring spectra (i.e., monoids in the model category) and strict module spectra are also preserved by homotopy idempotent functors, under similar restrictions. Our proof uses the fact that monoids may be viewed, up to homotopy, as algebras over a suitable cofibrant operad.

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## *Homology decompositions of spaces*

**Natália Castellana Vila** (Universitat Autónoma de Barcelona)

Homology decompositions are among the most useful tools in the study of the homotopy theory of classifying spaces. Roughly speaking, a homology decomposition for a space X is a recipe for glueing together specified spaces, desirably of a simpler homotopy type, such that the resulting space maps into X by a map which induces a homology isomorphism.

For finite and compact Lie groups homology decompositions have been constructed originally by Jackowski, McClure and Oliver and then more systematically for finite groups by Dwyer. They have been used for a variety of applications, both computational and theoretical. Common to all those decompositions is the use of group action techniques and the fact that they approximate BG by means of classifying spaces of proper subgroups of G.

The aim of this talk is to make the discussion of homology decompositions intrinsically homotopy theoretic. As a result we obtain different homology decompositions for p-compact groups in a systematic way, more or less analogous to Dwyer's discussion of homology decompositions for classifying spaces of finite groups.

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### *The normalizer of the torus in a compact Lie group*

**William Dwyer** (University of Notre Dame)

This is a preliminary report on joint work with C. Wilkerson. The normalizer of a maximal torus $T$ in a compact Lie group $G$ is an extension of the torus by the Weyl group of $G$. In general this extension is not split, and so corresponds to a nonzero element in $k \in H^2(W;T)$. Working with ideas of Tits, we show that $k$ is determined in a simple way by an explicit collection of elements in $T$ itself. In particular, we determine $k$ without calculating $H^2(W;T)$. The techniques also apply to 2-compact groups, and lead to a list of all objects $N$ which can appear as the normalizer of a torus in a connected 2-compact group. It turns out that any such $N$ is a product of two factors, one of which is derived from a connected compact Lie group, and the other of which is a power of the torus normalizer in the exceptional 2-compact group $DI(4)$.

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### *The Equivariant Topology of Cycles on Brauer-Severi Varieties*

**Pedro Ferreira dos Santos*** (Universidade Técnica de Lisboa)

**Paulo Lima-Filho**

We will describe the equivariant homotopy type of spaces of algebraic cycles on real Brauer-Severi varieties, under the action of the Galois group $\mathrm{Gal}(\mathbb{C}/\mathbb{R})$. Appropriate stabilizations of these spaces yield two equivariant spectra. The first one classifies Dupont/Seymour's quaternionic K-theory, and the other one classifies an equivariant cohomology theory $LH^*(-)$ which is a natural recipient of characteristic classes $KH^*(X) \to LH^*(X)$ for quaternionic bundles over Real spaces $X$.

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### *p-compact groups and p-local groups*

**Jesper Grodal** (University of Chicago)

$p$-compact groups and $p$-local finite groups are $p$-local homotopy theoretic analogs of compact (connected) Lie groups and finite groups respectively. The recently completed classification of $p$-compact groups, $p$ odd, states that connected p-compact groups are in 1-1-correspondence with finite $\mathbf{Z}_p$-reflection groups, i.e., a classification completely analogous to the classification of compact connected Lie groups.

It is natural to speculate whether (simple) $p$-local finite groups should admit a classification analogous to the classification of finite simple groups. In joint work with Broto, Castellana, Levi, and Oliver we examine the first step, namely we develop an extension theory of $p$-local (finite) groups. While much work still remain to be done before it is clear whether such a classification is even feasible, we are for instance able to give meaning to the term solvable and conclude that every solvable $p$-local finite group in fact comes from an actual finite group (which is in general not the case). My talk will be an introduction to these ideas.

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*Formality and symplectic geometry*

**Marisa Fernández** (Universidad del País Vasco)

**Vicente Muñoz Velázquez*** (Universidad Autónoma de Madrid)

In this talk, we shall introduce the concept of $s$-formality for a connected manifold, by using its $s$-stage minimal model. A main result establishes that a compact oriented manifold of dimension $2n$ or $2n-1$ is formal if and only if it is $(n-1)$-formal. This allows to study the formality of submanifolds $N \subset M$ of a formal manifold when the inclusion is $m$-connected for $m$ equal to half the dimension of $N$.

We apply this to study the formality of varios examples of symplectic manifolds constructed with the asymptotically holomorphic techniques introduced by Donaldson. We also discuss analogous statements about the hard Lefschetz property.

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*Elliptic spaces and a conjecture of Anick*

**Yves Félix** (Université Catholique de Louvain)

**Barry Jessup** (University of Ottawa)

**Aniceto Murillo*** (Universidad de Málaga)

An *elliptic* space is one whose rational homotopy and rational cohomology are both finite dimensional, and they may be less special than they appear. David Anick conjectured that any simply connected finite CW-complex $S$ can be realized as the $k$-skeleton of some elliptic complex as long as $k > \dim S$, or, equivalently, that any simply connected finite Postnikov piece $S$ can be realized as the base of a fibration $F \to E \to S$ where $E$ is elliptic and $F$ is $k$-connected, as long as $k > \dim S$. This conjecture is only known in a few cases that we shall describe. We also relate this conjecture with finiteness properties of rational spaces.

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*A proof of the Martino-Priddy conjecture*

**Bob Oliver** (Université Paris-Nord)

The Martino-Priddy conjecture gives an algebraic condition, in terms of fusion, for the $p$-completions of classifying spaces of two finite groups to be homotopy equivalent. The conjecture has recently been proven, using the classification of finite simple groups. We discuss some aspects of the proof.

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*On localizations of perfect groups and related topics*

**José Luis Rodríguez Blancas** (Universidad de Almeria)

In this talk we answer negatively a question posed by Libman and Casacuberta about the preservation of perfect groups under localization functors. Indeed, we show that the $p$-localization of Berrick's and Casacuberta's universal acyclic group is not perfect. Other new results and open questions related to localization of groups will be also described.

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*Homotopy G-Spheres*

**Jesper Grodal** (University of Chicago)

**Jeffrey Smith*** (Purdue University)

We study the function space of maps from the calassifying space to the classifying space of spherical fibrations. We compute $\pi_0$ of the function space and the fundamental group of each component. We also show that all homotopy groups of the function space are finitely generated and that almost are in fact finite.

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*Classification of p-local finite groups over the extraspecial group of order $p^3$ and exponent p*

**Albert Ruiz** (Universitat Autónoma de Barcelona)

**Antonio Viruel*** (Universidad de Málaga)

The concept of $p$-local finite group arises in the work of Broto-Levi-Oliver as a homotopy theoretical analogue of the classical concept of finite group. It is then natural to ask whether or not these $p$-local finite groups admit a classification similar to that of finite groups. Within such a programme, we fix our attention in the classification of $p$-local finite groups over extraespecial groups of order $p^3$ and exponent $p$, that appear as Sylow subgroup of many of the sporadic simple groups. As a consequence of that classification we obtain 3 new exotic (not coming from an honest finite group) 7-local finite groups.