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Abstracts

Session 6

Classical and Harmonic Analysis

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Index of Abstracts

(In this index, in case of multiple authors, only the speaker is shown)

Maria Angeles Alfonseca, *An almost-orthogonality principle for directional maximal functions* **3**

Jonathan Bennett, *Weighted Inequalities for the Fourier Extension Operator* **3**

Fernando Chamizo, *Some applications of large sieve* **3**

Martin Dindos, *Large solutions for Yamabe and similar problems on domains in Riemannian manifolds* **4**

M. Burak Erdoğan, *Mapping properties of the elliptic maximal function* **4**

Giacomo Gigante, *Estimates for oscillatory integrals and the Schrodinger equation* **5**

María José González, *Beltrami-type Operators and Geometry of Curves* **5**

Sanja Hukovic, *The Boundedness of the Hilbert Transform along Vector Fields* **5**

Alex Iosevich, *Variants of the Erdos and Falconer distance problems* **6**

José G. Llorente, *On the Fatou theorem for non-linear equations on trees* **6**

José María Martell, *L^p Bounds for Riesz Transforms and Square Roots Associated to Second Order Elliptic Operators* **6**

Joan Mateu, *Signed Riesz capacities* **7**

Camil Muscalu, *On the Carleson Hunt theorem in classical Fourier analysis* **7**

Osane Orue-Echevarría Fernández de la Peña, *Mixed norm inequalities for directional operators and k-plane transforms* **7**

Javier Parcet Hernández, *B-convex and K-convex operator spaces* **8**

Natasa Pavlovic, *Dyadic models for the equations of fluid motion* **8**

Stefanie Petermichl, *Bellman functions and continuous problems* **8**

Jill Pipher, *A multiparameter version of the Coifman-Meyer multilinear theorem* **8**

Malabika Pramanik, *A maximal operator associated to space curves* **9**

Zhongwei Shen, *Weighted Estimates for Elliptic Systems in Lipschitz Domains* **9**

Xavier Tolsa, *Bilipschitz maps, analytic capacity, and the Cauchy integral* **9**

Rodolfo Torres, *Some recent results about bilinear pseudodifferential operators* **9**

Rodrigo Trujillo González , ***On the Calderón-Zygmund Principle for some singular integral operators*** **9**

Sarah Ziesler, ***Maximal Function Estimates for the KP-I Equation*** **9**

An almost-orthogonality principle for directional maximal functions

Maria Angeles Alfonseca* (Universidad Autónoma de Madrid)

Fernando Soria (Universidad Autónoma de Madrid)

Ana Vargas (Universidad Autónoma de Madrid)

Given a set $\Omega \subset [0, \frac{\pi}{4})$, let us consider any ordered subject $\Omega_0 = \{\theta_1 > \theta_2 \ldots \theta_j > \ldots\}$ of Ω. We decompose Ω in several disjoint blocks, Ω_j, $j \geq 1$, separated by the elements of Ω_0. For each $j \geq 0$, we define the maximal operator associated to Ω_j as

$$M_{\Omega_j} f(x) = \sup_{x \in R \in B_j} \frac{1}{|R|} \int_R |f(y)|\, dy,$$

where B_j is the basis of all rectangles forming an angle $\theta \in \Omega_j$ with the x-axis.

We look for a relation between the norm of $M_\Omega f(x,y) = \sup_{j \geq 1} M_{\Omega_j} f(x,y)$ as an operator in $L^2(\mathbb{R}^2)$, and the norms of the M_{Ω_j}. By means of a covering lemma, we obtain an almost-orthogonality principle in L^2 for the family $\{M_{\Omega_j}\}$. In order to extend this result to other values of p, we need to introduce a Littlewood-Paley decomposition, and the associated square function.

As a corolary of these results, we are able to give a very simple proof of a recent result by Katz. Also, the almost-orthogonality principle allows us to extend some other results about directional maximal functions.

Weighted Inequalities for the Fourier Extension Operator

Jonathan Bennett* (Trinity College Dublin)

Antony Carbery (University of Edinburgh)

Fernando Soria (Universidad Autónoma de Madrid)

Ana Vargas (Universidad Autónoma de Madrid)

We discuss the manner in which Kakeya–type maximal functions may control the Fourier extension operator via L^2 weighted inequalities. Such estimates may be viewed as variants of a well–known conjecture of E. M. Stein concerning the Bochner–Riesz operators.

We give a solution to a corresponding problem for the extension operator restricted to circles in the plane. We include a brief discussion of our proof and its context in the literature.

Some applications of large sieve

Fernando Chamizo (Universidad Autónoma de Madrid)

Originally large sieve was a simple inequality created to deal with some specific problems in Number Theory. In this talk we shall discuss large sieve type inequalities in several geometric-analytic contexts. We shall focus mainly on the diversity of applications ranging over different topics.

Large solutions for Yamabe and similar problems on domains in Riemannian manifolds

Martin Dindos (Cornell University)

We are going to present both old and new results on the question of existence and uniqueness of positive solutions of the equation

$$\Delta u + hu - k\psi(u) = -f,$$

in Ω, $u(x) \to \infty$, with $\delta(x) = dist(x, \partial\Omega) \to 0$ on domains Ω with nonempty nonsmooth boundary in Riemannian manifolds M of dimension $n = \dim M \geq 2$. Here Δ is the Laplace-Beltrami operator given by the Riemannian metric on M. The function ψ is assumed to be well defined on all nonnegative numbers, vanishing at zero, increasing, convex and growing sufficiently fast as $u \to \infty$. As a typical example we can take $\psi(u) = u^\alpha$ for some $\alpha > 0$ and the case $\alpha = (n+2)/(n-2)$ arises in the problem of conformal change of metric in dimensions 3 and more and is known as the Yamabe problem.

Mapping properties of the elliptic maximal function

M. Burak Erdoğan (University of California, Berkeley)

The circular maximal theorem of Bourgain asserts that the circular maximal function,

$$Mf(x) = \sup_{t>0} \int_{S1} f(x - ts)\, d\sigma(s), \;\; x \in \mathbb{R}^2$$

is bounded in $L^p(\mathbb{R}^2)$ for $p > 2$. In this work, we consider a natural generalization of the circular maximal function by taking maximal averages over ellipses instead of circles. Let E be the set of all ellipses in $\mathbb{R}^2$ centered at the origin with axial lengths in $[1/2, 2]$. The *elliptic maximal function*, $M_E f$, is defined in the following way: Let $f : \mathbb{R}^2 \to \mathbb{R}$, then

$$M_E f(x) = \sup_{e \in E} \frac{1}{|e|} \int_e f(x - s)\, d\sigma(s), \;\; x \in \mathbb{R}^2,$$

where $d\sigma$ is the arclength measure on e and $|e|$ is the length of e. We are interested in the local L^p mapping properties of M_E. Simple counter-examples show that M_E can not be bounded in L^p if $p \leq 4$. We prove that for all $s > 1/6$, there is a constant $C = C(s)$ such that

$$\|Mf\|_4 \leq C\|f\|_{W^{4,s}} \,,$$

where $W^{4,s}$ is the Sobolev space with the norm

$$\|f\|_{W^{4,s}} = \|(1 - \Delta)^{s/2} f\|_4.$$

The proof of the theorem is geometric and employs a combinatorial method of Kolasa and Wolff.

Estimates for oscillatory integrals and the Schrodinger equation

Giacomo Gigante* (Università di Bergamo)

Fernando Soria (Universidad Autónoma de Madrid)

Let $u(x, t)$ be the solution to the linear Schrödinger equation with initial datum f, and $S^*f(x) = \sup_{t>0} |u(x, t)|$, $x \in \mathbb{R}^n$, the associated maximal operator. A fundamental problem in this setting is the validity of the inequality

$$\left(\int_{|x|\leq 1} |S^* f(x)|^2 \, dx \right)^{1/2} \leq C \|f\|_{H^\alpha}, \qquad f \in \mathcal{S}(\mathbb{R}^n),$$

where H^α is the Sobolev space. It is a well-known result that the above inequality holds in dimension $n = 1$ if and only if $\alpha \geq 1/4$. It remains an open problem to determine the best exponent α in the cases $n \geq 2$, the conjecture being $\alpha = 1/4$. In this work we test the conjecture (that $H^{1/4}$ suffices for the inequality to hold) on a smaller operator, namely on the maximal square function

$$Q^* f(x) = \sup_{t>0} \left(\frac{1}{\sigma(S^{n-1})} \int_{S^{n-1}} |u(|x|\omega, t)|^2 \, d\sigma(\omega) \right)^{1/2}.$$

Beltrami-type Operators and Geometry of Curves

K. Astala (University of Helsinki)

María José González* (University of Cádiz)

The main purpose of this talk is to show that certain curves, such as chord-arc curves,can be characterized by the boundedness and invertivility of a Beltrami-type operator in a weighted L^2 space.We will also show the relation between this result and the problem on the connectivity of chord-arc curves that remains open.

The Boundedness of the Hilbert Transform along Vector Fields

Sanja Hukovic* (University of Rochester)

Andrea Nahmod (University of Massachusetts)

The boundedness of the Hilbert transform along vector fields is a long standing problem. Different restrictions on curvature of the vector field (its well turning) have given sufficient conditions for the boundedness. We use the phase-space decomposition of the plane to define model operator for the Hilbert transform. We prove the boundedness of the model operator under a doubling condition on the vector field.

To obtain the boundedness of the model operator we use Lacey-Thiele technique of reducing the proof of the boundedness of the model operator to the proof of mass, size and tree restriction lemma. The crux of this technique is the proof of the mass lemma for any collection of tiles under the given doubling condition on the vector field.

Variants of the Erdos and Falconer distance problems

Alex Iosevich (University of Missouri)

Variants of the Erdos and Falconer distance problems The talk is based on two papers, one with Steve Hofmann and one with Izabella Laba. The Erdos distance conjecture says that the number of distances determined by by N points in $\mathbb{R}^d$ is at least $CN^{2/s}$. The Falconer distance conjecture says that if a compact set in $\mathbb{R}^d$ has Hausdorff dimension $> d/2$, then the set of distances has positive Lebesgue measure. We shall discuss variants of both conjectures for random metrics and a diophantine mechanism which allows one to convert Falconer type problems into appropriate variants of Erdos type problems.

On the Fatou theorem for non-linear equations on trees

R. Kaufman

José G. Llorente* (Universitat Autònoma de Barcelona)

J. M. Wu

Not much is known about the boundary behaviour of solutions of non-linear equations in divergence form in the unit ball. p-harmonic functions are the simplest model case of a more general class, the so called A-harmonic functions, which arise in a natural way associated to the theory of quasiregular maps.

Bounded p-harmonic functions in the unit ball have radial limits on a subset of the boundary of positive Hausdorff dimension, depending on p. Problems like the optimal determination of the size of this set, the quasiadditivity of p-harmonic measure or the existence of radial limits for bounded A-harmonic functions in the ball are widely open and difficult, except for the classical case (p=2). We introduce the corresponding classes of non-linear equations on trees, answer these questions and arrive to some unexpected properties.

L^p Bounds for Riesz Transforms and Square Roots Associated to Second Order Elliptic Operators

Steve Hofmann (University of Missouri)

José María Martell* (Universidad Autónoma de Madrid)

In joint work with S. Hofmann, we consider the Riesz transforms $\nabla L^{-1/2}$, where $L \equiv -\mathrm{div}\, A(x)\nabla$, and A is an $n \times n$ matrix with bounded measurable complex entries, defined on $\mathbb{R}^n$, and satisfying the "accretivity" condition

$$\lambda\,|\xi|^2 \leq \mathrm{Re}\, A\,\xi \cdot \bar{\xi} = \mathrm{Re} \sum_{j,k} A_{j,k}(x)\,\xi_k\,\bar{\xi_j},$$

for some $\lambda > 0$, and for all $\xi \in \mathbb{C}^n$. We establish boundedness of these operators on $L^p(\mathbb{R}^n)$, for the range $p_n < p \leq 2$, where $p_n = 2n/(n+2)$, $n \geq 2$, and we obtain a weak-type estimate at the endpoint p_n. The case $p = 2$ was already known: it is equivalent to the solution of the square root problem of T. Kato, recently obtained by P. Auscher, S. Hofmann, M. Lacey, A. McIntosh and P. Tchamitchian. We remark that our result is new only for $n \geq 3$: otherwise, Gaussian heat kernel bounds hold in general, in which case the L^p estimates follow immediately from the solution of the Kato problem, by a result of Calderón-Zygmund type due to Auscher and Tchamitchian.

Signed Riesz capacities

Joan Mateu* (Universitat Autònoma de Barcelona)

Laura Prat (Universitat Autònoma de Barcelona)

Joan Verdera (Universitat Autònoma de Barcelona)

In this talk we consider, as a natural extension of the analytic capacity, the capacity γ_α associated to the signed vector valued Riesz kernel $x/|x|^{\alpha+1}, 0 < \alpha < n$, in R^n. We establish the equivalence between $\gamma_\alpha, 0 < \alpha < 1$, and one of the well known Riesz capacities of non linear potential theory. As a consequence we obtain the semiaditivity of this capacity and also the Bilipschitz invariance.

On the Carleson Hunt theorem in classical Fourier analysis

Xiaochun Li

Camil Muscalu* (University of California at Los Angeles)

We are going to discuss some generalizations of the classical Carleson Hunt theorem in Fourier analysis.

Mixed norm inequalities for directional operators and k-plane transforms

Osane Orue-Echevarría Fernández de la Peña (Universidad del Pais Vasco)

In this work, we study mixed norm inequalities for certain directional operators $I_{\alpha,u}$. These operators appear when apply the Calderón and Zygmund method of rotations to operators with the homogeneity of Riesz potentials and variable kernel. If $\alpha = 1$ the operator is the X-ray transform, and when $\alpha = 0$ it is replaced by the directional Hardy-Littlewood maximal operator. Besides operators defined on straight lines, we consider the generalization to operators defined on k-planes and we study the mixed norm inequalities when the operator is applied to radial functions. Also, when the dimension of the k-plane is $n-1$, some inequalities for general functions are obtained, using knwon results about the Radon transform and about the maximal corresponding operator. In all cases, sharp results are obtained when the operator are restricted to radial functions, and also for general functions in a certain range of values of the parameter. The estimates for characteristic functions play an important role in some estimates, and they can be understood as geometric results regarding the measure of sections of certain sets.

B-convex and K-convex operator spaces

Javier Parcet Hernández (University de Autónoma de Madrid)

The theory of type and cotype lives in the interaction between orthonormal systems and Banach space geometry. In some recent works we have developed a non-commutative analog of this theory, where Banach spaces are replaced by operator spaces and the classical systems are substituted by quantized systems. In this new setting we have obtained non-commutative versions of classical results such as the theorem of Kwapień. After giving the main definitions of this new theory, we shall define the concept of B-convexity for operator spaces, which a priori depends on a set of parameters indexed by Σ. As we shall see, some of the classical characterizations of this geometric notion for Banach spaces also hold for operator spaces. However, the notion of B_Σ-convexity is no longer equivalent to the property of having non-trivial Σ-type, some counterexamples will be given. In particular, we can not expect to obtain an operator space version of the Maurey-Pisier theorem. Finally, we shall introduce K_Σ-convex operator spaces in order to analyze the independence of B_Σ-convexity with respect to Σ. This independence is equivalent to the non-commutative version of Pisier's theorem.

Dyadic models for the equations of fluid motion

Natasa Pavlovic (Princeton University)

This talk will be about dyadic models for the equations of fluid motion. In the talk we shall introduce a scalar dyadic model for the Euler and the Navier-Stokes equations. For the dyadic Euler equations we prove finite time blow-up. In the context of the dyadic Navier-Stokes equations with hyper-dissipation we prove finite time blow-up in case when the degree of dissipation is sufficiently small. Also we shall discuss how these results can be generalized to analogous results for a vector dyadic model. This talk is based on joint projects with Susan Friedlander and Nets Katz.

Bellman functions and continuous problems

Stefanie Petermichl (Brown University)

The method of Bellman functions originated in the theory of optimal control. Later it was used as a tool for dyadic estimates in harmonic analysis. It is now understood how to apply this method to some classical Calderon Zygmund operators by recognizing a certain analogy in various Greens theorems. We give an idea of this analogy and present some examples, such as weighted and unweighted bounds for Beurling, Riesz and Hilbert transform norms.

A multiparameter version of the Coifman-Meyer multilinear theorem

Jill Pipher (Brown University)

In this talk we give a multiparameter version of the Coifman-Meyer multilinear theorem.

A maximal operator associated to space curves

Malabika Pramanik (University of Kentucky)

We investigate the L^p regularity of an averaging operator and its corresponding maximal operator associated to space curves.

Weighted Estimates for Elliptic Systems in Lipschitz Domains

Zhongwei Shen (University of Kentucky)

Let Ω be a bounded Lipschitz domains in R^n, $n \geq 3$. We study the solvability of boundary value problems for elliptic systems in Ω with boundary data in the weighted L^2 space $L^2(\partial, \omega_\lambda d\sigma)$, where $\omega_\lambda(Q) = |Q - Q_0|^\lambda$, Q_0 is a fixed point on $\partial\Omega$, and $d\sigma$ denotes the surface measure on $\partial\Omega$. We obtain existence and unique results with optimal estimates in terms of nontangential maximal functions under certain restrications on λ.

Bilipschitz maps, analytic capacity, and the Cauchy integral

Xavier Tolsa (Universitat Autònoma de Barcelona)

Let $\varphi : \mathbb{C} \to \mathbb{C}$ be a bilipschitz map. We prove that if $E \subset \mathbb{C}$ is compact, and $\gamma(E)$, $\alpha(E)$ stand for its analytic and continuous analytic capacity respectively, then then $C^{-1}\gamma(E) \leq \gamma(\varphi(E)) \leq C\gamma(E)$ and $C^{-1}\alpha(E) \leq \alpha(\varphi(E)) \leq C\alpha(E)$, where C depends only on the bilipschitz constant of φ. Further, we show that if μ is a Radon measure on $\mathbb{C}$ and the Cauchy transform is bounded on $L^2(\mu)$, then the Cauchy transform is also bounded on $L^2(\varphi\mu)$, where $\varphi\mu$ is the image measure of μ by φ. To obtain these results, we estimate the curvature of $\varphi\mu$ by means of a corona type decomposition.

Some recent results about bilinear pseudodifferential operators

Arpad Benyi (University of Massachusetts)

Rodolfo Torres* (Kansas University)

We will describe several boundedness results for various natural classes of bilinear pseudodifferential operators acting on Lebesgue and other spaces functions.

On the Calderón-Zygmund Principle for some singular integral operators

Rodrigo Trujillo González (Universidad de La Laguna)

The Calderón-Zygmund Principle roughly states that any singular integral operator is controlled by a suitable maximal operator. The corresponding weighted estimate for Calderón-Zymund operators involves the Hardy-Littlewood maximal operator and is due to R. Coifman. In this talk we will briefly show how this kind of estimates make possible to deduce more general weighted inequalities. However, we will also proof that the corresponding Calderón-Zygmund Principle in the sense of Coifman does not hold for a wide class of singular integral operators, which include those with kernel satisfying the classical Hörmander's condition.

Maximal Function Estimates for the KP-I Equation

Sarah Ziesler (Dominican University)

I shall discuss joint work with C.E.Kenig, in which we give optimal (up to endpoint) maximal function type estimates for the solution of the linear initial value problem associated with the Kadomtsev-Petviashvili I equation. Applications to associated non-linear problems will also be discussed.