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Abstracts

Session 7

Combinatorics

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Index of Abstracts

(In this index, in case of multiple authors, only the speaker is shown)

Sergei Bezrukov, *Combinatorics of isoperimetric orders* **2**

Joseph E. Bonin, *Constructing matroids with the same Tutte polynomial* **2**

Javier Cilleruelo, *Sidon Sets* **2**

Sergi Elizalde, *Bijections for Refined Restricted Permutations* **3**

Alberto Márquez, *Convex sets in graphs* **3**

Anna de Mier, *Lattice path matroids* **4**

Melvyn B. Nathanson, *Representation functions of additive bases in combinatorial number theory* **4**

Ortrud R. Oellermann, *Steiner intervals and Steiner Geodetic Numbers in Distance-Hereditary Graphs* **4**

Josep Rifà, *The rank and kernel of several nonlinear codes* **5**

Francisco Santos, *Non-crossing graphs on a planar point set form the face poset of a polyhedron* **5**

Oriol Serra, *Sets with small sum in Z/pZ: beyond Vosper theorem* **6**

Brigitte Servatius, *Combinatorial pointed pseudo-triangulations* **6**

Combinatorics of isoperimetric orders

Sergei Bezrukov (University of Wisconsin - Superior)

It is a common practice to represent solutions to an isoperimetric problem on a graph $G = (V_G, E_G)$ in terms of initial segments of some total order on V_G. Such orders, if exist, are called isoperimetric. Not all graphs admit isoperimetric orders, but if they do then this fact is very beneficial for solving many other extremal graph problems.

We consider isoperimetric orders for the cartesian powers of a graph G. It turns out that the progress in finding isoperimetric orders for known graphs is not only due to the nice graph structure, but mostly due to the properties of the order itself. This moves the focus from studying of graphs to studying the total orders.

By contrast with the conventional approach, we start with a family of total orders $\mathcal{O}^n$ defined on the n-th cartesian power of the set $\{0, 1, \ldots, p-1\}$ for any $n \geq 1$ and look for graphs G of order p for which cartesian power G^n the order $\mathcal{O}^n$ is isoperimetric, for any $n \geq 1$. We show that under some natural assumptions all the orders that we have studied so far satisfy several common properties. This led to a powerful technique for attacking the isoperimetric problems. We present several approaches, some of them involve auxiliary optimization problems on posets. As an application we present a complete specification of graphs for several families of total orders.

Constructing matroids with the same Tutte polynomial

Joseph E. Bonin (The George Washington University)

We will show how to contstruct, for prime powers q exceeding 5, huge collections of nonisomorphic 3-connected matroids that are representable over $GF(q)$ and that have the same Tutte polynomial.

Sidon Sets

Javier Cilleruelo (Universidad Autónoma de Madrid)

A set A is called a Sidon set if for every s the equation $x + y = s$ has at most one solution (up to ordering) with $x, y \in A$.

We present an overview of recent results on Sidon sets including a joint work with I. Ruzsa about real and p-adic Sidon sequences.

Bijections for Refined Restricted Permutations

Sergi Elizalde* (MIT)

I. Pak (MIT)

The concept of pattern avoidance concerns permutations regarded as words $\pi = \pi_1\pi_2\cdots\pi_n$. On the other hand, concepts such as fixed points or excedances arise when we look at permutations as bijections $\pi : \{1, 2, \ldots, n\} \longrightarrow \{1, 2, \ldots, n\}$ (recall that i is a fixed point of π if $\pi_i = i$, and it is an excedance if $\pi_i > i$). It was not until recently that these two kinds of concepts were studied together.

In a recent paper, Robertson, Saracino and Zeilberger show that the classical result stating that the number of 321-avoiding permutations equals the number of 132-avoiding permutations can be refined by restricting to permutations having any given number of fixed points. Their proof is nontrivial and technically involved.

Here we give a simple combinatorial proof of a further refinement of this result, namely that it still holds when we fix not only the number of fixed points but also the number of excedances. We present a bijection between 321- and 132-avoiding permutations that preserves these two statistics.

Our bijection is a composition of two slightly modified known bijections into Dyck paths, and the result follows from a new analysis of these bijections. The Robinson-Schensted-Knuth correspondence is a part of one of them, and from it stems the difficulty of the analysis.

Convex sets in graphs

J. Cáceres (Universidad de Almería)

Alberto Márquez* (Universidad de Sevilla)

O. Oellermann (University of Winnipeg)

M. L. Puertas (Universidad de Almería)

The distance between pairs of vertices in a graph naturally gives rise to the notions of convex sets, convex hull, and extreme points in the vertex set of a graph. Here we have a look to some different notions of convex sets in graphs. The Steiner distance can be used to define a convexity in the vertex set of a graph, which is related to others already defined. A key point is the behavior in relation with the property that every convex set is the convex hull of its extreme points. The class of graphs where this property holds are characterized for different convexities. The results above lead us to pose a different question about the minimum amount of information needed to rebuild any graph using the convex hull operation. To this end, we define a vertex to be a contour vertex if the eccentricity of every neighbour is at most as large as that of the vertex. It is shown that every convex set of vertices in a graph is the convex hull of its contour vertices.

Lattice path matroids

J. E. Bonin (The George Washington University)

Anna de Mier* (Universitat Politècnica de Catalunya)

M. Noy (Universitat Politècnica de Catalunya)

Fix two lattice paths P and Q from $(0,0)$ to (m,r) that use East and North steps with P never going above Q. We show that the lattice paths that go from $(0,0)$ to (m,r) and that remain in the region bounded by P and Q can be identified with the bases of a particular type of transversal matroid, which we call a lattice path matroid. We consider a variety of enumerative aspects of these matroids and we study three important matroid invariants, namely the Tutte polynomial and, for special types of lattice path matroids, the characteristic polynomial and the β invariant.

Representation functions of additive bases in combinatorial number theory

Melvyn B. Nathanson (City University of New York, Lehman College)

Let A be a set of integers. For every integer n, let $r_{A,h}(n)$ denote the number of representations of n in the form $n = a_1 + a_2 + \cdots + a_h$, where $a_1, a_2, \ldots, a_h \in A$ and $a_1 \leq a_2 \leq \cdots \leq a_h$. The function $r_{A,h} : \mathbb{Z} \to \mathbb{N}_0 \cup \{\infty\}$ is the *representation function of order h for A.* The set A is called an *asymptotic basis of order h* if $r_{A,h}^{-1}(0)$ is finite, that is, if every integer with at most a finite number of exceptions can be represented as the sum of exactly h not necessarily distinct elements of A. It is proved that every function is a representation function, that is, if $f : \mathbb{Z} \to \mathbb{N}_0 \cup \{\infty\}$ is any function such that $f^{-1}(0)$ is finite, then there exists a set A of integers such that $f(n) = r_{A,h}(n)$ for all integers n. Moreover, the set A can be arbitrarily sparse in the sense that, if $\varphi(x) \to \infty$, then there exists a set A with $f(n) = r_{A,h}(n)$ such that $|\{a \in A : |a| \leq x\}| < \varphi(x)$ for all sufficiently large x.

Steiner intervals and Steiner Geodetic Numbers in Distance-Hereditary Graphs

Ortrud R. Oellermann* (University of Winnipeg)

Maria Luz Puertas (Universidad de Almería)

The Steiner distance of a set S of vertices in a connected graph G is the smallest number of edges in a connected subgraph of G that contains S. The Steiner interval $I(S)$ of S is the union of all the vertices of G that belong to some Steiner tree for S. If $S = \{u, v\}$, then $I(S) = I[u, v]$ is called the interval between u and v and consists of all vertices that lie on some shortest $u - v$ path in G. A set S of vertices is a geodetic set if $\cup_{\{u,v\}} I[u, v] = V(G)$. The smallest cardinality of a geodetic set is called the geodetic number and is denoted by $g(G)$. We show that the problem of finding the geodetic number of a graph is NP-hard. A set S of vertices of a connected graph G is a Steiner geodetic set if $I(S) = V(G)$ and the smallest cardinality of such a set is called the Steiner geodetic number of G and is denoted by $sg(G)$. We show that for distance-hereditary graphs $g(G) \leq sg(G)$ but that this inequality does not hold for graphs in general, thereby disproving a claim made by Chartrand and Zhang. We develop an efficient algorithm for finding the Steiner interval for a set of vertices in a graph and show how this result can be used to find a minimal Steiner geodetic set for a distance-hereditary graph.

The rank and kernel of several nonlinear codes

J. Borges

K. T. Phelps

Josep Rifà* (Universitat Autònoma de Barcelona)

V. A. Zinoviev

This presentation is a review of some recent results about two parameters, the rank and the dimension of the kernel, associated to nonlinear codes.

We say that a binary code of length n is additive if it is isomorphic to a subgroup of $Z_2^\alpha \times Z_4^\beta$, where the quaternary coordinates are transformed to binary by means of the usual Gray map and hence $\alpha + 2\beta = n$.

Given an extended 1-perfect code of length $n + 1 = 2^t$ the punctured code by deleting a Z_2 coordinate (if there is one) gives a 1-perfect additive code. We compute the possible parameters α, β, rank and dimension of the kernel for extended 1-perfect additive codes. A very special case is that of extended 1-perfect Z_4-linear codes.

Also in this paper we prove that any additive extended Preparata-like code always verifies $\alpha = 0$, i.e. it is always a Z_4-linear code. Moreover, we compute the rank and the dimension of the kernel of such Preparata-like codes and also the rank and the kernel of the Z_4-dual of these codes, i.e. the Z_4-linear Kerdock-like codes.

Non-crossing graphs on a planar point set form the face poset of a polyhedron

D. Orden (Universidad de Cantabria)

Francisco Santos* (Universidad de Cantabria)

For any finite set A of n points in $\mathbb{R}^2$, we define a $(3n-3)$-dimensional simple polyhedron whose face poset is isomorphic to the poset of "non-crossing marked graphs" with vertex set A, where a marked graph is defined as a geometric graph together with a subset of its vertices. The poset of non-crossing graphs on A appears as the complement of the star of a face in that polyhedron. The polyhedron has a unique maximal bounded face, of dimension $2n_i + n - 3$, where n_i is the number of points of A interior to $\mathrm{conv}(A)$. The vertices of this polytope are all the pseudo-triangulations of A, and the edges are flips of two types: the traditional diagonal edge-flips (in pseudo-triangulations) and the removal or insertion of a single edge.

Sets with small sum in Z/pZ: beyond Vosper theorem

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Oriol Serra* (Universitat Politècnica de Catalunya)

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Given two sets A, B in Z/pZ, p a prime, the well-known Cauchy-Davenport theorem states that $|A + B| \geq \min\{p, |A| + |B| - 1\}$. The corresponding inverse theorem by Vosper says that, if $|A| + |B| < p - 1$, then equality holds if and only if both A and B are arithmetic progressions with the same difference. A general inverse result was obtained by Freiman in this context by saying that small sets with small doubling in Z/pZ must be contained in not very large arithmetic progressions. We present precise generalizations of Vosper theorem giving the structure of sets which satisfy $|A + B| = |A| + |B| + m < p - (m + 3)(m + 4)/2$ and $|A|, |B| \geq m + 4$. The results illustrate the use of isoperimetric connectivity techniques which have been fruitful in different applications.

Combinatorial pointed pseudo-triangulations

Brigitte Servatius (WPI)

A combinatorial pointed pseudo-triangulation is an assignment of large and small angles to a plane graph such that every interior face has exactly 3 small angles, the outside face has only large angles, and at every vertex there is exactly one big angle. We show that not all combinatorial pointed pseudo- triangulations have geometric realizations. We prove that all planar minimally rigid graphs may combinatorially be pseudo-triangulated and the resulting pseudo-triangulations are geometrially realizable.