First Joint Meeting between the RSME and the AMS

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Abstracts

Session 08

Commutative Algebra: geometric, homological, combinatorial and computational aspects

Organizers:

Alberto Corso (University of Kentucky)
Philippe Gimenez (Universidad de Valladolid)
Santiago Zarzuela (Universidad de Barcelona)

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Some numerical invariants of local rings

J. Àlvarez Montaner (Universidad Politécnica de Cataluña)

Let R be a formal power series ring over a field of characteristic zero and $I \subseteq R$ be any ideal. The aim of this work is to introduce some numerical invariants of the local rings R/I by using theory of algebraic $\mathcal{D}$-modules. More precisely, we will prove that the multiplicities of the characteristic cycle of the local cohomology modules $H_I^{n-i}(R)$ and $H_{\mathfrak{p}}^p(H_I^{n-i}(R))$, where $\mathfrak{p} \subseteq R$ is any prime ideal that contains I, are invariants of R/I.

Saturation and Castelnuovo-Mumford regularity

I. Bermejo* (Universidad de La Laguna)

Ph. Gimenez (Universidad de Valladolid)

Let R be a polynomial ring in $n+1$ variables over an infinite field k, and let I be a homogeneous ideal of R. By avoiding the construction of a minimal graded free resolution of I, we provide an effective method for computing the Castelnuovo-Mumford regularity of I.

As an application, we obtain an explicit formula for the Castelnuovo-Mumford regularity of the defining ideal of a projective monomial subvariety $\mathcal{X}$ of $\mathbb{P}_k^n$ of codimension two.

All the results have been implemented in SINGULAR and will be included in the new version of our joint library with G.-M. Greuel, `mregular.lib`.

Multiplier ideals in exotic settings

A. Bravo* (Universidad Autónoma de Madrid)

K. E. Smith (University of Michigan)

We describe different approaches to develop a notion of multiplier ideal in the context of non-smooth varieties, and over fields of arbitrary characteristic.

Asymptotic Behaviour of Cohomology

M. Brodmann (University of Zuerich)

We present a few results on the asymptotic behaviour for $n << 0$ of the cohomology modules of the $n-th$ twist $F(n)$ of a coherent sheaf F on a projective scheme X (over an affine noetherian base scheme Y). If Y is either of dimension 1 or else semilocal and of dimension 2, the set of asymptotic primes of the mentioned cohomology modules eventually stabilizes for $n << 0$ (under mild additional hypotheses on Y). If Y is local and of dimension 1, various numerical invariants (multiplicities, Hilbert coefficients, number of generators, socle dimensions, torsion lengths) of the cohomology modules in question depend polynomially on n if $n << 0$. We relate our results to the examples of Singh and Katzman and discuss some open problems.

The structure of the Rao module and the geometry of schemes

M. Casanellas Rius (Universidad de Barcelona)

Using a technique introduced by J. Migliore, we will show how the structure of the Rao module of a scheme $X \subset \mathbb{P}^n$ determines part of the geometry of X. In particular we will prove that if X has dimension $d \geq 1$ and is r-Buchsbaum with $r > \max(codimX - d, 0)$, then X is contained in at most one variety of minimal degree.

Liaison of varieties of small dimension and deficiency modules

M. Chardin (CNRS and University of Paris VI)

Liaison relates the cohomology of the ideal sheaf of a scheme to the cohomology of the canonical module of its link. We here refer to Gorenstein liaison in a projective space over a field: each ideal is the residual of the other in one Gorenstein homogeneous ideal of a polynomial ring. Assuming that the linked schemes (or equivalently one of them) are Cohen-Macaulay, Serre duality expresses the cohomology of the canonical module in terms of the cohomology of the ideal sheaf. Therefore, in the case of Cohen-Macaulay linked schemes, the cohomology of ideal sheaves can be computed one from another: up to shifts in ordinary and homological degrees, they are exchanged and dualized. In terms of free resolutions this means that, up to a degree shift, they may be obtained one from another by dualizing the corresponding complexes (for instance, the generators of one cohomology module corresponds to the last syzygies of another cohomology module of the link). If the linked schemes are not Cohen-Macaulay, this property fails. Nevertheless, experience on a computer shows that these modules are closely related. We will explain in this talk this relation for the cases of surfaces and three-dimensionnal schemes.

Deformations of monomial ideals

A. Conca* (University of Genova)

A. Bigatti (University of Genova)

L. Robbiano (University of Genova)

There are many known ways of deforming a monomial ideal to another ideal so that the betti numbers are preserved. For instance, polarization, the so-called Hartshorne lifting and algebraic shifting (of strongly stable ideals) are deformations in this sense. I will present a simple and general deformation process (whose special cases are polarizations and Hartshorne liftings) and discuss various properties.

Poincaré series of resolution of surface singularities

S. D. Cutkosky* (University of Missouri)

J. Herzog (University GH-Essen)

A. Reguera (Universidad de Valladolid)

We associate a generally non-Noetherian graded ring to the exceptional divisors of a resolution of singularities of the spectrum of a two dimensional, complete normal local ring of dimension two with algebraically closed residue field of characteristic zero. We show that the Hilbert polynomial of this ring is rational if the Picard group is semi-abelian, but give examples showing that it can be rational in general.

Linearly Presented Ideals

D. Eisenbud* (MSRI/UC Berkeley)

C. Huneke (University of Kansas)

B. Ulrich (Purdue University)

Suppose that I is a homogeneous ideal in a polynomial ring S in n variables over a field. Let P be the maximal homogeneous ideal of S.

Theorem. If all the generators of I are in degree d, and all the relations are in degree $d+1$, and if I contains a power of P, then some power of I is equal to a power of P.

Conjecture. With hypotheses as in the theorem, $I^{n-1} = P^{d(n-1)}$.

I will explain the context and background of these ideas. Of particular note is result relating the shape of the free resolution of any homogeneous ideal to the monomials in the initial ideal with respect to a reverse lexicographic ordering.

Secant Varieties to Grassmann Varieties

A. V. Geramita* (University of Genova/ Queen's University)

M. V. Catalisano (University of Genova)

A. Gimigliano (University of Bolonia)

If $X \subset \mathbb{P}^n$ is a reduced and irreducible projective variety then by X^t we will denote the closure of the set of all secant $\mathbb{P}^{t-1}$'s to X, i.e. the closure of the set of all points on linear spaces spanned by t linearly independent points in $\mathbb{P}^n$. The expected dimension of X^t is $\min\{n, t(\dim X) + (t-1)\}$ and when this dimension is not reached the variety is *defective for* $t-1$*-secants*. It is a classical problem of projective geometry to identify the *defective* varieties.

We consider this problem in the case when X is a Grassmann variety. We show how to connect this problem to one involved in finding the dimension of a piece of an ideal in an exterior algebra. The ideal involved is of a very special type: it is the intersection of squares of ideals generated by families of linear forms in the exterior algebra. We find this dimension in many cases and, as a consequence, can show (among other things) that no secant line variety to any Grassmannian is defective (except for the well known exceptions of the Grassmannian of two dimensional subspaces of an $n+1$-dimensional vector space). We also find some new defective cases.

Our results use both combinatorial and commutative algebra methods.

Positive combinatorial formulae for quiver polynomials

E. Miller* (MSRI, Berkeley)

A. Knutson (UC Berkeley)

M. Shimozono (Virginia Tech.)

This talk concerns homological and geometric properties of ideals generated by minors in products of two or more matrices filled with independent variables. These ideals arise naturally in representation theory of quivers, and their cohomological invariants are polynomials that give universal topological data on degeneracy loci for sequences of vector bundle morphisms. I will present explicit combinatorial formulae for these cohomological invariants. The formulae generalize in many directions the Giambelli-Thom-Porteous formula, which amounts to the classical theorem that multiplicities of ideals generated by fixed size minors in rectangles count semistandard Young tableaux.

Monomial ideals and their core

C. Polini* (University of Notre Dame)

B. Ulrich (Purdue University)

M. Vitulli (University of Oregon)

The *core* of an ideal I is the intersection of all (minimal) reductions of I. The core of a monomial ideal I is always monomial even though I may not have any minimal reduction which is monomial. In this talk, I will present some cases where the core of monomial ideals has a nice geometric interpretation and it is connected with the *multiplier ideal*. I will also investigate the relationship between the core of an ideal I and the core of the integral closure of I, always focusing on the case of monomial ideals.

The equality $I^2 = QI$ in Buchsbaum rings

H. Sakurai (Meiji University)

Let Q be a parameter ideal in a Noetherian local ring A with the maximal ideal $\mathfrak{m}$ and let $I = Q : \mathfrak{m}$. In my talk the problem of when the equality $I2 = QI$ holds true is explored. When A is a Cohen-Macaulay ring, this was completely solved by A. Corso, C. Huneke, C. Polini, and W. Vasconcelos, while almost nothing is known when A is not a Cohen-Macaulay ring. My purpose is to show that within a huge class of Buchsbaum local rings A the equality $I2 = QI$ holds true for all parameter ideals Q and that when A is a Buchsbaum local ring, the equality $I2 = QI$ holds true, if $\mathrm{e(A)} = 2$ and depth $\mathrm{A} > 0$. These results give ample examples of ideals I, for which the Rees algebras $\mathrm{R(I)} = \bigoplus_{\mathrm{n} \geq 0} \mathrm{I^n}$, the associated graded rings $\mathrm{G(I)} = \mathrm{R(I)/IR(I)}$, and the fiber cones $\mathrm{F(I)} = \mathrm{R(I)}/\mathfrak{m}\mathrm{R(I)}$ are all Buchsbaum rings with certain specific graded local cohomology modules.

On birational Macaulayfications and Cohen-Macaulay canonical modules

P. Schenzel (University of Halle)

Let $(A, \mathfrak{m})$ denote a local domain. One motivation of the present talk is the following question:

Does there exists a birational extension ring $A \subseteq B \subseteq Q$, ($Q$ denotes the field of quotients) such that B is finitely generated as an A-module and a Cohen-Macaulay ring?

We call such an extension ring a birational Macaulayfication of A.

Theorem *Suppose that A is the factor ring of a Gorenstein ring. Then A possesses a birational Macaulayfication B if and only if the canonical module $K(A)$ is a Cohen-Macaulay module. Moreover, in this case B is uniquely determined up to isomorphisms and $B \simeq \operatorname{Hom}_A(K(A), K(A))$.*

This has to do with the notion of sequentially Cohen-Macaulay modules (as introduced by R. P. Stanley) resp. Cohen-Macaulay filtered modules (as introduced by the author). That means, the dimension filtration $\{M_i\}$ of a finitely generated A-module M (consisting of all maximal submodules M_i such that $\dim M_i \leq i$) has the property that the quotients M_i/M_{i-1} are either zero or an i-dimensional Cohen-Macaulay module. Let $K^i(M), 0 \leq i < \dim M$, denote the modules of deficiency of M, measuring the Cohen-Macaulay deviation of M.

Typical examples that fulfill the requirements of Theorem are affine semigroup rings of codimension two studied by M. Morales resp. I. Peeva and B. Sturmfels.

Ideals of linear type in Cremona maps

A. Simis (Universidade Federal de Pernambuco)

Hulek and Schreier introduced the idea of looking into syzygies in order to understand Cremona maps. This has now developed into a more precise theory where other algebraic features come to play a role. I will talk about some of these aspects and relate them to a curious question as to whether or when the defining ideal of the Rees algebra is a minimal prime of a certain graded algebra (besides the symmetric algebra). In the opportunity I will revise criteria for Cremona maps and pose some further questions.

A formula for the core of ideals

B. Ulrich* (Purdue University)

C. Polini (University of Notre Dame)

We give a formula for the core of certain ideals that are not necessarily equimultiple. The *core* of an ideal is the intersection of all reductions of the ideal. The concept is related to Briançon-Skoda type theorems and to a conjecture by Kawamata on the existence of non-trivial sections of certain line bundles, as discovered recently by Hyry and Smith.

Divisors of Integrally Closed Modules

J. Hong (Rutgers University)

S. Noh (Ewha Womans University, Korea)

W. V. Vasconcelos* (Rutgers University)

There is a beautiful theory of integral closure of ideals in regular local rings of dimension two, due to Zariski, several aspects of which were later extended to modules. Our goal is to study integral closure of modules over normal domains by attaching divisors/determinantal ideals to them. They will be of two kinds: the ordinary Fitting ideal and its divisor, and another 'determinantal' ideal obtained through Noether normalization. One or the other are useful in studying completeness, or even normality, in some classes of modules over singular rings.

Resolution of Singularities; computational Aspects

O. Villamayor (Universidad Autónoma de Madrid)

There are two main theorem in resolution of singularities: embedded desingularization of reduced schemes, and Log-resolution of ideals in a smooth scheme. Both theorem proved by Hironaka for the case of excellent schemes over a field of characteristic zero. We intend to sketch an alternative and constructive proof, always over fields of characteristic zero, but defined in terms of an algorithm. We will also discuss some computational aspects of the Bodnar-Schicho implementation of this algorithm.

Monomial ideals and normality

R. H. Villarreal (Centro de Investigación y de Estudios Avanzados del IPN, México)

In this talk we present various recent results about the normality of ideals and algebras associated to monomials. Those results include obstructions for the normality of a monomial ideal and for the normality of a monomial subring.