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Abstracts

Session 09

Computational Methods in Algebra and Analysis

Organizers:

Francisco Jesús Castro-Jiménez (Universidad de Sevilla)
Eduardo Cattani (University of Massachusetts)

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Poincaré Series, Topology and Cyclotomic Factors

A. Campillo (Universidad de Valladolid)

The topology of an irreducible plane curve singularity can be described by means of the so-called semigroup of values of the singularity, which is an invariant associated with the only divisorial valuation naturally associated to its local ring. More precisely, one has a natural filtration on that local ring given by the valuation ideals whose associated Poincaré series coincides with the zeta function of the monodromy. Above Poincaré series, which coincides with that of the toric curve given by the semigroup of values, is therefore an alternating product of cyclotomic polynomials because of A'Campo zeta function formula. Recent results together with F.Delgado and S. Gussein-Zade show that precise formulae for analogous Poincaré series can be also obtained for quite more general objets (equipped with some associated valuations) than irreducible curve singularities. The computation of those Poincaré is done by integrating with respect to Euler characteristics some generalized topological spaces. Again, the Poincaré series sometimes becomes directly related to the topology or the geometry of the considered objet. On the algebraic side, the Poincaré series sometimes becomes an alternating product of cyclotomic type polynomials. We present a detailed account of statements as above.

Balanced Configurations and Rational Hypergeometric Functions

E. Cattani* (University of Massachusetts)

A. Dickenstein (Universidad de Buenos Aires)

We are interested in the problem of classifying gkz-rational toric subvarieties of projective space; that is, varieties whose associated discriminant appears in the denominator of a rational A-hypergeometric function in the sense of Gelfand, Kapranov and Zelevinsky. In the codimension-two case this leads to the combinatorial notion of *balanced* configurations of vectors. We classify balanced configurations of seven plane vectors up to $GL(2,\mathbb{R})$-equivalence and deduce that the only gkz-rational toric four-folds in $\mathbb{P}^6$ are those varieties associated with an essential Cayley configuration. In this case, we study a suitable hyperplane arrangement and show that all rational A-hypergeometric functions may be described in terms of toric residues.

Classical and modern approaches to bivariate hypergeometric functions

A. Dickenstein* (Universidad de Buenos Aires)

L. Felicia Matusevich (Harvard University)

T. Sadykov (Krasnoyarsk State University, Russia)

Classically, there have been two main directions in the study of hypergeometric functions. The first of these is to study the properties of a particular series. The other classical avenue of research is to find differential equations that hypergeometric functions satisfy. In the multivariate case, these Horn systems have not been studied systematically from the point of view of D-module theory.

We make a complete analysis of the bivariate case. We give a formula for the rank of a Horn system of two hypergeometric equations in two variables when the parameters are generic. We also show that the appearance of Puiseux polynomial solutions is a commonplace phenomenon.

One of the main tools in our approach is the theory of A-hypergeometric systems, one of the most recent efforts towards generalizing the definition of hypergeometric function to the multivariate realm.

How to use exterior algebras in elimination theory

D. Eisenbud* (MSRI/UC Berkeley)

F. -O. Schreyer (Saarland University)

I'll speak about recent work of mine with Frank–Olaf Schreyer, where we show how to use free resolutions over an exterior algebra to solve some elimination theory problems. The theory suggests a number of new problems whose answers might lead to further elimination methods. Some of them are explained further in my talk in Session 8.

Computation of Ratliff-Rush closure

J. Elias (Universidad de Barcelona)

Let R be a Cohen-Macaulay local ring with maximal ideal m. In this talk we present a procedure for computing the Ratliff-Rush closure of a m-primary ideal $I \subset R$.

Computational K-theory for polynomial rings over Dedekind domains

J. Gago-Vargas* (Universidad de Sevilla)

S. Arias de Reyna Domínguez (Universidad de Sevilla)

Let D be a Dedekind domain and consider a $n \times n$ matrix $A = (f_{ij}), n \geq 3$ of determinant 1 with elements of the polynomial ring $D[x_1, \ldots, x_m]$. Suslin's Stability Theorem asserts that A can be written as a product of elementary matrices. There exist algorithms to compute such decomposition when D is a field or an Euclidean domain. We present here a procedure when D is the ring of algebraic integers of a number field, as for example $\mathbb{Z}[\sqrt{-5}]$.
Let P be a finitely generated projective module over $D[x_1, \ldots, x_m]$. The Quillen-Suslin Theorem says that P is extended from D. It is known how to decide the freeness of P. As a corollary of the previous method, we obtain an algorithm to compute a free basis of P if there exists any.
Gröbner basis techniques are used in the implementation of these algorithms.

Computer Algebra and Computer Aided Geometric Design

L. González-Vega (Universidad de Cantabria)

In this talk we will show how symbolic algorithms (together with numerical techniques) can be very useful to deal, in a very efficient way, with applied problems coming mainly from Computer Aided Geometric Design.

Explicit calculation of slopes in hypergeometric systems

M. I. Hartillo Hermoso (Universidad de Cádiz)

Algebraic Analysis studies in an algebraic way systems of linear partial differential equations. We can always consider such a system as a $\mathcal{D}$-module. The classical notion of irregular singular point has its analogous in the theory of $\mathcal{D}$-modules with the concept of slope of a $\mathcal{D}$-module. A slope of a $\mathcal{D}$-module is a gap of the Gevrey filtration of the module, this concept comes from the irregular sheaf associated to the module, due to Mebkhout. With that new point of view the explicit calculation of slopes is possible (Assi, Castro and Granger) but in practice it is not very efficient. In the set of hypergeometric systems we can find all the slopes of those systems with new tools based in commutative algebra.

This work is partially supported by BFM2001-3164 and FQM-813.

Determinantal Formulas for Resultants in Low Dimension

A. Khetan (University of California Berkeley)

The resultant of $n + 1$ polynomials in n variables is a single polynomial in the coefficient which vanishes when the system has a common zero. Resultants are therefore important in elimination theory and polynomial system solving. We present new exact determinantal formulas for any unmixed resultant in 2 or 3 variables. This generalizes the known formulas for classical multivariate resultants. The main techniques used are sheaf cohomology on toric varieties and resolutions over an exterior algebra.

Hypergeometric systems and local cohomology

L. F. Matusevich (Harvard University)

We report recent progress, joint with Ezra Miller, on a conjecture that explicitly describes the exceptional set of a matrix A in terms of the local cohomology of the underlying toric ideal I_A.

Computing with locally effective matrices

J. Rubio García (Universidad de la Rioja)

In this work, we start from the naif notion of integer *infinite matrix* (that is to say, the set of functions $\mathbb{Z}^{\mathbb{N}\times\mathbb{N}} = \{f : \mathbb{N} \times \mathbb{N} \to \mathbb{Z}\}$). Then, several undecidability results are established, leading to a convenient data structure for effective machine computations. We call this data structure *locally effective matrix*. We study when (and how) the standard matrix calculus (Ker and CoKer computations) can be extended to the infinite case. We find again several *undecidability barriers*. When these limitations are overcome, we describe effective procedures for computing in the locally effective case. Finally, the role of these data structures in the development of real symbolic computation systems for Algebraic Topology (based on Sergeraert's notion of *effective homology*) is illustrated.

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Finite dimensional representations of invariants under tori of the Weyl algebra

S. L. Rueda Pérez (Universidad de Almería)

Let k be an algebraically closed field of characteristic zero and $V = k^s$. Let G be the torus of dimension l acting diagonally on V. Let $\mathcal{D}(V)^G$ be the ring of invariants under G of the Weyl algebra.

We show how to construct Y an open subset in V such that $Y//G$ is a toric variety $T_N emb(\Delta)$. There is some flexibility in this construction; the fan Δ determines Y but we require only a condition on the one dimensional cones of Δ.

We consider $\mathcal{D}(V)^G$-modules of covariants $\mathcal{O}(Y)_m$, $m \in \mathbb{Z}^l$. We show that each $\mathcal{O}(Y)_m$ is finite dimensional if and only if the fan Δ is not contained in a halfspace. Our main result says that $V^G = 0$ if and only if $\mathcal{D}(V)^G$ has enough simple finite dimensional representations, in the sense that the intersection of the kernels of all the simple finite dimensional representations is zero. In particular if $V^G = 0$ there is a Fourier transform $\mathcal{F}_I$ such that $\mathcal{F}_I\mathcal{O}(Y)_m$ is finite dimensional for all $m \in X(G)$.

Computational Methods for Rational Curves and Surfaces

J. Rafael Sendra Pons (Universidad de Alcalá)

The theoretical development of scientific computation (numerical-symbolic), and the computing feasibility provided by the recent technological advances in software and hardware, have implied an increasing interest in the field of application for the fundamentals and algorithms for curves and surfaces. This phenomenon has motivated a reciprocal relationship of interest between the fields of applications and development of constructive methods in algebraic geometry.

In this talk we plan to survey on symbolic algorithms for dealing with rational algebraic curves and surfaces as well as on some applications. More precisely, the talk will have two main parts. In the first one we will briefly report on the basic ideas of some of the symbolic algorithmic methods to conversion algorithms for rational curves and surfaces (i.e. parametrization algorithms, implicitization algorithms as well as inversion algorithms). The second part of the talk will illustrate how rational curves and surfaces can be used in some applications in the frame of computer aided geometric design. In particular, we will discuss the problem of offsetting, that basically consists in computing parallel curves and surfaces, and the problem of blending where algebraic surfaces providing a smooth transition among distinct geometric features of an object are computed. These problems will be treated under the assumption that the input is parametrically represented.

Algorithms for the ring of differential operators and their applications

N. Takayama (Kobe University)

In this talk, I will survey on studies on algorithms for the ring of differential operators D and their applications with demonstrations by a computer. After explaining several division algorithms for D, I will discuss about applications to A-hypergeometric systems, computational algebraic geometry (localization, b-function, de Rham cohomology), asymptotic analysis and a digital formula book project.

On Bernstein-Sato ideals

F. J. Castro Jiménez (Universidad de Sevilla)

J. M. Ucha Enríquez* (Universidad de Sevilla)

Let $(f_1, \ldots, f_p)$ be a vector of non-zero polynomials in $\mathbf{C}[x_1, \ldots, x_n]$ and $\mathcal{B}$ be the *Bernstein-Sato ideal* of $(f_1, \ldots, f_p)$ consisting of polynomials $b(s) \in \mathbf{C}[s] = \mathbf{C}[s_1, \ldots, s_p]$ such that there exists a linear differential operator $P(s)$ with coefficients in $\mathbf{C}[x_1, \ldots, x_n, s]$ satisfying

$$b(s) f_1^{s_1} \cdots f_p^{s_p} = P(s) f_1^{s_1+1} \cdots f_p^{s_p+1}.$$

In this talk we compare the approach of Briançon-Maisonobe to compute Bernstein-Sato ideals —based on computations in a Poincaré-Birkhoff-Witt algebra— to the available method of Oaku-Takayama. We show that it can manage interesting examples intractable so far.

Stratifications induced by homomorphisms between holonomic sheaves

U. Walther (Purdue University)

Let M be a holonomic module on $\mathbb{C}^d$. We give an algorithm to stratify $\mathbb{C}^d$ such that on each stratum X each restriction (derived inverse image) module $H^i(\rho_{X,\mathbb{C}^d}(M))$ is a connection. For regular holonomic modules this stratifies the solution complex $\mathbb{R}Hom_{\mathcal{D}}(\mathcal{M}, \mathcal{O})$. We consider some special cases of this stratification for GKZ-systems.