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Abstracts

Session 10

Constructive Approximation Theory

Organizers:

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(In this index, in case of multiple authors, only the speaker is shown)

On q-polynomials on the exponential lattice

R. Alvarez-Nodarse (University of Sevilla)

The main aim of this talk is to present some results concerning the q-polynomials of the Hahn class and their applications in the construction of some discrete models.

The Schwarzian Derivative and Hyperbolically Convex Functions

R. Barnard (University of South Carolina)

We discuss our verification of Mejía-Pommerenke's conjectures in their article *On Hyperbolically Convex Functions*. These results give the sharp bound for the Schwarzian derivative of a hyperbolically convex function. This problem was also recently mentioned in a paper by Ma and Minda. We use variational techniques and Special Function computations.

Information entropies of special functions and orthogonal polynomials

J. S. Dehesa (University of Granada)

The present knowledge of the information entropies of the special functions of mathematical physics as well as its applications to analytically solvable quantum-mechanical problems will be reviewed. Emphasis will be done in the broad class of generalized hypergeometric-type functions, which encompasses all the classical orthogonal polynomials as well as the most common special functions (Airy, Bessel, Macdonald, ...). A number of new results will be also critically discussed. The notion of entropy of a special function, which plays a relevant role in the approximation theory and other branches of mathematics, is shown to be a very useful tool in the study of the internal disorder of physical systems as well as for the theoretical interpretation of numerous physical and chemical phenomena in the framework of the modern density-functional theory.

The Mathematics of Analogue to Digital encoding

R. DeVore (University of South Carolina)

Digital Signal Processing (DSP) has revolutionized the storage and transmission of audio and video signals in consumer electronics and also in scientific settings. The main advantage of DSP is its robustness: although all of the operations have to be implemented (by necessity) in not quite ideal hardware, the a priori knowledge that all correct outcomes must lie in a very restricted set of well separated numbers makes it possible to recover them by round off appropriately.

However, many signals (audio signals e.g.) are not digital but are rather analog in nature. For this reason the first step in any digital processing of such signals is a conversion of the analog signal to the digital world. The question is then what is the most efficient method to do such a conversion. A first mathematical look would conclude the problem to be trivial: sample at Nyquist rate and encode these samples in binary. However, this is generally not done in practice. Rather engineers use a quite unexpected encoding consisting of high oversampling of the signal followed by very coarse (e.g. one bit) quantization. Such methods of encoding lead to an array of interesting mathematical questions.

This talk will discuss one bit quantization methods with an eye to explaining why engineers prefer this method. The talk requires no background in signal processing and little mathematical sophistication.

Asymptotic expansions of integrals: application to singular perturbation problems

J. L. López (Public University of Navarra)

Asymptotic expansions of solutions of important ordinary differential equations in science (special functions) may be derived by Olver's method. Asymptotic expansions of special functions may be derived also by using asymptotic methods for integrals when an integral representation is available (as it usually happens). In the case of partial differential equations (boundary value problems) the situation is more complicated: a powerful method similar to Olver's method is not available.

In this work we try to seed light on the applications of the asymptotic techniques for integrals to the approximation of solutions of boundary value problems. For this purpose we consider a famous topic in partial differential equations, convection-diffusion with singular perturbation: $-\epsilon\Delta u + \overrightarrow{v} \cdot \overrightarrow{\nabla} u = 0$ in $\Omega \subset \mathbb{R}^2$, $u|_{\partial\Omega} = f(\tilde{x})$ with $\tilde{x} \in \partial\Omega$, $0 < \epsilon << 1$ and $\overrightarrow{v} \in \mathbb{R}^2$. For several domains Ω and several discontinuous Dirichlet conditions $f(\tilde{x})$ we derive integral representations of the solution. Then we apply asymptotic methods for integrals to obtain asymptotic expansions of $u(x)$ when: a) the singular parameter $\epsilon \to 0^+$ and b) we approach the discontinuity point, $r \to 0^+$. The asymptotic expansion at $\epsilon = 0$ is derived by a classical method, whereas the derivation of the expansion at $r = 0$ requires the distributional approach. The first term of the $\epsilon \to 0^+$ expansion is a combination of error functions.

Advances on Hermite-Padé approximation of Nikishin systems

G. López Lagomasino (University Carlos III of Madrid)

Hermite-Padé or simultaneous Padé approximation of vector valued functions play an important role in applications to number theory of rational approximation. Such approximants are vectors of rational functions with common denominator built on the basis of interpolatory conditions. Unfortunately, convergence results are hard to obtain. For so called Nikishin systems, a general theory is fairly well developed. Such systems consist of vectors of Markov type functions all of which are supported on the same interval. In this talk we will report on recent advances in this theory regarding normality of indices, that is to say the uniqueness of the interpolating rational function defined through the interpolation requirements. We will discuss extensions to the so called multi-point case when interpolation is carried on according to a predetermined table of points. We will consider the question of convergence in the uniform norm and in logarithmic capacity. Finally, we see the connection of this construction with that of simultaneous schemes of quadrature formulas.

Orthogonal Polynomials for Exponential Weights

D. Lubinsky (Georgia Institute of Technology)

We discuss orthogonal polynomials for exponential weights $|x|^a \exp(-Q)$ on $[0, d)$ where d may be finite or infinite. In particular, we present bounds and asymptotics for the orthogonal polynomials, and some features of their zeros, weighted Markov-Bernstein inequalities, and Christoffel functions. Estimates for Laguerre polynomials, "one-sided" Pollaczek, and Erdos weights are special cases.

Some applications will also be presented: the size of the smallest eigenvalues of some Hankel matrices considered by Szego, Widom, Wilf ... ; also how ideas from orthogonal polynomial ideas might be used in investigating Bernstein's constant.

Recent trends in Sobolev Orthogonal Polynomials

F. Marcellán Español (University Carlos III of Madrid)

We summarize some recent work on the analytic properties of sequences of polynomials orthogonal with respect to a Sobolev inner product. We will cover the following topics

1.- Strong Asymptotics of orthogonal polynomials with respect to measures of bounded support [5].
2.- Plancherel-Rotach Asymptotics for Freud-Sobolev orthogonal Polynomials [4].
3.- Generalized coherent pairs of measures [3].
4.- Best polynomial approximation in weighted Sobolev spaces [2].
5.- Spectral properties of the multiplication operator in weighted Sobolev spaces and zeros of orthogonal polynomials [1, 5]. Finally, some open problems will be considered.

References.

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[5] A. Martínez Finkelshtein: "Bernstein-Szegö´s Theorem for Sobolev Orthogonal Polynomials". Constr. Approx. 16 (2000) 73-84.
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Jacobi polynomials with general parameters

A. Martínez Finkelshtein* (University of Almería)

A.B.J. Kuijlaars (Katholieke Universiteit Leuven, Belgium)

R. Orive (University of La Laguna, Spain)

The Jacobi polynomials $P_n^{(\alpha,\beta)}$, with $\alpha, \beta > -1$, constitute probably the most "classical" family of orthogonal polynomials, and their main properties are well known. Nevertheless, once we allow for arbitrary real values of the parameters, new features appear.

We establish orthogonality conditions satisfied by $P_n^{(\alpha,\beta)}$ on a generic closed contour on a Riemann surface when α and β are not necessarily > -1. This, depending on the parameters, leads to either full orthogonality conditions on a single contour in the plane, or to multiple orthogonality conditions on a number of contours in the plane.

These results provide the necessary setting for the application of Riemann-Hilbert analysis which gives us full and uniform asymptotics of the sequences $P_n^{(\alpha_n,\beta_n)}$ as $n \to \infty$, assuming the existence of the limits

$$\lim_n \frac{\alpha_n}{n} \in \mathbb{R} \quad \text{and} \quad \lim_n \frac{\beta_n}{n} \in \mathbb{R}\,.$$