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Abstracts

Session 12

Differential Galois Theory

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Application of a criterium for testing the non complete integrability of a hamiltonian system

D. Boucher (IRMAR-Université de Rennes I)

In this talk we recall a criterium of non complete integrability for hamiltonian systems deduced from the theorem of J.-J. Morales and J.-P. Ramis. Using tools from Computer Algebra, we apply it to prove the non complete integrability of the modele of a satellite in the general case.

A Reduction for Regular Differential Systems

M. Bronstein* (INRIA)

B. M. Trager (IBM Watson Research Center)

We propose a definition of regularity of a linear differential system with coefficients in a monomial extension of a differential field, as well as a global and truly rational (*i.e.* factorisation–free) iteration that transforms a system with regular finite singularities into an equivalent one with simple finite poles. We then apply our iteration to systems satisfied by bases of algebraic function fields, obtaining algorithms for computing the number of irreducible components and the genus of algebraic curves.

Connections on vector bundles, differential equations and Fuchs' relations

E. Corel (Universidad de Barcelona)

We give a formal algebraic notion of exponents for linear connections on a vector bundle over a Riemann surface, as eigenvalues of the residue of an attached canonical regular connection on a maximal logarithmic subbundle (that we call the *Levelt lattice*). In particular, we give estimates on the sum of these exponents that can be seen as generalizations of the classical Fuchs relation for differential equations.

As an application, we construct for every fuchsian differential equation on a compact Riemann surface, a holomorphic vector bundle which is logarithmic with respect to the connection attached to the equation, and has the same set of exponents.

Differential Galois realization of covers

T. Crespo Vicente (Universidad de Barcelona)

Z. Hajto* (Universidad de Barcelona)

We present an explicit construction of linear differential equations of order 3 with Galois group the Valentiner group. Our method is based on group representation and symmetric products of differential equations.

Generic Picard–Vessiot extensions and examples

L. Juan (Texas Tech University)

Let G be a linear algebraic group over the algebraically closed field C. A generic Picard-Vessiot extension $\mathcal{E} \supset \mathcal{F}$ with group G is a Picard–Vessiot extension with G as its differential Galois group and with the property that any Picard–Vessiot extension $E \supset F$ with G as its differential Galois group as well can be obtained by a suitable specialization of the derivation in $\mathcal{E}$. We will show how such extensions can be constructed for connected linear algebraic groups and present some examples.

The differential Galois theory of strongly normal extensions

J. J. Kovacic (The City University of New York)

Differential Galois theory, the theory of strongly normal extensions, has unfortunately lanquished, possibly due to its reliance on Kolchin's elegant, but not widely adopted, axiomatization of the theory of algebraic groups. Here we attempt to revive the theory using affine differential schemes. We also avoid Weil's "group chunks" and get the Galois group canonically identified (not merely birationally) with the closed points of an affine differential scheme. We do not need a universal differential field, but use a certain tensor product instead. This tensor product is a Sweedler coring in a natural way and the comultiplication translates into the Galois group operation. Diffspec of this tensor product splits, i.e. is obtained by base extension from a (not differential, not necessarily affine) group scheme. And the converse also holds. If diffspec of a certain tensor product splits then it comes from a strongly normal extension. Thus we have a geometric characterization of the notion of strongly normal extension.

Lamé differential operators with finite monodromy

R. Litcanu (Université de Lille 1)

A classical theorem of Klein asserts that a second order differential operator on an algebraic curve has finite projective monodromy group if and only if it is a pull-back, via a rational function f, of a hypergeometric operator in the "Schwarz list". Under some additional hypothesis, f has at most three branching points - it is a so-called "Belyi cover". We shall explain how the combinatics associated to such a cover can be used for describing the Lamé differential operators with a full set of algebraic solutions.

Non-integrability of the heavy top problem. A differential Galois approach

A. J. Maciejewski* (University of Zielona Gora)

M. Przybylska (Nicholaus Copernicus University)

In his celebrated papers Ziglin [1, 2] proved that the symmetric heavy top problem does not admit an additional complex meromorphic first integral except the known cases of Euler, Lagrange, Kovalevskaya and Goryachev-Chaplygin (in the last case the integral exist only on the zero level of the area integral) . Later on Ziglin [3] shows that except the mentioned cases the system does not admit a real meromorphic additional first integral. We present new, purely algebraic proofs of these theorems which are based on a differential Galois approach.

References

[1] S. L. Ziglin.*Functional Anal. Appl.*, 16:181–189, 1982.

[2] S. L. Ziglin. *Functional Anal. Appl.*, 17:6–17, 1983.

[3] S. L. Ziglin. *Funktsional. Anal. i Prilozhen.*, 31(1):3–11, 95, 1997.

The Ring of All Solutions of All Linear Differential Equations and Categorical Duality

A. R. Magid (University of Oklahoma)

The category of finite dimensional modules for the proalgebraic differential Galois group of the differential Galois theoretic closure of a differential field F (with algebraically closed characteristic zero constant field) is equivalent to the category of finite dimensional F spaces with an endomorphism extending the derivation of F (see van der Put, M. and Singer, M. *Differential Galois Theory* (to appear), `http://www4.ncsu.edu/~singer/ms_papers.html`).

This paper presents an expository proof of this fact using the ring S of all solutions of all linear differential equations over F which is an injective ind–object in both categories.

Algebraic Solutions of the Lamé Equations

R. S. Maier (University of Arizona)

One of the simplest nontrivial second-order Fuchsian differential equations on $\mathbf{P}^1(\mathbf{C})$ is the Lamé equation $L_{\ell,B;e_1,e_2,e_3}u = 0$. It has four singular points e_1, e_2, e_3, ∞, with exponent differences $1/2, 1/2, 1/2, \ell + 1/2$. Here $\ell, B \in \mathbf{C}$ are the degree and accessory parameters. If an elliptic curve E_{g_2,g_3}, a double cover of $\mathbf{P}^1(\mathbf{C})$, is defined by $y^2 = 4x^3 - g_2x - g_3 = \prod_i(x - e_i)$, the Lamé equation can be pulled back to it, yielding $L_{\ell,B;g_2,g_3}u = 0$, the simplest nontrivial second-order equation on an elliptic curve (one singular point). For both equations, it is unsolved for which parameter values all solutions are algebraic over $\mathbf{C}(x)$, resp. $\mathbf{C}(x, y)$.

We correct previous work on necessary conditions for each finite subgroup of $PGL(2, \mathbf{C})$ to occur as the projective monodromy and Galois group, G. We use the Baldassarri–Dwork modernization of Klein's pullback theorem, which says that if a second-order equation on an algebraic curve has finite projective monodromy, it must be a hypergeometric pullback. It was thought $G(L_{\ell,B;e_1,e_2,e_3})$ could be octahedral only if $\ell \in \mathbf{Z} \pm 1/4$, and $G(L_{\ell,B;g_2,g_3})$ could not be octahedral. We correct this, and show how each possible polyhedral group can occur.

Model Theory and Differential Galois Theory

D. Marker (University of Illinois at Chicago)

Poizat showed how model theoretic methods could be used to give a new approach to Kolchin's Galois theory of strongly normal extensions. Pillay extended this approach to develop a Galois theory of extensions where arbitrary finite dimensional differential algebraic groups arise as Galois groups. I will describe Pillay's work and some partial work on inverse problems in this setting.

Inverse differential problems

C. Mitschi (Université Louis Pasteur)

The talk will present different possible formulations of inverse problems for differential equations over the field $C(x)$ of rational functions whit either a general algebraic closed field C of characteristic zero or $C = \mathbf{C}$, the complex numbers, focusing in the second case on the type of singularities occurring in the differential equations solving the inverse problem.

Differential Galois Theory and Integrability

J. J. Morales-Ruiz (Universidad Politécnica de Cataluña)

We give a review of the approach to the integrability of the complex analytical Hamiltonian systems by means of the Differential Galois Theory. More concretely, by a joint result with J.-P. Ramis, if the Hamiltonian system is completely integrable with meromorphic first integrals, then the identity component of the differential Galois group of the variational equation along a particular integral curve must be abelian. Furthermore, we explain a work in progress with J.-P. Ramis about the extension of this result to the variational equation along Lagrangian Manifolds.

D-modules and irreducible plane curves

O. Neto* (Universidade de Lisboa)

P. Silva (Universidade de Lisboa)

Preliminary report.

Pursuing the line of research started in "A Microlocal Riemann-Hilbert Correspondence, Compositio Mathematicae 127, 2001, pp. 229-241" we study the linear representations of the local fundamental group of the germ of an irreducible plane curve that define D-modules with characteristic variety equal to the union of the zero section with the conormal of the curve. We extend to this situation the notion of rigid local system on a Riemann surface (cf. N. Katz, Rigid local systems, P.U.P. 95) and classify these local systems under some reasonable hypothesis.

Principal invariant ideals for some polynomial derivations

A. Nowicki (Nicolaus Copernicus University)

Let d be a derivation of a polynomial algebra $k[X] = k[x_1, \ldots, x_n]$, where k is a field of characteristic zero. A polynomial $F \in k[X] \setminus k$ is called a Darboux polynomial of d if $d(F) = PF$ for some $P \in k[X]$. We present properties of Darboux polynomials. Moreover, we present some new classes of homogeneous derivations of $k[X]$ without Darboux polynomials.

Picard-Vessiot theory in positive characteristic

A. Pillay (University of Illinois at Urbana-Champaign)

We use the model theory of separably closed fields with an iterative Hasse derivation to give an account of the galois theory of linear iterative differential equations (existence and uniqueness of the Picard-Vessiot extension as well as the Galois correspondence).

Invariant Theory and Galois Theory for q-difference Equations

J.-P. Ramis (Toulouse University and Institut Universitaire de France)

I will describe the recent complete solution of the "Birkhoff Program" about the classification of linear analytic q-difference equations in the complex domain. (Generalizations of the Riemann-Hilbert problem.)

I will insist on the fuchsian case and its relation with the differential case and explain the relations between the (algebraic and transcendental) invariants and the Galois theory.

Differential jet schemes for PDEs

T. Scanlon (University of California, Berkeley)

Building on recent work of Pillay and Ziegler we develop various analogues of jet and arc spaces for differential schemes over partial differential fields. Analyzing these spaces we reduce the general study of the Zilber trichotomy for regular types (given by systems of potentially non-linear equations) in partial differential fields of characteristic zero to the case of additive groups defined by linear differential equations having regular generic types.

[Some of the work reported was performed jointly with R. Moosa.]

On a characterization of the Painlevé equations based on differential Galois theory of infinite dimension

H. Umemura (Nagoya University)

Today we know several definitions of the Painlevé equations other than the original definition: the differential equations $y'' = f(t, y, y')$ without movable singular points that are not integrable by the so far known functions, where f is a rational function of the independent variable t, dependent variable y' and the derivative y'

We analyze an idea of J.Drach in 1914 of characterizing the sixth Painlevé equation. The characterization depends on the differential Galois theory of infinite dimension which is an object of discussions. We replace his differential Galois theory of infinite dimension by ours proposed in 1996.

The multidimensional Jouanolou system

H. Zoladek (Warsaw University)

It is shown that the Jouanolou system $\dot{x}_1 = x_2^s, \ldots, \dot{x}_{n-1} = x_n^s, \dot{x}_n = x_1^s$ has no invariant hypersurfaces for $n \geq 3$ and $s \geq 2$.