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Abstracts

Session 13

Differential structures and homological methods in commutative algebra and algebraic geometry

Organizers:

Gennady Lyubeznik (University of Minnesota)
Luis Narváez-Macarro (Universidad de Sevilla)

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Bousfield localization on schemes

Leovigildo Alonso Tarrío* (Universidade de Santiago)

Ana Jeremías López (Universidade de Santiago)

Let X be a separated noetherian scheme and $\mathbf{D}(X)$ its derived category of quasi-coherent sheaves. For X affine, A. Neeman has established a bijection between Bousfield localizations of $\mathbf{D}(X)$ and subsets of X. We extend this results to general X. Subsets of X correspond to *rigid* localizations in $\mathbf{D}(X)$, i.e. those whose associated localizing subcategories are stable for derived tensor products. Moreover, we show that if the subset is an arbitrary union of closed subsets Z, then the associated colocalization is cohomology with supports in Z, $\mathbf{R}\varGamma_Z$. If the subset is an arbitrary intersection of open sets $\bigcap U_i$ then the ideal sheaves associated to the closed subsets $X \setminus U_i$ define a linear topology $\mathcal{T}$ in the structural sheaf $\mathcal{O}_X$ and the associated localization is left-derived completion $\mathbf{L}\varLambda_{\mathcal{T}}$. For $Z = X \setminus \bigcap U_i$, the functors $\mathbf{R}\varGamma_Z$ and $\mathbf{L}\varLambda_{\mathcal{T}}$ are adjoint. This fact is a generalization in the noetherian case of "Greenlees-May duality", a result from previous work with J. Lipman.

Meromorphic functions with respect to a locally quasi-homogeneous free divisor

Francisco Javier Calderón Moreno (Universidad de Sevilla)

I expose a recent joint work with Luis Narváez. We obtain a free resolution of the $\mathcal{D}$-modules

$$M_k = \mathcal{D}/\mathcal{D}F1(\mathrm{Ann}_{\mathcal{D}} f^{-k}),$$

where f is a local reduced equation of a locally quasi-homogeneous free divisor, and $k >> 0$. In particular, we obtain a free resolution of $\mathcal{D}f^{-k}$.

In the case of a quasi-homogeneous curve f, all the quotients M_k are isomorphic to the ring of the meromorphic functions with poles along the curve. We pretend to prove that, in the case of a locally quasi-homogeneous free divisor, all the quotients M_k are isomorphic to the ring of the meromorphic functions with poles along the divisor. So, we will have a resolution of this ring, as in the case of curves.

A p-local approach to cohomology

Eric Friedlander (Northwestern University)

This is a report of on-going research with Julia Pevtsova in which we study the cohomology and representation theory of various Artin algebras by restriction to subalgebras isomorphic to the group algebra of the cyclic group Z/pZ.

On the formal stationary phase

Ricardo García López (Universitat de Barcelona)

Let $\mathbb{M}$ be a holonomic $\mathcal{D}$-module over the affine line. If $\mathbb{M}$ is of exponential type (in the sense of Malgrange, i.e. its formal slopes at infinity are smaller or equal than one) then the formal germ at infinity of the Fourier transform of $\mathbb{M}$ can be described in terms of the formal microlocalizations of $\mathbb{M}$ at its singular points, the formula obtained is a so-called stationary phase formula.

In this talk we will explain how some calculations of differential Galois groups can be performed using such formulas (transposing to this setting a computation of ℓ-adic monodromy groups due to N. Katz) and how to obtain a formal stationary phase formula for modules with arbitrary slopes at infinity, for this purpose one has to define a transformation which plays the rôle of the local Fourier transform for ℓ-adic sheaves labelled (∞, ∞') by Laumon.

Riemann Roch and the exponential map

Henri Gillet (University of Illinois at Chicago)

In this talk I will discuss joint work with my student Fatih Mehmet Unlu, on how one may apply ideas of Toledo, Tong and Kapranov to give an explicit proof of the Riemann-Roch theorem using “formal” methods.

Explicit models for perverse sheaves

Félix Gudiel Rodríguez (Universidad de Sevilla)

This is part of a joint work with L. Narvaez. Perverse sheaves are topological objects that generalize local systems and are equivalent,by means of the Riemann-Hilbert correspondence, to holonomic regular D-modules. In ”Faisceaux Pervers”, Beilinson-Bernstein-Deligne extracted the notion of t-structure over a triangulated category and proved that the category of analytic constructible perverse sheaves (”classical perverse sheaves”), can be obtained by a process of ”gluing” t-structures, that makes sense in general.

This point of view allows us to work with different perversities and to establish some precise relations between them. The main idea consists of constructing a functor relating d-perverse and (d-1)-perverse sheaves and, by iteration, with 0-perverse sheaves, which are usual sheaves. Construction of functor and many other results in this work are inspired by the formalism of vanishing cycles and the gluing of classical perverse sheaves of Deligne-Verdier and MacPherson-Vilonen.

The main result is an equivalence theorem, from which we deduce that any (generalized) perverse sheaf, and then any classical one, has a canonical model. Then, the category of (generalized) perverse sheaves is equivalent to a non full (resp. full) abelian subcategory of the category of usual bounded complexes (resp. up to homotopy).

Cohomological descent and weight filtration

Francisco Guillén Santos (Universidad de Barcelona)

In this talk we present a joint-work with V. Navarro Aznar (Publ. Math. IHES'02) which is a sequel of our previous theory of cubical hyperresolutions (Springer LNM, n.1335, '88), itself based in Hironaka's theorem of resolution of singularities.

In this work, using a precise version of Hironaka's theorem, we give, over a field of characteristic zero, a general criterion to extend cohomological functors defined on the category of non-singular algebraic varieties to the category of all algebraic varieties.

Essentially, our result states that if the category where the cohomological functors take values is a descent category, which means a variant of a derived category, then an extension exists when the functor satisfies the standard long exact sequence of a blowing up with a smooth centre.

Among the different applications of this criterion, we show the existence of a filtration L in the cohomology groups of an algebraic variety, which is part of the weight filtration in Deligne's sense. Despite its definition using the resolution of singularities, we can prove that the filtration L is independent of the resolution and it is functorial. In particular, we apply this results to the singular cohomology with integer coefficients, Hodge-De Rham cohomology and motivic cohomology.

Algorithmic proofs of two theorems of Stafford

Anton Leykin (University of Minnesota)

Two classical results of Stafford say that every (left) ideal of the n-th Weyl algebra A_n can be generated by two elements, and every holonomic A_n-module is cyclic, i.e. generated by one element. We modify Stafford's original proofs to make the algorithmic computation of these generators possible.

On the Residue Theorem for Formal Schemes

Joseph Lipman (Purdue University)

On formal schemes, local and global Grothendieck duality unify into a single theory. Algebraic local duality is "equivalent" to duality on affine formal schemes; under this equivalence, algebraic residues are closely related to the global trace map. The Residue Theorem describes a canonical instantiation of the theory via differential forms. The classical result for curves has been generalized to varieties over perfect fields–Asterisque 117, 1984, and further to certain proper maps between noetherian schemes by Sastry-Hubl –Amer. J. Math. 115, 1993. The generalization to maps between formal schemes draws one through a rich vein of functorial relationships, some of which will be exposed, as time permits.

The p-adic exponents of a differentiel equation

Zoghman Mebkhout (Université Paris VII - C.N.R.S.)

In this lecture we explain how the p-adic exponents of a differentiel equation lead to an analogue of the Riemann existence theorem on curve and how the rationality of theses exponents caracterise the existence of a Frobenius structure on a differentiel module.

Nonabelian Hodge Theory in Characteristic p

Arthur Ogus (University of California at Berkeley)

Simpson's nonabelian Hodge theory establishes a (partial) equivalence between the category of representations of the fundamental group of a compact Kahler manifold X and the category of Higgs bundles on X. In joint work with Vadim Vologodsky, we provide an analog of Simpson's theory in characteristic p, in which the p-curvature plays the role of the Higgs field. As in Simpson's theory, our correspondence is compatible with cohomology. We thus obtain a generalization of the Hodge decomposition of Deligne and Illusie to the case of modules with suitably nilpotent integrable connection.

The Hodge filtration for the de Rham complex of higher level

Bernard Le Stum (University Rennes)

Adolfo Quirós Gracián* (Universidad Autónoma de Madrid)

Let X be a proper and smooth scheme over a (nice) p-adic base S. Using the notion of transversal crystal, Ogus has proved a version with coefficients in an F-crystal of Mazur's theorem on the link between the eigenvalues of Frobenius and the Hodge filtration. We later used Berthelot's theory of crystals of level m ($m = 0$ is the classical case) to study transversal m-crystals. To carry out calculations, we constructed [J. of Algebra 240 (2001)] a de Rham complex of higher level, $\Omega^{\bullet}_{X/Sm}$. It satisfies a Poincaré lemma and can be used to calculate the crystalline cohomology of level m of an m-crystal.

A closer look at the construction we used to define $\Omega^{\bullet}_{X/Sm}$ (we call it the Berthelot-Lieberman construction) allows us to prove a filtered Poincaré lemma.

Theorem: *Let $u_X : (X/S)^{(m)}_{cris} \to X_{Zar}$ be the projection from the crystalline site of level m to the Zariski site. If E is a transversal m-crystal, there is a canonical isomorphism in the filtered derived category $Ru_{X*}E \simeq E_X \otimes \Omega^{\bullet}_{X/Sm}$.*

When looking at the classical crystalline Dieudonné module, the Hodge filtration is determined by the invariant differentials. Using the Berthelot-Lieberman construction, we can show that the Hodge filtration is also related to invariant differentials in level $m > 0$. But the relation is slightly more subtle than for $m = 0$.

Differential Operators on Toric Schemes

William Traves* (U.S. Naval Academy)

Mutsumi Saito (Hokkaido University, Japan)

The ring of differential operators on a toric variety has a nice combinatorial structure, investigated by many authors. It finds application in the study of A-hypergeometric differential equations: the ring of differential operators is anti-isomorphic to the symmetry algebra that determines equivalent differential systems. For this application, it is important to study differential operators on toric *schemes*, schemes determined by binomial equations. This led to our interest in the rings $D(R)$ and $GrD(R)$ for $R = \mathbb{C}[\mathbb{N}A]$. We characterize when these rings are finitely generated.