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Abstracts

Session 14

Discrete and Computational Geometry

Organizers:

Ferran Hurtado (Universitat Politècnica de Catalunya)
William Steiger (Rutgers, the State University of New Jersey)

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On Parallel Diagonal Flips in Triangulations

P. Bose* (Carleton University)

J. Czyzowicz (U. Quebec en Outaouais)

Z. Gao (Carleton University)

P. Morin (Carleton University)

D. R. Wood (Carleton University)

Let G be a *triangulation*; that is, a simple planar graph with a fixed plane (combinatorial) embedding such that every face consists of three edges. Let vw be an edge of G. Let x and y be the vertices such that vw is incident to the faces vwx and wvy. If xy is not an edge of G then vw is *flippable* and the operation of replacing vw with xy is a (*diagonal*) *flip*.

A set S of edges in a triangulation G is *flippable* if S contains no two consecutive edges, and no parallel edges are produced by flipping every edge in S. Flipping every edge of S in parallel is called a *parallel* (*diagonal*) *flip*.

We review some results on parallel diagonal flips in triangulations. We then show that every triangulation with at least six vertices has a parallel diagonal flip into a 4-connected triangulation. This result is used to prove that for any two n-vertex triangulations, there exists a sequence of $O(\log n)$ parallel flips to transform one into the other.

Jacobi submanifolds of multiple Morse functions

H. Edelsbrunner (Duke University)

The *contour* of a smooth surface and a viewing direction in three-dimensional space can be defined as the set of points at which the gradient of the two coordinate functions of the projection are parallel. The image of this set is also known as the *fold* of the two functions. More generally, we define the *Jacobi submanifold* of $k+1$ Morse functions over a common d-manifold as the set of points where the $k+1$ gradients are linearly dependent. Generically and for $k+1 \leq d$, this submanifold is a smoothly embedded k-manifold in the d-manifold.

We present a combinatorial algorithm that constructs the piecewise linear analogue of the Jacobi submanifold for $k+1$ functions defined over a common triangulation. The running time of the algorithm depends on the definition of a critical point of a piecewise linear function. Using the Euler characteristic of the lower link the time is proportional to the number of simplices in the triangulation, using Betti numbers modulo 2 the time is cubic, and demanding non-contractibility leads to undecidable subproblems.

Delaunay complexity of points on surfaces

J. Erickson (University of Illinois)

In the last few years, several algorithms have been developed that provably reconstruct an unknown smooth surface from an unorganized set of sample points. Almost all these algorithms start by constructing the three-dimensional Delaunay triangulation of the sample points. Despite the well-known fact that the Delaunay triangulation of n points in 3-space can have complexity $\Omega(n2)$, these algorithms are remarkably efficient in practice. For all practical purposes, three-dimensional Delaunay triangulations appear to have linear complexity.

This talk will survey several recent results that attempt to explain this frustrating discrepancy between theory and practice. The worst-case Delaunay complexity of n sample points on a surface is either $\Theta(n2)$, $\Theta(n^{3/2})$, $\Theta(n \log^c n)$, or $\Theta(n)$, depending on exactly how the question is asked: whether the surface is considered fixed or allowed to vary with n; whether we consider generic smooth surfaces, arbitrary smooth surfaces, or polyhedra; whether the sample points are chosen randomly or deterministically; and how much we assume about the density of the sample points. These results imply that super-linear Delaunay complexity is possible, even for very well-behaved point sets, but only if both the surface and the sample points are chosen maliciously.

A lower bound for the rectilinear crossing number of the complete graph

B. Ábrego (California State University, Northridge)

S. Fernández* (California State University, Northridge)

Given a set P of n points in the plane, no three of them collinear, draw all straight line segments joining pairs of points in P. This will give a drawing of the complete graph with n vertices. We denote by $\overline{cr}(P)$ the number of edge-crossings in such a drawing (a crossing is given by a pair of edges intersecting in their interior). Let $\overline{cr}(n)$ be the minimum $\overline{cr}(P)$ over all such sets P.The number $\overline{cr}(n)$ also represents the minimum number of convex quadrilaterals determined by n points (no three collinear) in the plane. We prove that $\overline{cr}(n) \geq \frac{1}{4} \left\lfloor \frac{n}{2} \right\rfloor \left\lfloor \frac{n-1}{2} \right\rfloor \left\lfloor \frac{n-2}{2} \right\rfloor \left\lfloor \frac{n-3}{2} \right\rfloor$ for all $n \in \mathbb{Z}^+$.

Games on Triangulations

O. Aichholzer (Graz University of Technology)

D. Bremner (University of New Brunswick)

E. D. Demaine (Massachusetts Institute of Technology)

F. Hurtado* (Universitat Politècnica de Catalunya)

E. Kranakis (Carleton University)

H. Krasser (Graz University of Technology)

S. Ramaswami (Rutgers University)

S. Sethia (Oregon State University)

J. Urrutia (Universidad Nacional Autónoma de México)

Let S be a set of n points in the plane, which we assume to be in general position, i.e., no three points of S lie on the same line. A *triangulation* of S is a simplicial decomposition of its convex hull having S as vertex set.

In this work we consider several games involving the vertices, edges (straight-line segments) and faces (triangles) of some triangulation. We present games where two players $\mathcal{R}$(ed) and $\mathcal{B}$(lue) play in turns, as well as *solitaire* games for one player. In some *bichromatic* versions, player $\mathcal{R}$ will use red and player $\mathcal{B}$ will use blue, respectively, to color some element of the triangulation. In *monochromatic* variations, all players (maybe the single one) use the same color, green.

Games on triangulations come in three main flavors: *constructing*, *transforming* and *marking* (a triangulation). For each of the variety of games we are interested in characterizing who wins the game, and designing efficient algorithms to determine the winner and compute a winning strategy.

Proximate Planar Point Location

J. Iacono (Polytechnic University)

S. Langerman* (Université Libre de Bruxelles)

A new data structure is presented for planar point location that executes a point location query quickly if it is spatially near the previous query. Given a triangulation T of size n and a sequence of point location queries $A = q_1, \ldots q_m$, the structure presented executes q_i in time $O(\log d(q_{i-1}, q_i))$. The distance metric, d, that is used is a two dimensional generalization of rank distance that counts the number of triangles in a region from q_{i-1} to q_i. The data structure uses $O(n \log \log n)$ space.

Sets in Euclidean position in 2-orbifolds

M. Abellanas (Universidad Politécnica de Madrid)

C. Cortés (University of Sevilla)

G. Hernández (Universidad Politécnica de Madrid, Spain.)

A. Márquez* (University of Sevilla)

J. Valenzuela (University of Extremadura)

Intuitively, a set of sites on a surface is in Euclidean position, if points are so close each other that planar algorithms can be easily adapted in order to solve most of classical problems on Computational Geometry. In this work we: a) formalize a definition of the term "Euclidean position" for a relevant class of metric spaces, the Euclidean 2-orbifolds, b) present methods to compute whether a set of sites has this property or not, and c) give methods for finding sets in Euclidean position with maximal cardinality.

Delaunay graphs of order k

M. Abellanas (Universidad Politécnica de Madrid)

P. Bose (Carleton University)

J. García López (Universidad Politécnica de Madrid)

F. Hurtado (Universitat Politècnica de Catalunya)

P. A. Ramos* (Universidad de Alcalá)

Given a set of points S in the plane in general position and $p, q \in S$, we say that the edge pq has *order* k if there exists a circle passing through p and q containing $k-1$ points of S and there is no circle containing less than $k-1$. The Delaunay graph of order k is defined as the set of order k edges. If $k = 1$, we obtain the well known Delaunay triangulation but, up to our knowledge, the structure has not been previously studied for $k \geq 2$.

We use the concept of order of an edge to define the order of a triangulation T of S. If we denote by E the set of edges of T, we can define

$$\text{order}(T) = \max_{e \in E} \text{order}(e).$$

Let $TG(k)$ be the graph whose vertices are the triangulations of order at most k and the adjacency is given by the flip operation. We show that $TG(2)$ is connected and conjecture that the same is true for arbitrary k.

A note on contracting edges on convex decompositions

F. Hurtado (Universitat Politècnica de Catalunya)

E. Rivera-Campo* (Universidad Autónoma Metropolitana-I.)

Let P be a set of points in general position in the plane. A set Π of convex polygons with vertices in P and with pairwise disjoint interiors is a *convex decomposition* of P if their union is the convex hull $CH(P)$ of P and no point of P lies in the interior of any polygon in Π.

Let Q be the set of points of P in the boundary of $CH(P)$. In this talk we prove that for each convex decomposition Π of P there is a sequence $P_0, P_1, \ldots, P_s$ of subsets of P with $P_0 = P$, $P_s = Q$ and a sequence $\Pi_0, \Pi_1, \ldots, \Pi_s$ such that $\Pi_0 = \Pi$ and, for $i = 1, 2, \ldots, s$, Π_i is a convex decomposition of P_i obtained from Π_{i-1} by either deleting or by contracting one edge of Π_{i-1}.

Small point sets whose graph of triangulations is not connected

F. Santos Leal (Universidad de Cantabria)

We present the construction of small (50 and 26 points, respectively) point sets in dimension 5 whose graphs of triangulations are not connected. The example improves our construction in *J. Amer. Math. Soc.* **13**:3 (2000), 611–637 not only in size, but also in the following two aspects:

- They can be perturbed into convex position, yielding 5-dimensional polytopes whose graphs of triangulations are not connected.
- They have unimodular triangulations in non-regular components of the graph of trianglations, which has important algebraic implications: the *toric Hilbert schemes* canonically associated to the point ses are not connected (in the 26-points example, a non-homogeneous embedding of the configuration is needed for this to be true).

Relative Geometric Inequalities

A. Cerdán (Universidad de Alicante)

U. Schnell (Universität Siegen)

S. Segura* (Universidad de Alicante)

Let G be a bounded, convex, open set, and let $E \subset G$; a relative geometric inequality is an inequality of the type

$$\frac{\mu(E)}{P(E,G)} \leq C,$$

where $P(E,G)$ is the relative perimeter (the measure of the part of the boundary of E which lies in G), μ is any geometric measure and C is a constant.

We summarize the classical Dido-type inequalities and present new results considering μ as the diameter and as the inradius, and characterize in both cases the maximizers describing some of their geometric properties.

Almost-Delaunay Tetrahedra and Applications to Protein Structure Analysis

D. Bandyopadhyay (University of North Carolina at Chapel Hill)

A. Tropsha (University of North Carolina at Chapel Hill)

J. Snoeyink* (University of North Carolina at Chapel Hill)

For a given finite sets of points, the Voronoi diagram, or its dual Delaunay tessellation, captures proximity and identifies some sets of points as neighbors. When the point positions are uncertain, we may wish to enlarge the collection of neighbor sets: we define the *almost-Delaunay simplices* to be the simplices that can become part of a Delaunay tessellation if the point coordinates are perturbed by at most $\epsilon \geq 0$. We relate the problem of finding almost-Delaunay simplicies to a minimum-width annulus problem, and use this to compute these simplices in 2d and 3d. We also use them to estimate the probability of simplices being Delaunay in this dimension, when point coordinates have a normal distribution. Delaunay tessellations of points representing protein atoms or residue positions have been used to compute molecular surfaces and protein volumes, to define cavities and pockets, to analyze and score packing interactions, and to find structural motifs. We use almost-Delaunay tetrahedra explore the robustness of the Delaunay four-body potential functions and derive a new method to detect structural motifs. `http://www.cs.unc.edu/~debug/papers/AlmDel`

The Convex Hull of Vertices in Random Hyperplane Arrangements

W. Steiger (Rutgers University)

Let $S = \{\pi_1, \ldots, \pi_n\}$ be a set of hyperplanes in general position in $R3$. The vertex set $V = \{\pi_i \cap \pi_j \cap \pi_k, i < j < k\}$ of this arrangement has size $O(n3)$ and we are interested in |Conv(V)|, the number of extreme points of its convex hull.

Suppose the planes are *chosen uniformly at random.* The specific model we use is that the planes in S are the duals of n points chosen uniformly and independently from $[0,1]^3$, under the familiar duality that maps a point $P = (x, y, z)$ to the plane $TP = \{(u, v, w) : w = xu + yv + z\}$. We prove that there is a constant $c > 0$ so that $E(|\text{Conv(V)}|) < \text{c}$.

Graph Embeddings with partial Oriented Matroid constraints

I. Streinu (Smith College)

We study questions of realizability for graph embeddings in the plane, under certain constraints of an oriented matroid nature. In general, such realizability questions are as hard as the existential theory of the reals. But for minimally rigid graphs, we offer a simple algorithmic solution. We will also discuss several applications, in particular for planar graph embeddings for which the vertex rotations are given as signed local (hyperline) sequences.

On Polyhedra Induced by Point Sets in Space

F. Hurtado (Universidad Politecnica de Catalunya)

G. Toussaint* (McGill University)

J. Trias (Universidad Politecnica de Catalunya)

Given a set S of n points in the plane (not all on a line) it is well known that it is always possible to *polygonize* S, i.e., construct a simple polygon P such that the vertices of P are precisely the given points in S. In 1994 Grünbaum showed that an analogous theorem holds in 3-dimensional space. More precisely, if S is a set of n points in space (not all of which are coplanar) then it is always possible to *polyhedronize* S, i,e., construct a simple (sphere-like) polyhedron P such that the vertices of P are precisely the given points in S. Grünbaum's constructive proof may yield *Schonhardt* polyhedra that cannot be triangulated. In this paper we propose several alternative algorithms for constructing such polyhedra induced by a set of points. Our methods yield polyhedra which not only may always be triangulated, but which enjoy several other useful properties as well. Such properties include polyhedra that are star-shaped, have hamiltonian skeletons, and admit efficient point location queries. Furthermore, we show that *polyhedronizations* with a variety of such useful properties can be computed efficiently in $O(n \log n)$ worst-case time.

Spanning trees, cycles and minimum weight spanning trees for colored point sets on the plane

J. Urrutia (Univ. Nacional Autńoma de México)

Let P_n be a set of points on the plane in general position such that its elements are colored with k colors $c_1, ..., c_k$. In this talk we will show that for each chromatic class of P_n we can find a spanning tree such that the total number of times the edges of these spanning trees intersect is at most $(k-1)n$. Similar results will be presented for matchings, spanning cycles and minimum weight spanning trees.