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Abstracts

Session 15

Dynamical Systems

Organizers:

George Haller (Massachusetts Institute of Technology)
Xavier Jarque (Universitat Autònoma de Barcelona)
Zbigniew Nitecki (Tufts University)
Enrique Ponce (Universidad de Sevilla)
Tere M. Seara (Universitat Politècnica de Catalunya)

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Sets of periods for tree maps: a characterization

Lluís Alsedà Soler* (Universitat Autònoma de Barcelona)

David Juher (Universitat de Girona)

Pere Mumbrú (Universitat de Barcelona)

In this talk we will describe a characterization of the set of periods of continuous tree maps, except for a finite set which is explicitly bounded. The characterization is given in terms of initial segments of the orderings introduced by Baldwin in the study of the sets of periods of star (n-od) maps.

Persistence of homoclinic orbits for billiards and twist maps

Sergey Bolotin* (University of Wisconsin-Madison)

Amadeu Delshams (Universitat Politècnica de Catalunya)

R. Ramírez-Ros (Universitat Politècnica de Catalunya)

We consider billiards inside n-dimensional ellipsoids with one diameter. The diameter is a hyperbolic periodic orbit with an n-dimensional family of homoclinic orbits. We give estimates for the number of primary homoclinic orbits for billiards in a domain which is a small perturbation of an ellipsoid. The estimate runs between six for prolate ellipsoids to $8n$ for generic ones, and is optimal for generic ellipsoids. The proof relies on a general result for perturbations of twist maps with hyperbolic periodic orbits whose stable and unstable manifolds have clean intersection along submanifolds verifying certain compactness conditions. We discuss also bifurcations of secondary homoclinic orbits for perturbations of integrable twist maps, in particular billiards in ellipsoids.

Geometric mechanisms for diffusion in Hamiltonian systems

Rafael de la Llave Canosa* (University of Texas at Austin)

Amadeu Delshams (Universitat Politècnica de Catalunya)

Tere M. Seara (Universitat Politècnica de Catalunya)

We will present several mechanisms for diffusion in Hamiltonian systems. They are based on identifying geometric objects and how they fit together.

This talk is a follow–up of the talk of A. Delshams.

A geometric mechanism for diffusion in Hamiltonian systems overcoming the large gap problem

Rafael de la Llave Canosa (University of Texas at Austin)

Amadeu Delshams* (Universitat Politècnica de Catalunya)

Tere M. Seara (Universitat Politècnica de Catalunya)

We introduce a geometric mechanism for diffusion in a priori unstable nearly integrable dynamical systems. It is based on the observation that resonances, besides destroying the primary KAM tori, create secondary tori and tori of lower dimension. We argue that these objects created by resonances can be incorporated in transition chains taking the place of the destroyed primary KAM tori.

We establish rigorously the existence of this mechanism in a simple model that has been studied before.

The main technique is to develop a toolkit to study in a unified way tori of different topologies and their invariant manifolds, their intersections as well as shadowing properties of these bi-asymptotic orbits. This toolkit is based on extending and unifying standard techniques. A new tool is the scattering map of normally hyperbolic invariant manifolds.

An attractive feature of the mechanism is that the transition chains are shorter in the places where the heuristic intuition and numerical experimentation suggests that the diffusion is strongest.

Cycles and systems of iterated functions: the role of secondary bifurcations

Lorenzo J. Díaz* (PUC-Rio de Janeiro)

Jorge Rocha (University of Porto)

Bianca Santoro (Massachusetts Institute of Technology)

We consider bifurcations via heterodimensional cycles (i.e., cycles associated to points having different indices). We see that in many cases the dynamics following these bifurcations is given by a system of iterated functions. Using such systems, we study the role of secondary bifurcations in this setting and we see how these bifurcations may lead to the phenomenon of collision, collapse and explosion of homoclinic classes (i.e., two different homoclinic classes corresponding to points of different indices at this bifurcation exhibit and intersection and thereafter explode and coincide)

High frequency perturbations of classical 1-DOF Hamiltonian systems with a parabolic fixed point

Inmaculada Baldomá (Universitat de Barcelona)

Ernest Fontich Julià* (Universitat de Barcelona)

We consider Hamiltonian systems $h_0(x,y) = \dfrac{y^2}{2} + V(x)$, where $V(x)$ is an analytic function of order $n \geq 3$ and we assume that h_0 has a homoclinic orbit, associated to the equilibrium point $(0,0)$. Notice that the linearization of the corresponding differential equations at the fixed point is $\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$.

We consider perturbations $\mu\varepsilon^p h_1(x,y,t/\varepsilon,\mu,\varepsilon)$ that are 2π-periodic with respect to t/ε and analytic with respect to (x,y). Let P^{t_0} be the Poincaré map from t_0 to $t_0 + 2\pi\varepsilon$. We denote by A the area of the lobe generated by its stable and unstable perturbed manifolds between two consecutive homoclinic points. We provide conditions such that the asymptotic expression for the splitting of separatrices is given by the Melnikov function.

The proof requires a careful analysis of the (complexified) invariant manifolds associated to the parabolic point, suitable flow-box coordinates as well as some tools developed by Lazutkin and Delshams-Seara.

Maximum number of limit cycles through Bendixson-Dulac Criterion

Armengol Gasull Embid* (Universitat Autònoma de Barcelona)

Hector Giacomini (CNRS et Universitè de Tours)

The Bendixson-Dulac Criterion for ℓ-connected sets is one of the well known methods used to find upper bounds for the number of limit cycles of planar ordinary differential equations, ODE for short. Recall that, essentially, this criterion reduces the problem to the study of the sign of a function of two variables. In this talk we consider two classes of planar ODE for which this final problem can be reduced to the study of a function of just one variable.

The first class contains Liénard differential equations. In particular, we present a polynomial Liénard system with exactly three limit cycles.

The second class is formed by the ODE of the form $\dot{x} = y$, $\dot{y} = h_0(x) + h_1(x)y + h_2(x)y^2 + h_3(x)y^3$. A remarkable fact in this last case is that the one variable function whose sign we have to control is solution of a linear ODE. Some concrete examples are also developed.

Patterns and minimal dynamics for graph maps

Lluís Alsedà Soler (Universitat Autònoma de Barcelona)

François Gautero (Université de Lille I)

John Guaschi* (Université Toulouse III)

Jérôme E. Los (Université d'Aix-Marseille I)

Francesc Mañosas (Universitat Autònoma de Barcelona)

Pere Mumbrú (Universitat de Barcelona)

We study the rigidity problem for periodic orbits of graph self-maps belonging to the same homotopy equivalence class. We first define a notion of *pattern* which enables us to compare periodic orbits of self-maps of homotopy-equivalent spaces. This definition unifies the known notions of pattern for other spaces (maps of the interval and the circle, and surface homeomorphisms). Our main results are as follows. First, given a free group endomorphism, we study the persistence under homotopy of the periodic orbits of its topological representatives. Secondly, in the case that the given endomorphism is also irreducible, we prove the minimality (within the homotopy class) of the set of periodic orbits of its efficient (in the sense of Bestvina-Handel) representatives. As a consequence, two efficient representaives of an irreducible endomorphism have (with at most $10(n-1)$ exceptions) the same number of periodic orbits of any pattern, n being the rank of the given free group.

Invariant manifolds in quasiperiodic systems: theory, computation and applications

Rafael de la Llave Canosa (University of Texas at Austin)

Àlex Haro Provinciale* (Universidad de Barcelona)

We explain the parameterization method to prove the existence of (normally hyperbolic) invariant tori and their whiskers in quasi-periodic systems. The method not only proves the existence of the usual stable and unstable manifolds, but also of non-resonant ones.

It also provides an effective algorithm to compute these manifolds, that we illustrate with different examples, as quasi-periodic perturbations of the Henon map and the standard map.

Results about the unfolding of the nilpotent singularity of codimension three

Santiago Ibáñez Mesa (Universidad de Oviedo)

Results will be given related to the different dynamical behaviours and bifurcations displayed by the family

$$\begin{cases} x' = y \\ y' = z \\ z' = \lambda + \mu y + \nu z + x^2, \end{cases}$$

where $\lambda^2 + \mu^2 + \nu^2 = 1$. Mainly two reasons motivate our interest in such a family. First one is that it plays an essential role in the understanding of the generic unfoldings of the nilpotent singularity of codimension three in $\mathbb{R}^3$. Equally relevant is the fact that, putting $\nu = 0$, $\lambda < 0$ and $\mu < 0$, we get a system which is linked to the study of the Kuramoto-Sivashinsky equation.

The current state of the problem, pointing out several open questions, will be described. For certain parameter values, in particular when $\nu = 0$, $\lambda < 0$ and $\mu < 0$, the system has two hyperbolic saddles P and Q with $\dim(W^u(P)) = \dim(W^s(Q)) = 2$. We will pay special attention to new results about the existence and transversality of intersections between both manifolds and their dynamical consequences: heteroclinic cycles, Shil'nikov homoclinic orbits, strange attractors, ...

On the Hamiltonian-Hopf bifurcation

Angel Jorba* (University of Barcelona, Spain)

Mercè Ollé (Universitat Politècnica de Catalunya)

In this talk we will give a numerical description of the neighbourhood of a fixed point of a 4D symplectic map undergoing a transition from linear stability to complex instability, i.e., the so called Hamiltonian-Hopf bifurcation. We have considered both the direct and inverse cases.

The study is based on the numerical computation of the Lyapunov families of invariant curves near the fixed point. We will show how these families, jointly with their invariant manifolds and the invariant manifolds of the fixed point organise the phase space around the bifurcation.

On Newhouse's phenomenon

Vadim Kaloshin (California Institute of Technology)

In the early 70's S. Newhouse disproved a conjecture by R. Thom which said that generically a diffeomorphism of a compact manifold can not have infinitely many attracting periodic points (sinks). Newhouse exhibited a Baire generic (residual) set of surface diffeomorphisms having infinitely many coexisting sinks; this is now called the *Newhouse phenomenon.* Later J. Palis stated a conjecture that even though the above phenomenon is generic topologically it has "probability zero", moreover, infinitely many coexisting attractors have probability zero. In the talk we report on some progress toward Palis's conjecture for Newhouse's phenomenon, generalizing results of Tedeschini-Lalli-Yorke.

Parametrized dynamics of the Weierstrass elliptic function

Jane Hawkins (University of North Carolina at Chapel Hill)

Lorelei Koss* (Dickinson College)

We study the parameter spaces of some families of Weierstrass elliptic $\wp$ functions. If we restrict our attention to one fundamental period for the modular group, then each point in that region gives rise to a parameter space corresponding to one shape of lattice. Many of our results are concentrated on specific shapes, namely square and triangular lattices. We discuss some results on the connectivity of the Julia set for certain lattice shapes.

The cyclicity of the elliptic segment loops of reversible quadratic Hamiltonian systems under quadratic perturbations

Chengzhi Li* (Peking University)

Robert Roussarie (Université de Bourgogne)

We denote by Q_3^H and Q_3^R the Hamiltonian class and the reversible class of quadratic integrable systems. There are several topological types for systems belonging to $Q_3^H \cap Q_3^R$. One of them is the case that the corresponding system has two heteroclinic loops, sharing one saddle–connection, called the elliptic segment loops. The paper by Chow, Li and Yi (*Ergodic Theory Dynamical Systems* **22** (2002), 349–374) studied the cyclicity of the period annuli bounded by the elliptic segment loops. But the conclusion does not apply to the loops. In the present work, we study the maximal number of limit cycles which bifurcate from the loops under quadratic perturbations, and we prove this number is two. We also give the bifurcation diagram.

On the inverse integrating factor and its applications

Jaume Llibre Saló (Universitat Autònoma de Barcelona)

In recent works it has been shown that a new method based in an inverse integrating factor $V(x, y)$ can be used to study the limit cycles of C^1 differential systems

$$\dot{x} = P(x, y), \qquad \dot{y} = Q(x, y),$$

defined in an open subset U of $\mathbb{R}^2$. A C^1 function $V(x, y) : U \to \mathbb{R}$, that satisfies the equation

$$P\frac{\partial V}{\partial x} + Q\frac{\partial V}{\partial y} = \left(\frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y}\right) V$$

is called an *inverse integrating factor* of the system because the function $R = 1/V$ is an integrating factor of the system in $U \setminus \{V = 0\}$. These functions V are important because:
(1) We shall show that the best way to understand when a planar differential system is integrable is through its inverse integrating factors.
(2) The curve $\{V = 0\}$ contains the limit cycles of the system which are in U. This fact allows us to study the limit cycles which bifurcate from periodic orbits of a center (Hamiltonian or not) under a perturbation and to compute their shape.
(3) Using the function V we shall prove that any possible configuration of limit cycles is realized by a polynomial differential system.

Accessible points in the Julia set for stable exponentials

Mónica Moreno Rocha (Tufts University)

In this talk we consider the question of accessibility of points in the Julia set of complex exponential functions in the case where the exponential admits an attracting cycle. In the case of an attracting fixed point it is known that the Julia set is a Cantor bouquet and that the only points accessible from the basin are the endpoints of the bouquet. In case the cycle has period two or greater, there are many more restrictions on which points in the Julia set are accessible. In this talk we give a precise condition for a point to be accessible in terms of the kneading sequence of the cycle.

Minimal subsets of a class of convex monotone skew-product semiflows

Rafael Obaya García (Universidad de Valladolid)

We study almost automorphic and almost periodic dynamics in a class of convex monotone skew-product semiflows which arise in the qualitative study of nonautonomous ordinary, partial and functional differential equations. We asume the existence of two strongly ordered minimal subsets and give a complete description of the long-time behaviour of the trajectories. We determine the global picture of the dynamics in terms of the upper Lyapunov exponents.

Asymptotic randomization of sofic shifts by linear cellular automata

Marcus Pivato* (Trent University)

Reem Yassawi (Trent University)

If $\mathcal{A}$ is a finite abelian group, then $\mathcal{A}^{\mathbb{Z}}$ is a compact abelian group. A *linear cellular automaton* (LCA) is a topological group endomorphism $\Phi : \mathcal{A}^{\mathbb{Z}} \longrightarrow \mathcal{A}^{\mathbb{Z}}$ that commutes with all shift maps. If μ is a probability measure on $\mathcal{A}^{\mathbb{Z}}$, then Φ *asymptotically randomizes* μ if weakk$^* - \lim_{N\to\infty} \frac{1}{N} \sum_{n=1}^{N} \Phi^n \mu = \eta$, where η is the Haar measure on $\mathcal{A}^{\mathbb{Z}}$.

Recent work shows that LCA asymptotically randomize a wide variety of measures having *full support* (i.e., all cylinder sets get nonzero measure). We will show that LCA also asymptotically randomize many measures supported on *sofic shifts* (subshifts generated by finite-state machines). This includes, for example, any image (under a block map) of a mixing Markov chain.

Applications of bounded homeomorphisms

David Richeson* (Dickinson College)

Jim Wiseman (Swarthmore College)

A homeomorphism $f : X \to X$ is called *bounded* if there exists a compact set W with the property that the forward orbit of every point $x \in X$ intersects W. We've shown that every bounded homeomorphism has such a set W that is forward invariant. In this talk we give applications of this result. We discuss applications to fixed point theorems, minimal homeomorphisms, and forward expansive maps.

Quantitative estimates on the size of Herman rings of the complex standard family using geometrical methods

Núria Fagella (Universitat de Barcelona)

Tere M. Seara (Universitat Politècnica de Catalunya)

Jordi Villanueva Castelltort* (Universitat Politècnica de Catalunya)

We consider the complexification of the Arnold standard family of circle maps given by $F_{\alpha,\varepsilon}(z) = z\mathrm{e}^{\mathrm{i}\alpha}\mathrm{e}^{\frac{\varepsilon}{2}(z-\frac{1}{z})}$, with $\alpha = \alpha(\varepsilon)$ chosen so that $F_{\alpha(\varepsilon),\varepsilon}$ restricted to the unit circle has a prefixed rotation number belonging to the set of (irrational) Brjuno numbers. In this case, it is known that $F_{\alpha(\varepsilon),\varepsilon}$ has a a Herman ring U_ε around the unit circle. The conformal radius R_ε of U_ε goes to infinity as $\varepsilon \to 0$, but one can ask for its asymptotic behavior. Then, we have proved that $R_\varepsilon = \frac{2}{\varepsilon}(R + \mathcal{O}(\varepsilon \log \varepsilon))$, where R is the maximal radius for which is defined the normalized linearization map of the Siegel disk of the complex semistandard map $G(z) = z\mathrm{e}^{\mathrm{i}\omega}\mathrm{e}^{z}$. In the proof we have used a very explicit quasiconformal surgery construction to relate $F_{\alpha(\varepsilon),\varepsilon}$ and G, and hyperbolic geometry to obtain the quantitative result.

Graphs of NMS Flows on S^3 without heteroclinic trajectories

Beatriz Campos (Universitat Jaume I)

José Martínez Alfaro (Universitat de València)

Pura Vindel* (Universitat Jaume I)

Morse-Smale flows form a dense open subset of differentiable vector fields and they are the structurally stable flows on 2-dimensional manifolds. M. M. Peixoto studied Morse-Smale flows on compact orientable two-dimensional manifolds; he associated a graph endowed with finite combinatorial structure to every flow of such type and proved that these graphs are in $1-1$ correspondence with the topological equivalence classes of Morse-Smale flows.

For the three–dimensional case, M. Wada has made a topological characterization of the set of the periodic points of NMS flows on the 3-sphere, in terms of knots and links. Nevertheless, different flows can be characterized by the same link.

Our aim is to obtain the dual graphs for the NMS systems on S^3 characterized by Wada operations and with no heteroclinic trajectories connecting two saddles orbits. We also define different ways of gluing graphs and prove that these kinds of NMS flows are obtained from the gluing of dual graphs corresponding to each operation. Moreover, we show that these types of NMS flows can be reproduced from these dual graphs.

Symbolic dynamics from arbitrary matrices

Jim Wiseman (Swarthmore College)

We consider a method for assigning symbolic dynamics (a sofic shift) to a square matrix with arbitrary real entries by associating to it a directed graph with some vertices labeled 1 and the rest 2 (in applications the choice of labeling should be natural). We can obtain an estimate for the topological entropy of this sofic shift by comparing the characteristic polynomial of the original matrix to those of the matrices for the restrictions of the shifts to each piece (1 and 2). Our main application is to the use of the discrete Conley index to detect symbolic dynamics in isolated invariant sets.

Functional differential equation on a Banach space

Fadila Nigro (University of Tlemcen)

Mustapha Yebdri* (University of Tlemcen)

We prove an existence theorem for the solutions of the functional differential equation $\dot{x}(t) = F(t, x_t)$, where $x_t(\theta) = x(t + \theta)$ for all $\theta \in [-r, 0]$ and $F : [0, A] \times X_2 \to X$, where X is a Banach space and X_2 is the Banach space of continuous function defined on $[-r, 0]$ with values in X.