First Joint Meeting between the RSME and the AMS

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Abstracts

Session 16

Effective Analytic Geometry over Complete Fields

Organizers:

Luis Miguel Pardo (Universidad de Cantabria)
J. Maurice Rojas (Texas A&M University)

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P-adic integration and exponential sums

R. Cluckers (University of Leuven)

Working with the field of p-adic numbers, one can define (globally) subanalytic sets and subanalytic functions similar as over the real numbers. Consider the algebra generated over $\mathbb{Q}$ by the norm and the valuation of subanalytic functions. This algebra is recently proven by the author to be closed under integration against the Haar measure. A similar result for semialgebraic functions was proven before by Denef. I will talk about various kinds of algebras closed under p-adic integration, also considering oscillatory factors in the integrand, of the form

$$\exp(2\pi i f(x)),$$

where $f : \mathbb{Q}_p^n \to \mathbb{Q}_p$ is a subanalytic function. Such integrals are often directly related to classical exponential sums like Kloosterman sums mod p^m. Standard methods (used by e.g. Igusa) to look at these sums are: Fourier transforms, Gelfand-Leray differential forms, local singular series, and so on. We will speak of two new methods: parametrized local singular series and p-adic cell decomposition. The main application is a decay rate for exponential sums modulo p^m when m tends to infinity. This represents a multivariate analogue of a result by Igusa on exponential sums.

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Betti numbers of definable sets

A. Gabrielov (Purdue University)

Joint work with N. Vorobjov (Bath, UK) and T. Zell (Purdue)

A spectral sequence associated with a surjective continuous map provides upper bounds on the Betti numbers of a set defined by an expression with quantifiers, in terms of the Betti numbers of some auxiliary sets defined by quantifier-free formulas. Applications include sub-Pfaffian and "sub-algebraic" sets. For a "limit set" defined by the relative closure operation on a semi-Pfaffian family (in particular, for a Hausdorff limit of a family of semi-Pfaffian sets) Betti numbers can be estimated in terms of the Betti numbers of auxiliary semi-Pfaffian sets associated with the fibers of the family.

A P-adic Nullstellensatz

S. Kochen (Princeton University)

In analogy with the real Nullenstellensatz, which follows from Artin's representation of positive definite rational functions over real fields as sums of squares (Hilbert's 17th Problem), we describe a p-adic Nullstellensatz that is a consequence of a representation theorem for integral definite rational functions over p-adic fields.

Non-Archimedean Semi-analytic and Subanalytic Sets

L. Lipshitz* (Purdue University)

Z. Robinson (East Carolina University)

We will present some uniformity results on non-Archimedean semi-analytic and subanalytic sets.

Average bit length of initial points in linear homotopy polynomial solving

L. M. Pardo (Universidad de Cantabria)

In this talk I show estimates on the bit length of initial points of linear homotopy deformation methods when applied to solve systems of multivariate polynomial equations.

An Introduction to Ultrametric Fewnomial Theory

J. M. Rojas (Texas A&M University)

Rene Descartes stated no later than June of 1637 that any real univariate polynomial with exactly m monomial terms has at most $m-1$ positive roots — an upper bound totally independent of the degree. Finding a sharp generalization to multivariate polynomial systems would have many applications in dynamical systems and engineering, but has eluded us now for close to four centuries.

We sketch an advance in a slightly different direction: an arithmetic multivariate analogue of Descartes' bound — now for the number of geometrically isolated roots over any finite algebraic extension of the ordinary or p-adic rationals — which is asymptotically near optimal. The upper bound is

$$1 + (Cn(m-n)3 \log m)^n$$

where m is the total number of distinct exponent vectors, n is the number of variables, and C is a constant depending only on the underlying field. This result generalizes and simplifies earlier work of Denef, Hendrik W. Lenstra, Jr., Lipshitz, and van den Dries, and provides the foundation for an arithmetic analogue of Khovanski's theory of fewnomials. We also highlight some connections to amoebae and complexity theory, including a variant of the $\mathbf{P} \stackrel{?}{=} \mathbf{NP}$ question.

Analytic Difference Rings

T. Scanlon (University of California, Berkeley)

We describe a quantifier elimination theorem for certain valued fields (including Laurent series fields of characteristic zero and the field of fractions of the Witt vectors of an algebraically closed field of positive characteristic) given together with analytic functions and an endomorphism. The key to the proof is a uniform version of the Weierstrass preparation theorem which we shall describe in detail.

Solving linear differential equations in H-fields

L. van den Dries (University of Illinois at Urbana-Champaign)

I will report on joint work in progress with Matthias Aschenbrenner and Joris van der Hoeven. An H-field is a field with an ordering and a derivation subject to some compatibilities. Hardy fields and fields of transseries are H-fields. An optimistic conjecture is that each H-field can be embedded, as ordered differential field, into a field of transseries, analogous to the fact that each valued field of equal characteristic zero can be embedded as valued field into a field of generalized power series. As a step towards this conjecture we wish to understand which algebraic differential equations over a given H-field can be solved in suitable H-field extensions. This understanding can be achieved for linear differential equations.