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Abstracts

Session 17

Geometric Methods in Group Theory

Organizers:

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Strong rigidity in even Coxeter groups

Patrick Bahls (University of Illinois at Urbana-Champaign)

Recall that a Coxeter system is a pair (W, S) where W is a group with fundamental generating set S for which there is a presentation $\langle S|R \rangle$, where

$$R = \{(s_i s_j)^{m_{ij}} = 1 \mid m_{ij} \in N, m_{ij} = m_{ji}, \text{ and } m_{ij} = 1 \Leftrightarrow i = j\}$$

A Coxeter group is said to be strongly rigid if any two fundamental generating sets S_1 and S_2 for W (corresponding to different systems for W) are conjugate to one another. We characterize all strongly rigid even Coxeter groups with a Coxeter diagram $\mathcal{V}$ of one of the following forms:

1. $\mathcal{V}$ has no edges labeled 2,
2. $\mathcal{V}$ has no simple cycles of length less than 5.

We also indicate how techniques similar to the proof of the above classifications can be used to compute the automorphism group $Aut(W)$ for certain Coxeter groups W.

Measured laminations and equations over free groups

Mladen Bestvina* (Utah University)

Mark Feighn (Rutgers University)

In this talk I will discuss the recent work of Zlil Sela on Tarski's problems about first order theory of free groups. Concretely, I will focus on proving the following theorem. Let $G \supset H$ be a pair of finitely generated groups and suppose that for some fixed $k > 1$ every homomorphism from H to the free group F_k of rank k can be extended to G. Then the same holds with any other nonabelian free group as target. This theorem is an example of the fact, asked by Tarski and proved by Sela, that nonabelian free groups of different ranks are "elementary equivalent". Our proof of the above statement is different from Sela's and centers on the consideration of measured laminations (on finite complexes). If there is time, I will discuss generalizations of the above theorem (but still special cases of Sela's theorem) in which one allows "inequalities", e.g. for a fixed element $g \in G$ one requires the extensions to send g nontrivially. Surprisingly, this small generalization complicates the proof significantly; however, it still follows the same broad outline as the "positive sentences" case.

An analogue of a Magnus' theorem for surface groups and some other one-relator groups

Oleg Bogopolski (Institute of Mathematics, Novosibirsk and Utah University)

In 1930 W. Magnus published a very important article where he proved the so-called *Freiheitssatz* and the following

Theorem (Magnus). *If elements u and v of a free group have the same normal closures, then u is a conjugate of $v^{\pm 1}$.*

We will say that a group G possesses *the Magnus' property*, if for any two elements u, v of G with the same normal closures we have that u is a conjugate of $v^{\pm 1}$.

Not any one-relator group has the Magnus' property. For example, the The Baumslag-Solitar group $\langle a, b \,|\, a^{-1}b^n ab^m \rangle$, where $|n|, |m| \geqslant 2$; or the noncyclic one-relator groups with torsion.

Note that each one-relator group of type $\langle X \mid R^k \rangle$, $k \geqslant 5$, is hyperbolic by a result of I. Kapovich. We prove the following

Theorem. *The fundamental group of a closed orientable surface possesses the Magnus' property.*

Earlier O. Bogopolski, E. Kudrjavtseva and H. Zieschang proved a very partial case of this theorem using coverings of surfaces. The second proof in the partial case by O. Bogopolski involves simple arguments from 3-dimensional topology. However this does not work in general. Now the proof returns to Magnus' original ideas.

Some unsolved related problems and new results, concerning "roots" in free groups will be discussed also.

Small cancellation and non-positive curvature

Noel Brady* (University of Oklahoma)

Jon McCammond (University of California, Santa Barbara)

We show that the small cancellation $C'(1/4) - T(4)$ groups are the fundamental groups of non-positively curved cubical complexes.

Properly 3-realizable groups

Manuel Cárdenas* (Universidad de Sevilla)

Francisco F. Lasheras (Universidad de Sevilla)

A finitely presented group G is said to be 3-properly realizable if there exists a compact 2-polyhedron K with $\pi_1(K) \cong G$ and whose universal cover $\tilde{K}$ has the proper homotopy type of a (p.l.) 3-manifold with boundary. We enumerate some results on this class of groups regarding their behavior with respect to the free product and certain amalgamated free product (HNN-extensions) over a cyclic group. The question of whether or not every finitely presented group is properly 3-realizable still remains open.

Thompson's group F is not almost convex

Sean Cleary* (City College of City University of New York)

Jennifer Taback (University at Albany)

Almost convexity is a property of generating sets of groups which allows algorithmic construction of increasingly-large balls in the Cayley graph of the group. We show that Thompson's group is not almost convex with respect to the standard two element generating set for F. We use the remarkable method of Fordham for computing the word length of a tree pair diagram representing an element of F. I will also discuss other metric phenomena in the Cayley graph of F, including a complete classification of "dead-end" elements through which geodesics from the identity cannot be extended.

Geometry of the Word Problem: Space and Time as Geometric Notions

Steve Gersten (University of Utah)

Time, or the number of steps in solving the word problem for a finitely presented group, has long been related to the area, or number of 2-cells, of a van Kampen diagram. Space, or memory, is more recently related to the geometric notion of *filling length*. We show now that filling length can also be read off a van Kampen diagram as the *gallery length*. This result has group theoretic consequences, including bounds on isoperimetric functions for central extensions (implying a new proof of the "$c+1$-theorem" for nilpotent groups) and examples of class c nilpotent groups with isoperimetric polynomials of degree c for all $c \geq 2$.

Braids and Nielsen-Thurston theory

Juan González-Meneses López (Universidad de Sevilla)

The braid group can be seen as the group of automorphisms of a punctured disc, up to isotopy fixing the boundary. Hence one can use Nielsen-Thurston theory to study braids. This turns to be a powerful tool for studying conjugacy in braid groups, and led us to show the following conjecture due to G. Makanin: if the k-th root of a braid exists, then it is unique up to conjugacy.

Minimal almost convexity

Susan Hermiller (University of Nebraska)

A group satisfies minimal almost convexity if for all sufficiently large natural numbers R, and for any two group elements in the sphere of radius R a distance at most 2 apart, there is a path of length at most $2R-1$ in the Cayley graph which joins the elements and is contained inside the ball of radius R. In this talk I will discuss specific examples of groups that illustrate properties of this condition.

Generic properties of the Whitehead Algorithm, of stabilizers in $Aut(F_k)$ and of one-relator groups

Ilya Kapovich* (University of Illinois at Urbana-Champaign)

Paul Schupp (University of Illinois at Urbana-Champaign)

Vladimir Shpilrain (The City College of New York)

We show that the "hard" part of the Whitehead algorithm for solving the automorphic conjugacy problem in a fixed free group F_k terminates in linear time (in terms of the length of an input) on an exponentially generic set of input pairs and that the algorithm has strongly linear-time generic-case complexity. The reason for this is that generic elements of F_k are already "strongly minimal" in their $Aut(F_k)$-orbits. We also prove that the stabilizers of generic elements of F_k in $Aut(F_k)$ are cyclic groups generated by inner automorphisms. We apply these results to generic one-relator groups and show, in particular, that they have trivial outer automorphism groups.

Algorithms for fully residually free groups

Olga Kharlampovich (McGill University)

We discuss different algorithms for finitely generated fully residually free groups, in particular, the algorithm to solve equations (joint result with A. Miasnikov and D. Serbin) and the algorithm to find the abelian JSJ decomposition (joint result with A. Miasnikov).

Automorphisms of canonical splittings

Gilbert Levitt (Université Paul Sabatier, Toulouse)

Let Γ be a finite graph of groups decomposition of a group G such that edge groups are rigid (their outer automorphism group is finite). We describe the group of automorphisms of G preserving Γ, by comparing it to direct products of suitably defined mapping class groups of vertex groups. This is applied to hyperbolic groups, generalized Baumslag-Solitar groups, limit groups.

Examples of groups acting freely on non-archimedean trees

Armando Martino* (University College Cork)

Shane O Rourke (University College Cork)

We show that various groups are $\mathbb{Z}^n$-free. In particular we show that almost every surface group is $(\mathbb{Z} \times \mathbb{Z})$-free as are the groups of Liousse which act freely and affinely on $\mathbb{R}$-trees. We also demonstrate that the class of $\mathbb{Z}^n$-free groups is closed under taking amalgamated free products over an infinite cyclic group as long as it is maximal abelian in each vertex group. It follows that a large class of hyperbolic groups is $\mathbb{Z}^n$-free.

Existence of CAT(0) structures for finite type Artin groups

Jon McCammond (University of California Santa Barbara)

In recent years, Tom Brady and Daan Krammer independently constructed new Eilenberg-MacLane spaces for finite type Artin groups. Since it has been shown that all of the Brady-Krammer spaces for Artin groups with at most 3 generators support CAT(0) metrics, it has been natural to conjecture that all of them have this property. In this talk I will describe how to prove that the spaces of type A_4 and B_4 support CAT(0) metrics along with recent work by my graduate student, Woonjung Choi, showing that the spaces of type D_4 and F_4 do not.

The prefix membership problem for one-relator groups

John Meakin* (University of Nebraska - Lincoln)

Zoran Šunić (University of Nebraska - Lincoln)

Let w be a fixed cyclically reduced word over $X \cup X^{-1}$ and let $G = \langle X | w = 1 \rangle$ be the corresponding one-relator group. By recent results of Ivanov, Margolis and Meakin the word problem for the inverse monoid having the same presentation as the group G is reduced to the membership problem in the submonoid M of G generated by the set P of prefixes of w.

A basic approach that we use for solving this membership problem is to construct a homomorphism $G \to H$ such that the elements in M that have large length with respect to the generating set P, map to elements in H that have have large word length with respect to some finite generating set for H. In many cases we successfully use this approach by constructing non-faithful linear representations of G in $GL_2(\mathbb{C})$ or in $GL_3(\mathbb{C})$. For example, we are able to use this method to solve the prefix membership problem in surface groups, solvable Baumslag-Solitar groups, and more generally in many Adian one-relator groups.

Groups acting on trees and infinite words

Alexei Miasnikov (City University of New York)

I am going to discuss a new technique to study groups acting freely on Lambda-trees. The main tool is representation of elements of such groups by "non-standard" infinite words.

Representations of the braid group by automorphisms of groups

John Crisp (Université de Bourgogne)

Luis Paris* (Université de Bourgogne)

From a group H and a non-trivial element h of H, we define a representation $\rho : B_n \to \mathrm{Aut}(*^n H)$, where B_n denotes the braid group on n strands and $*^n H$ denotes the free product of n copies of H. Such a representation shall be called the Artin type representation associated to the pair (H, h). If $H = \mathbb{Z}$ and $h = 1$, then $*^n H = F_n$ is the free group of rank n and ρ is the classical representation introduced by Artin in 1925.

The goal of this lecture is to present different aspects of these representations. We show how to use them to define new invariants for links, we prove that they are faithful, and we study some combinatorial properties of the semi-direct product $(*^n H) \rtimes_\rho B_n$.

Amenability and Random Walks

Christophe Pittet (Université de Toulouse)

The degree of amenability of a finitely generated group can be evaluated through geometric methods involving isoperimetric inequalities. Spectral theory of the discrete Laplacian relates isoperimetric inequalities with random walk properties. We will illustrate this link with examples, focusing on finitely generated solvable groups.

Counting elements in automorphic orbits

Vladimir Shpilrain (The City College of New York)

Let F_r be the free group of a finite rank $r \geq 2$ with a fixed set X of free generators. Counting elements of a given length (with respect to the basis X) is important for understanding, among other things, the complexity of various algorithms, including the Whitehead algorithm. In this talk, we shall give rather tight bounds for the number $P(r, n)$ of primitive elements of a given length n in F_r, improving previously known bounds due to Burillo and Ventura and Borovik, Myasnikov, and Shpilrain. We shall also report results on counting elements in other automorphic orbits.

Shift automorphisms of free groups

Edward Turner (State University of New York at Albany)

A basic building block for Rational Canonical Form for linear transformations of a vector space is a map whose effect on a basis $\{v_1, \ldots, v_n\}$ is

$$v_1 \to v_2 \to \cdots \to v_n \to a_1 v_1 + \cdots + a_n v_n.$$

The analogous notion for automorphisms of a free group $F(x_1, \ldots, x_n)$ is an automorphism whose effect on the basis is

$$x_1 \to x_2 \to \cdots \to x_n \to w$$

fore some word $w \in F(x_1, \ldots x_n)$.

We will discuss the problem of classifying shift automorphisms, particularly those of finite order.

Foldings and the rank problem of Fuchsian groups

Richard Weidmann (Universität at Frankfurt)

The *rank* of a group is the minimal number of elements necessary to generate the group. In 1970 Zieschang shows how to use Nielsen methods to compute the rank of Fuchsian groups (NEC-groups) that do not contain reflections. The case of Fuchsian groups containing reflections is much more involved and the only substantial result is the classification of 2-generated Fuchsian group by Sakuma and Klimenko.

We will show how to use foldings as developed by Stallings, Bestvina, Feighn and Dunwoody to solve the rank problem for planar Coxeter groups which present the most difficult case for the general problem. Although the proof is somewhat technical, its main idea is geometric and, as an offshot, yields an analogue of Grusko's Theorem for splittings over finite groups.

If time permits we will further discuss how the rank problem of fundamental groups of graph manifolds can be solved. This solution implies that whenever the fundamental group of a totally orientable graph manifold has rank smaller than the Heegaard genus then the manifold is of a very specific type. This class of manifolds contains all knows examples where the rank is different from the genus.