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Abstracts

Session 19

Homological Methods in Banach Space Theory

Organizers:

Jesus M. F. Castillo (Universidad de Extremadura)
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Nonlinear centralizers in homology, with applications.

Félix Cabello Sánchez (Universidad de Extremadura)

The talk is intended as a homological guide to Kalton's Memoir *Nonlinear Commutators in Interpolation Theory.* Starting with Kalton's description of L_∞-extensions

$$0 \to Y \to X \to Z \to 0$$

by centralizers $\Omega : Z \to Y$, we prove that, under rather mild assumptions on the function modules Z and Y, given an L_∞-extension $0 \to Y \to X \to Z \to 0$ and another module V, the "tensorized" sequence

$$0 \to Y \circledast_{L_\infty} V \to X \circledast_{L_\infty} V \to Z \circledast_{L_\infty} V \to 0$$

makes sense and is exact. In some cases, it can be proved that this new extension splits if, and only if, so does the starting one. Then we apply an "adjoint" procedure to show that if Y^* is a K-space, then $\mathrm{Ext}_{L_\infty}(Z,Y) = \mathrm{Ext}_{L_\infty}(Z \circledast_{L_\infty} Y^*, L_1)$. For (Banach) sequence spaces we have the sharp result that if Y^* is a K-space, then

$$\mathrm{Ext}_{\ell_\infty}(Z,Y) = \mathrm{Ext}_{\ell_\infty}(Z \circledast_{\ell_\infty} Y^*, \ell_1) = \mathrm{Ext}(Z \circledast_{\ell_\infty} Y^*, \mathbb{K})$$

and so $\mathrm{Ext}_{\ell_\infty}(Z,Y) = 0$ if, and only if, $Z \circledast_{\ell_\infty} Y^*$ is a K-space. It follows that if Z is super-reflexive, then $\mathrm{Ext}_{\ell_\infty}(Z,Z) = \mathrm{Ext}(\ell_1, \mathbb{K})$.

On the Pełczyński-Lindenstrauss-Rosenthal-Johnson-Zippin-Kalton cycle of ideas (A homological vision of the hexagonal garden)

Jesús M. F. Castillo (Universidad de Extremadura)

This talk is about a series of quite classical results in Banach space theory. Among others, let us mention the Lindenstrauss-Pełczyński theorem about the extension of $C(K)$-valued operators from subspace os c_0; the Lindenstrauss-Rosenthal theorem about extension and lifting of automorphisms in c_0 and l_1; the Johnson-Rosenthal decomposition of a subspaces of c_0; the Johnson-Zippin theorem about the extension of $\mathcal{L}_\infty$-valued operators from duals of quotients of c_0; Zippin's theory of extension of $C(K)$-valued operators; Kalton's results about spaces X for which $Ext(X, C(K)) = 0$, etc.

We want to show how all these results share a common homological nature. And then, to show how the using of homological techniques leads to new problems and aplications to classical Banach spaces.

On the splitting of long exact sequences

Paweł Domański (Adam Mickiewicz University)

The talk will contain a survey of the results on the splitting of long exact sequences of the form:

$$0 \longrightarrow \ker T_0 \longrightarrow X \xrightarrow{T_0} X \xrightarrow{T_1} X \xrightarrow{T_2} \dots, \tag{1}$$

where T_n denote linear continuous operators and X denotes a natural space like: $H(\Omega)$ the Fréchet space of holomorphic functions, $C^\infty(\Omega)$ the Fréchet space of smooth functions, $\mathcal{D}'(\Omega)$ the space of distributions etc. We will give splitting criteria based on results of Vogt, Poppenberg and Vogt, Vogt and the author, and Jakubczyk and the author. We will explain the Banach space theory component in the proof of the splitting results.

The complexes of the form (1) appear naturally, for instance, in the theory of partial differential equations, where T_n are linear partial differential operators or as Koszul complexes, where T_n are exterior multiplications of differential forms.

Local complementation on Banach spaces and some applications

Ricardo García González (Universidad de Extremadura)

We study the relationship between the local complementation and the following problems in Functional Analysis:

1. The extension of multilinear forms.
2. The relationship between the bidual of the projective tensor product of Banach spaces and the tensor product of their biduals.
3. The compact extension property for quotient spaces.

Representations of the dual of a Banach space

Manuel González* (Universidad de Cantabria)

Antonio Martínez-Abejón (Universidad de Oviedo)

We consider three properties of a subspace Z of the dual X^* of a Banach space X which correspond to increasing stages in the accuracy of the representation of X^* by means of Z: namely, Z norming, X^* finitely dual representable in Z, and Z a local dual of X. The *principle of local reflexivity* is equivalent to saying that X, as a subspace of X^{**}, is a local dual of X^*.

We present global characterizations, local characterizations, and characterizations in terms of ultrapowers of these properties. We also give examples of local dual spaces for several classes of Banach spaces that include some classical Banach spaces. For example, the canonical copy of $L_1[0,1]$ in $M[0,1]$ is a local dual of $C[0,1]$.

Domination by strictly singular and co-singular operators

Julio Flores (Universidad Rey Juan Carlos)

Francisco L. Hernández* (Universidad Complutense de Madrid)

Recent results on the domination problem for the class of positive strictly singular operators will be presented. Let E and F be Banach lattices and two positive operators $S, T : E \to F$ such that $0 \leq S \leq T$, which conditions on E and F guarantee that T strictly singular implies that S is strictly singular? The special cases of endomorphisms or rearrangement invariant spaces will also be discussed as well as similar questions for the class of strictly co-singular operators.

Stochastic approximation properties in Banach spaces

Vladimir P. Fonf (Ben-Gurion University of the Negev)

William B. Johnson* (Texas A & M University, College Station)

Gilles Pisier (Texas A & M University, College Station, and Université Paris VI)

David Preiss (University College London)

We show that a Banach space X has the stochastic approximation property if, and only if, it has the stochasic basis property, and these properties are equivalent to the approximation property if X has non trivial type. If for every Radon probability on X, there is an operator from an L_p space into X whose range has probability one, then X is a quotient of an L_p space. This extends a theorem of Sato's which dealt with the case $p = 2$. In any infinite dimensional Banach space X there is a compact set K so that for any Radon probability on X there is an operator range of probability one that does not contain K.

Uniform liftings of quotient maps

Nigel Kalton (University of Missouri)

We will discuss the problem of whether a given quotient map $Q : X \to Y$ admits a lifting which is (a) uniformly continuous or (b) uniformly continuous relative to the unit ball.

Lipschitz quotients and metric trees

William B. Johnson (Texas A & M University, College Station)

Joram Lindenstrauss* (Hebrew University of Jerusalem)

David Preiss (University College London)

Gideon Schechtman (Weizmann Institute of Science)

It is proved that a separable metric space Y is a Lipschitz quotient of a Banach space X which contains a copy of ℓ_1 if and only if Y is complete and λ-metrically convex for some $\lambda < \infty$. In particular any separable Banach space is a Lipschitz quotient of $C(0, 1)$. The main tool used in proving this result is the notion of a metric tree. The proof uses some rather canonical constructions.

Existence of hermitian operators on finite-dimensional Banach spaces: geometrical consequences

Miguel Martín (Universidad de Granada)

We characterize those finite-dimensional real Banach spaces X admitting a non-zero operator T, such that iT is hermitian in the complexification of the Banach algebra $L(X)$ of all linear operators. Equivalently, we characterize those finite-dimensional real Banach spaces with numerical index zero.

The main result is the following.

Theorem. *Let $(X, \|\cdot\|)$ be a finite-dimensional real Banach space. Then the following are equivalent:*

(i) There is a non-zero operator T on X such that iT is hermitian in $L(X)_{\mathbb{C}}$.

(ii) The numerical index of X is zero.

(iii) There are a real *vector space X_0,* complex *vector spaces $X_1, \ldots, X_n$, and numbers $p_1, \ldots, p_n \in \mathbb{N}$, such that $X = X_0 \oplus X_1 \oplus \cdots \oplus X_n$ and*

$$\left\|x_0 + e^{ip_1\rho}x_1 + \cdots + e^{ip_n\rho}x_n\right\| = \|x_0 + x_1 + \cdots + x_n\|$$

for all $\rho \in \mathbb{R}$, $x_j \in X_j$ $(j = 0, 1, \ldots, n)$.

As a consequence, we obtain that:
- The only 2-dimensional space satisfying the above conditions is the Hilbert space.
- If a 3-dimensional space X satisfies the above conditions, then X is an absolute sum of the 2-dimensional real Hilbert space and $\mathbb{R}$.

We also give a 4-dimensional example showing that the number of complex vector spaces in (iii) cannot be reduced to one.

Convex sets which are intersections of closed balls

José Pedro Moreno Díaz (Universidad Autónoma de Madrid)

In this talk we aim to present different questions concerning the stability of the set $\mathcal{M}$ of all intersections of closed balls in a normed space. We are mainly concerned with: (i) the stability of $\mathcal{M}$ under the closure of the vector sums; (ii) the stability under the addition of balls. We will prove that (i) and (ii) are different properties which have strong connections with the geometry of the space. They have interest both in finite and infinite dimension. In the former case, there is a link with linear programming theory. We also study two more stability properties leading to a new characterization of the well-known Binary Intersection Property. Mazur sets and Mazur spaces will be introduced, as a natural family satisfying (i). We will prove that every two dimensional normed space is a Mazur space, a result which distinguishes dimension $d \leq 2$ from dimension $d \geq 3$. We will also discuss the connections between Mazur spaces and porosity.

On the Lindenstrauss-Rosenthal theorem

Yolanda Moreno Salguero (Universidad de Extremadura)

This talk is about a homological principle that governs the behaviour of certain exact sequences of Banach and quasi-Banach spaces. Let us show two appications: i) A unifying method of proof for the results of Lindenstrauss-Rosenthal, Kalton, Kalton-Peck and Kislyakov about the extension and lifting of isomorphisms in c_0, l_∞, l_p and L_p for $0 < p \leq 1$; ii) New results on the extension of $C(K)$-valued operators.

A perturbative characterization for non-convergent martingale preserving operators

Manuel González (Universidad de Cantabria)

Antonio Martínez-Abejón (Universidad de Oviedo)

Javier Pello* (Universidad de Oviedo)

For every operator ideal, it is possible to define some operator classes, called operator semigroups by Aiena, González and Martínez-Abejón, with opposite properties. For instance, the class of Fredholm operators is an operator semigroup associated to compact operators, as the class of tauberian operators is with respect to weakly compact operators.

Following the axiomatization introduced for operator semigroups by the aforementioned authors, we study the operator semigroup associated to the ideal of Radon-Nikodým operators; an operator belongs to this semigroup when it preserves non-convergence of martingales.

We prove that the operator semigroup associated to the dual ideal of Radon-Nikodým operators admits a perturbative characterization: an operator belongs to this semigroup if and only if each of its compact perturbations has an Asplund cokernel.

Zeros of quadratic functionals on nonseparable Banach space

Anatolij Plichko (Kirovograd State Pedagogical University)

We prove that if a real Banach space X has the strong separable complementation property and admits no positive quadratic functional, then every quadratic functional on X vanishes on some nonseparable subspace. It is open question, whether this statement is valid for any X. This question is connected with the following three space problem: Let there exist bounded linear injective operators from X/Y into a Hilbert space and from Y into a Hilbert space. Does there exists a bounded linear injective operator from X into a Hilbert space?

An extension of the Krein-Šmulian Theorem

Antonio Suárez Granero (Universidad Complutense de Madrid)

Let X be a Banach space, $z \in X^{**}$, $A \subset X^{**}$ and denote $d(z, X) = \inf\{\|z + x\| : x \in X\}$ and $d(A, X) = \sup\{d(a, X) : a \in A\}$. The Krein-Šmulian Theorem asserts that every w*-compact subset $K \subset X^{**}$ with $d(K, X) = 0$ (i.e. $K \subset X$ is a weakly compact subset of X) satisfies $d(\overline{\text{co}}^{w^*}(K), X) = 0$ (i.e. the closed convex hull $\overline{\text{co}}(K)$ of K in X is weakly compact). We extend this result in the following way: if $K \subset X^{**}$ is a w*-compact subset then, in general, $d(\overline{\text{co}}^{w^*}(K), X) \leq 5d(K, X)$ and, moreover, if $X \cap K$ is w*-dense in K, then $d(\overline{\text{co}}^{w^*}(K), X) \leq 2d(K, X)$. However, there is a lot of situations in which $d(K, X) = d(\overline{\text{co}}^{w^*}(K), X)$, for example, if $\ell_1 \not\subset X^*$, if X has w*-angelic dual unit ball (for example, if X is WCG or WLD), if $X = \ell_1(I)$, if K is fragmented by the norm of X^{**}, etc. We also construct under CH a w*-compact subset $K \subset B(X^{**})$ such that $K \cap X$ is w*-dense in K, $d(K, X) = \frac{1}{2}$ and $d(\overline{\text{co}}^{w^*}(K), X) = 1$.

A local property of projections' norms and its applications

Mordecay Zippin (Hebrew University of Jerusalem)

Let $X = (\mathbb{R}^N, \|\ \ \|)$ be an N dimensional Banach space. A projection P on X is called orthogonal if the matrix representing P with respect to the standard basis of $\mathbb{R}^N$ is symmetric. An orthogonal projection P of X onto a subspace E is called approximately minimal approximately minimal if there exist positive constants D and δ such that, for every $0 < \alpha < \delta$, the ball $B(P, \alpha)$ in the operators space contains no orthogonal projection Q of norm $\|Q\| \leq \|P\|(1 - D\alpha 2)$. We prove that an orthogonal projection P of X onto E is approximately minimal if, and only if, the inequality $\|P\| \leq \frac{1}{2}\|L + L^t\|$ holds for every projection L of X onto E, where L^t denotes the operator on X which is represented by the transposed matrix of L.