First Joint Meeting between the RSME and the AMS

Sevilla, June 18–21, 2003

Abstracts

Session 20

Homotopy Algebras

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Chicken or egg... Homotopy BV algebra or homotopy G algebra?

Fusun Akman (Coastal Carolina University)

Following the work of Kimura-Voronov-Zuckerman (KVZ) in 1996, we were able to reduce the defining identities of a certain homotopy Gerstenhaber algebra (HGA) structure on a topological vertex operator algebra (TVOA) to a statement about the composition of mega-operators. We are now in a position to introduce a similar mega-identity for the intriguing homotopy Batalin-Vilkovisky algebra (HBVA) on a TVOA, discovered several years before by Lian and Zuckerman (LZ) and published in 1993. True to the chronological order, it has been conjectured that the anti-symmetrization of the HBVA maps of LZ will lay some fresh HGA maps. But it turns out that the egg is the parent of the chicken after all; we can identify some components of the lower identities in LZ as bona fide HGA operators (disguised as homotopy maps). In fact, every explicit map introduced by LZ as part of the HBVA structure is simply the bracket of the anti-ghost operator b_0 with the HGA maps. Then why not define a HBVA to be in general an algebra obtained by bracketing HGA maps by a suitable odd operator, and then convert the mega-identity of the latter into a mega-identity for the former by bracketing? The correspondence is visible in LZ once each identity is deciphered as "sums of compositions of maps equals zero". This multi-faceted chicken also gives us an idea as to how to construct some, perhaps all, of the HGA maps on a TVOA explicitly by induction.

Transferring TTP-structures up to homology equivalence

Víctor Álvarez Solano* (Universidad de Sevilla)

José Andrés Armario (Universidad de Sevilla)

María Dolores Frau (Universidad de Sevilla)

Pedro Real (Universidad de Sevilla)

Let $A \otimes_t C$ be a *twisted tensor product* of an algebra A and a coalgebra C, by means of a *twisting cochain* $t : C \to A$. By means of what is called the *tensor trick* and under some nice conditions, Gugenheim, Lambe and Stasheff proved in the early 90s that $A \otimes_t C$ is homology equivalent to objects $M \otimes_{t'} C$ and $A \otimes_{t''} N$, M and N being strong deformation retracts of A and C, respectively.

We attack here this problem from the point of view of contractions. We find explicit contractions from $A \otimes_t C$ to $M \otimes_{t'} C$ and $A \otimes_{t''} N$. Moreover, we describe another way for deriving an A_∞-structure from a given (co)algebra, distinct from the *tensor trick* one. Applications to the study of the structure of homological models for semidirect products and central extensions of abelian groups are given.

Linearization through generalized Seiberg-Witten map in the Poisson-Sigma model

Glenn Barnich* (Université Libre de Bruxelles)

Pietro A. Grassi (C.N. Yang Institute for Theoretical Physics, SUNY at Stony Brook)

Andrea Quadri (Max-Planck-Institute for Physics (Werner-Heisenberg-Institute, Munich))

In the context of the Poisson-Sigma model, the linearization problem for Poisson structures is related to the existence of a generalized Seiberg-Witten map. In particular, for Poisson structures whose linearized approximation is semi-simple, the explicit construction of the linearization through homological methods is discussed.

HPT and the miniversal deformation

Johannes Huebschmann (Universite des Sciences et Technologies de Lille)

In a joint paper with J. Stasheff [Forum mathematicum 14 (2002) 847–868, `math.AG/9906036`], by means of homological perturbation theory, a solution of the master equation is given for a general, not necessarily formal, dg Lie algebra. The purpose of the talk is to show that this solution yields the miniversal deformation, in a suitable sense, for any deformation problem controlled by a dg Lie algebra. This includes the ordinary miniversal deformations in deformation theory (deformations of complex manifolds, of germs of complex hypersurfaces, of singularities, of flat vector bundles, etc.) usually obtained under additional hypotheses (e.g. of the kind that the dg Lie algebra under consideration be formal). The approach also applies to the formality conjecture.

A(∞)-structures and Inversion Theory

María José Jiménez Rodríguez* (Universidad de Sevilla)

Pedro Real (Universidad de Sevilla)

In the first part of this talk, an A(∞)-algebra (resp. an A(∞)-coalgebra) M is described in terms of an explicit special homotopy equivalence, called contraction, from the DG-algebra $\bar{\Omega}(\tilde{B}(M))$ (resp. $\bar{B}(\tilde{\Omega}(M))$) to the DG-module M. Then, taking into account the classical perturbation results of Gugenheim, Lambe and Stashef about A(∞)-structures and contractions (or SDR-data, in their terminology) [V.K.A.M. Gugenheim, L.A. Lambe and J.D. Stasheff."Perturbation theory II". Illinois Journal of Math., vol. 35, n. 3, 357-373(1991)], it is possible to conclude that operations on A(∞)-structures can be done in terms of contractions. In the second part, we focus our interest in the category of commutative differential graded algebras and we study the problem of computing the A(∞)-coalgebra structure of its 1-homology. For doing this, we use perturbation techniques such as the Tensor Trick (Gugenheim, Lambe , Stasheff 1991) and the Inversion Technique [P. Real. "HPT and associativity". Homology, Homotopy and Applications, v. 2, n. 5, 2000, 51-88 (2000)].

Homotopy algebra structure on homology

Tornike Kadeishvili (Georgian Academy of Sciences)

We show that if a dg module C is an algebra over a dg operad P then the homology $H(C)$ is a *homotopy P-algebra*, that is (equivalently): (i) there exists a it strong homotopy operadic map from P to the endomorphism operad $End(H(C))$; (ii) there exists a *twisting cochain* from the bar construction BP to $End(H(C))$; (iii) $H(C)$ is an algebra over the cobar bar operad ΩBP; (iv) $H(C)$ is an algebra over a free resolution $F \to P$ of P.

Finite Dimensional L_∞ Algebras, Preliminary Report

Tom Lada (North Carolina State University)

We will discuss several examples of finite dimensional L_∞ algebras that arise in joint work with M. Daily. These examples are related to our study of L_∞ modules as well as to our study of the algebra of gauge transformations in the Berends, Burgers, van Dam context.

Invariance of Homotopy Algebras

Martin Markl (Czech Academy of Sciences)

The aim of the talk is to show that various strongly homotopy algebras (A-infty algebras, strongly homotopy Lie algebras, C-infty algebras, etc.) are homotopy invariant concepts and discuss consequences of this fact. Our approach will be a 'constructive one', that is, based on specific models of colored operads describing spaces of maps, thus giving rise to a very explicit notion of homotopy equivalence. The talk will more or less follow preprint math.AT/9907138 "Homotopy algebras are homotopy algebras" (to appear in Forum Mathematicum).

HPT and cohomology operations

Rocio González–Díaz (Universidad de Sevilla)

Pedro Real* (Universidad de Sevilla)

In this talk, we explain a method for translating the classical work of cohomology operations into the language of the "recent" area named Homological Perturbation Theory. This translation allows us to describe primary and higher cohomology operations at the cochain level in terms of permutations and component morphisms of a given Eilenberg-Zilber contraction. A final step of simplification of these simplicial formulae gives us a combinatorial picture of some cohomology operations, in which only face operators are involved. The resulting formulation allows us to avoid partially the problem of computability of cohomology operations on an arbitrary simplicial set.

Algebraic Models for Homotopy Types

Julio Rubio (Universidad de La Rioja)

Francis Sergeraert* (Université Grenoble I)

Obtaining algebraic models for $\mathbb{Z}$-homotopy types is a major problem. The *statement* of the problem itself is a constant source of strong and regrettable ambiguities. We explain why the adjective *algebraic* is in fact inappropriate, the right one being *effective* (or constructive, computable, ...). The notion of simplicial set with effective homology (SSEH), due to the lecturers, is a complete solution, very simple from a theoretical point of view, once the possibilities of functional programming are understood. This solution led to an interesting concrete computer work. Other solutions are based on the operadic techniques, and they are now intensively studied. The key point is the notion of E_∞-operad; the main results so obtained are described and compared with the SSEH solution. The current result is that the SSEH solution is, for concrete work, much more simple and efficient. The good point of view for future work is probably a mixture of SSEH's and operadic techniques, the last ones being to be considered as mandatory tools to understand and improve the results obtained so easily through SSEH's.

Homotopy Theory of algebras over "n-dimensional little cube" operad

Vladimir Smirnov (Moscow State Pedagogical University)

Here we consider the homotopy theories of algebras over operads and in particular over the "n-dimensional little cubes" operads E_n, $1 \leq n \leq \infty$. The main results are the following.

Theorem 1 If X is an $n-1$-connected topological space then there is the isomorphism

$$\pi_*^{E_n}(X) \cong S^n P H_*(\Omega^n X),$$

where $\pi_*^{E_n}(X)$ denotes the homotopy groups of X in the category of E_n-algebras, S^n denotes n-fold suspension, P denote the primitive elements, $H_*(\Omega^n X)$ denotes the homology of the n-fold loop space over X.

Theorem 2 There is the spectral sequence (of the Adams type) which converges to the homotopy groups $\pi_*^{E_n}(X)$. The first term of this spectral sequence is isomorphic to the module $S^n R_n - 1 L_n - 1 S^- n H_*(X)$, where $R_n - 1$ denotes the submodule of the Dyer-Lashof algebra generated by allowable sequence of excess less than n, $L_n - 1$ denotes the free Lie algebra.

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2. Smirnov V.A. Homotopy theory of coalgebras. Izv. Ac. Nauk (Russia) 49 (1985), 1302-1321.

The Biderivative Operator and A-infinity Hopf Algebras

Ron Umble (Millersville University of Pennsylvania)

An A_∞-Hopf algebra is a dgm A equipped with structurally compatible operations $\left\{\omega^{i,j} : A^{\otimes i} \to A^{\otimes j}\right\}$. Structural compatibility is controlled by the biderivative operator, defined in terms of certain cochain algebras of pemutahedra over the universal PROP. In particular, $\left(A, \omega^{i,1}\right)$ is an A_∞-algebra and $\left(A, \omega^{1,j}\right)$ is an A_∞-coalgebra. We prove that the homology of every dg Hopf algebra over a field is an A_∞-Hopf algebra. In particular, there exists an A_∞-Hopf algebra structure on the homology of a loop space that specializes to the A_∞-(co)algebra structures observed by Gugenheim and Kadeishvili.

Brane Topology

Alexander A. Voronov (University of Minnesota)

String Topology was introduced a few years ago by M. Chas and D. Sullivan, who defined a new algebraic structure, that of a BV-algebra, on the homology of the free loop space in a compact manifold. This structure describes interaction of strings (loops) in the manifold and mimics the Gromov-Witten invariants in a purely topological setting. I will discuss String Topology, as well as a higher-dimensional generalization of it, where strings get replaced by spheres. This generalization is related to Hochschild cohomology and a conjecture of Kontsevich. I will present a computation of the homology of sphere spaces generalizing Burghelea-Fiedorowicz's results on the homology of loop spaces. Some of this is joint work with Dennis Sullivan.