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Abstracts

Session 21

Interpolation Theory, Function Spaces and Applications

Organizers:

Fernando Cobos (Universidad Complutense de Madrid)
Pencho Petruchev (University of South Carolina)

Index of Abstracts

(In this index, in case of multiple authors, only the speaker is shown)

Weighted L^p estimates for classical operators using analytic families of operators

Maria Jesus Carro Rossell (Universitat de Barcelona)

We study a variant of Stein's interpolation theorem on analytic families of operators and apply it to the case of weighted Lebesgue spaces. The results turn out to be very useful to obtain weighted estimates for many kind of operators such as convolution operators, Fourier multipliers, operators associated to flat curves, discrete operators, ...

Entropy Function Spaces

Joan Cerdá Martín* (Universitat Autonoma de Barcelona)

Joaquim Martín (Universitat de Barcelona)

We associate to every (quasi–)Banach function space E and to every entropy function (such as Shannon entropy) two new "extremal" spaces, $\Lambda(E)$ and $M(E)$. In the case of a Banach function space, as happens with the classical Lorentz and Marcinkiewicz spaces associated to a rearrangement invariant space, $\Lambda(E) \subset E \subset M(E)$ and these inclusions are isometric when restricted to characteristic functions of measurable sets. If E is Shannon entropy, $\Lambda(E)$ is the corresponding entropy space. Necessary and sufficient conditions for $\Lambda(E)$ and $M(E)$ to be normed spaces are proved, and several applications to functional properties of several variants of Lorentz spaces and entropy spaces are given.

Optimal domains for the kernel operator associated with Sobolev's inequality

Guillermo Curbera Costello* (Universidad de Sevilla)

Werner J. Ricker (Eichstätt Universitat)

Refinements of the classical Sobolev inequality lead to optimal domain problems in a natural way. This is made precise in recent work of Edmunds, Kerman and Pick, where the fundamental technique is to identify equivalence of the (generalized) Sobolev inequality with boundedness of an associated kernel operator on [0,1]. We study the optimal domain for this kernel operator when the values are taken in rearrangement invariant spaces, providing various characterizations of it, and the properties of the kernel operator when it is extended to act in its optimal domain. We identify the maximal rearrangement invariant space inside the optimal domain. We consider the cases when the values are taken in Lorentz or Mancinkiewicz spaces.

Recent Theorems on Interpolation of Operators

Ronald DeVore (University of South Carolina)

We shall discuss new interpolation theorems that were developed for problems in partial differential equations. In the process we shall introduce new techniques for obtaining interpolation results.

Inclusion indices of function spaces and applications

Luz Fernández–Cabrera (Universidad Complutense de Madrid)

We investigate inclusion indices for general function spaces, not necessarily symmetric. Using then, we estimate the grade of proximity between two spaces $E \hookrightarrow F$ when we have certain information on the inclusion. The results are based on ideas from interpolation theory.

The Gustavsson-Peetre Method for Several Banach Spaces

Pedro Fernández Martínez (Universidad de Murcia)

We show an extension of the Gustavsson-Peetre method to the context of N-tuples of Banach spaces. We present estimates for the norm of the interpolated operator, and apply the method to tuples of weighted L_p spaces and to tuples of Orlicz spaces, identifying the outcoming spaces in both cases.

An axiomatic approach to function spaces and spectral synthesis

Lars Inge Hedberg* (inköping University)

Y. Netrusov (University of Bristol)

We define a class of function spaces which includes the Besov and Lizorkin–Triebel classes as special cases, and prove atomic representations and spectral synthesis in this setting.

Compactness of Certain Hardy-Type Operators

Pankaj Jain (University of Delhi)

A characterization will be given for the compactness of the "Hardy-Steklov operator"

$$(Tf)(x) = \int_{a(x)}^{b(x)} f(t)\,dt$$

between suitable weighted Lebesgue spaces where a and b are certain increasing differentiable functions on $[0, \infty]$. This result will be used to characterize the compactness of the n-dimensional Hardy-Steklov operator

$$(T_E f)(x) = \int_{b(|x|)S_x \setminus a(|x|)S_x} f(t)\,dt, \qquad x \in E$$

where E is a "cone" in $\mathbf{R}^n$ and S_x is a part of E with "radius"$\leq |x|$. The compactness of T_E includes, as a special case, the compactness of the operator in which the integral is taken over anulii.

St. Louis interpolation method and weak compactness

Andrzej Kryczka (Universidad Complutense de Madrid)

We consider certain seminorm Γ which measures the deviation of bounded linear operators from weak compactness (in particular, $\Gamma(T) = 0$ if and only if T is a weakly compact operator). We present a logarithmically convex estimate on Γ for operators acting between St. Louis interpolation spaces with respect to finite families of Banach spaces. This estimate is a quantitative extension to weakly noncompact operators of some results on behavior of weakly compact operators under complex interpolation.

Interpolation properties of a scale of spaces

Elijah Liflyand (Bar-Ilan University, Ramat-Gan, Israel)

A scale of function spaces is studied which proved to be of considerable importance in analysis. These are the spaces $A_{p,r}$ endowed with the norms

$$\|f\|_{A_{p,r}} = \Big(\int_0^\infty \Big(u^{-1}\int_{u\le|t|\le 2u} |f(t)|^p dt\Big)^{r/p} du\Big)^{1/r} \quad (1 \le p, r < \infty),$$

$$\|f\|_{A_{\infty,r}} = \Big(\int_0^\infty \Big(\sup_{u\le|t|\le 2u} |f(t)|\Big)^{r} du\Big)^{1/r} \quad (1 \le r < \infty),$$

$$\|f\|_{A_{p,\infty}} = \sup_{u>0}\Big(u^{-1}\int_{u\le|t|\le 2u} |f(t)|^p dt\Big)^{1/p} \quad (1 \le p < \infty).$$

The case $p = r = \infty$ is naturally reduced to $A_{\infty,\infty} = L^\infty$; moreover for any $1 \le p < \infty$ we have $A_{p,p} = L^p$. We denote $A_{p,1} = A_p$.

Interpolation properties of these spaces are established by means of the real interpolation method. The main result consists in demonstrating that this scale is interpolated in a way different from that for L^p spaces, namely, the interpolation space is not from this scale.

Interpolation by J- and K-methods of certain closed operator ideals

Fernando Cobos (Universidad Complutense de Madrid)

Luz M. Fernández–Cabrera (Universidad Complutense de Madrid)

Antonio Manzano Rodríguez* (Universidad de Burgos)

Antonio Martínez (Universidad de Vigo)

The behavior under interpolation of certain closed operator ideals by using the general J- and K-methods is studied. In particular, the results apply to weakly compact operators, Rosenthal operators and Banach-Saks operators. Our techniques allow to extend previous results established by some authors for general K-spaces, as well as to give the corresponding novel versions for general J-spaces.

Interpolation of Entropy Function Spaces

Joan Cerdá (Universitat Autonoma de Barcelona)

Heribert Coll (Universitat Autonoma de Barcelona)

Joaquim Martín Pedret* (Universitat Autonoma de Barcelona)

As for the rearrangement invariant case, to every function space and to every entropy function E, two new extremal spaces $\Lambda(E)$ and $M(E)$ similar to the classical Lorentz and Marcinkiewick spaces can be associated. In the case of a Banach function space, $\Lambda(E) \subset E \subset M(E)$, and these inclusions are isometric when restricted to characteristic functions of measurable sets. As expected, they have an important role in interpolation theory. A complete analysis of real interpolation with L^∞, and applications to the case of block and Lorentz spaces are given.

Complex interpolation, minimal methods and compact operators

Fernando Cobos (Universidad Complutense de Madrid)

Luz M. Fernández–Cabrera (Universidad Complutense de Madrid)

Antonio Mártínez Martínez* (Universidad de Vigo)

We characterize compact operators between complex interpolation spaces in terms of a weaker compactness condition and convergence of certain sequence of operators. Similar results for other minimal methods are also obtained.

Best possible compactness results of Lions–Peetre type for N-tuples

Raúl Romero Martín (Universidad Complutense de Madrid)

Let A be an intermediate space with respect to the N-tuple $\overline{A} = (A_1, \ldots, A_N)$. We investigate whether or not for any $T \in \mathcal{L}(\sum_{j=1}^{N} A_j, B)$ with $T : A_j \mapsto B$ compact for all $1 \leq j \leq N$ with $j \neq i, i+1$, it holds that $T : A \mapsto B$ is compact. The dual case is studied too, as well as the extension of these results to other operator ideals.

Smoothness spaces and multiresolution analysis with redundant basis elements

B. Karaivanov (University of South Carolina)

P. Petrushev (University of South Carolina)

Robert C. Sharpley* (University of South Carolina)

A scale of smoothness spaces (B-spaces) is studied and several proposed nonlinear approximation methods are shown to be achieve optimal rates in this setting. Scalable algorithms are developed for nonlinear n-term Courant element approximation of functions in $L_p(0 < p \leq \infty)$ on bounded polygonal domains in $R2$.

The B-spaces are defined through a multiresolution analysis based on redundant collections of Courant elements, generated by multilevel nested triangulations allowing arbitrarily sharp angles. These spaces are closely related to Besov spaces, but are more natural for approximations required for efficient computational visualization of surfaces. A primary motivating application is the real time approximation of Digital Terrain Maps and associated imagery. In a certain sense, this may be viewed as the analogure of multiresolution wavelet analysis and its application to image compression.

Simple thresholding criteria enable approximation of a target function f to optimally high asymptotic rates which are determined and automatically achieved by the inherent smoothness of f. The algorithms provide direct approximation estimates and, using modifications of interpolation theoretic methods, permit utilization of a general Jackson-Bernstein machinery to characterize n-term nonlinear Courant element approximation.

Form boundedness of Schrödinger operators and related function spaces

Igor Verbitsky (University of Missouri-Columbia)

A scale of function spaces associated with the form boundedness problem for the Schrödinger operator $H = -\Delta + V$ on $L2(\Omega)$, where Ω is a domain in R^n, is studied. The class of all real- or complex-valued distributional potentials V such that the quadratic form inequality

$$|\langle Vu, u\rangle| = \left| \int_\Omega |u|^2 \, V \, dx \right| \leq C \, (||\nabla u||^2_{L2(\Omega)} + ||u||^2_{L2(\Omega)}), \quad \forall u \in C_0^\infty(\Omega),$$

holds is characterized. A similar problem for the relativistic Schrödinger operator $H = \sqrt{-\Delta + m2} - m + V$, as well as L^p-analogues in terms of multipliers on a pair of Sobolev spaces $V : W^{m,p}(\Omega) \to W^{-m,p}(\Omega)$ will be discussed.