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Abstracts

Session 22

Lorentzian Geometry and Mathematical Relativity

Organizers:

Luis J. Alias (Universidad de Murcia)
Gregory J. Galloway (University of Miami)

Index of Abstracts

(In this index, in case of multiple authors, only the speaker is shown)

Gowdy phenomenology in scale free variables

Lars Andersson (University of Miami)

I will describe the phenomenology of Gowdy spacetimes in the direction of the singularity. Using Hubble normalized variables, it is possible to extract an asymptotic dynamical system which models the dynamics near the singularity. The asymptotic attractor is identified, and used to explain the behavior near spike points.

Relativistic particles with rigidity and torsion

Manuel Barros Díaz (Universidad de Granada)

The simplest models describing spinning particles with rigidity and torsion, both massive and massless, are considered. The moduli spaces of solutions are completely exhibited in Lorentzian backgrounds with constant curvature. In particular, in models with only rigidity of order one, while spinning massive particles evolve fully along helices in any three-dimensional background, spinning massless particles need anti De Sitter background to be consistent. The main machinery used to determine those moduli in AdS_3 is provided by a pair of natural Hopf mappings. Therefore, Hopf tubes, B-scrolls and specially the Hopf tube constructed on a horocycle in the hyperbolic plane, play a principal role in this program.

Conformal Killing spinors in Lorentzian geometry

Helga Baum (Humboldt University Berlin)

Conformal Killing spinors on a Lorentzian spin manifold are spinor fields which satisfy the so-called Penrose twistor equation. A special property of such a spinor field is that its square root is a future-directed causal conformal vector field. In the lecture we discuss special Lorentzian geometries which admit conformal Killing spinors. In particular, we give a local classification of all solutions up to dimension 7. The occuring geometries include Fefferman spaces, Einstein- Sasaki-manifolds, Brinkman spaces with parallel spinors, special Kaehler flags and certain warped product constructions. They are closely related to the properties of the conformal vector field associated to the spinor.

The Penrose inequality and null hypersurfaces

Göran Bergqvist (Linköpings Universitet)

In general relativity the Penrose inequality states the area A of a trapped surface and the total mass m of spacetime are related by $A \leq 16\pi m2$. Various different approaches, e.g. by Huisken-Ilmanen, Bray and Malec-Mars-Simon, have recently led to proofs of important special cases and progress towards a general proof of the inequality. In general one studies monotonicity properties of quasilocal mass expressions along hypersurfaces. Here we describe some null hypersurface approaches and discuss advantages and disadvantages compared with other methods.

Integral formulae for spacelike hypersurfaces in conformally stationary spacetimes and applications

Luis J. Alias (Universidad de Murcia)

Aldir Brasil Jr. (Universidad Federal do Ceará)

Antonio Gervasio Colares* (Universidad Federal do Ceará)

We obtain general Minkowski-type integral formulae for compact spacelike hypersurfaces immersed into Lorentz manifolds possessing a timelike vector field. Theese ambients are said to be conformally stationary and examples of such spacetimes are the De Sitter Spaces and the Generalized Robertson-Walker spacetime. As an applicattions it is studied the umbilicity of compact spacelike hypersurfaces in terms of its r-mean curvatures. Several uniqueness re- sults are proved, as for instance: compact spacelike hypersurfaces are totally umbilical if either some of its r-mean curvature are linearly related or one of them is constant.

This work is to appear in Proceed. of the Edinburgh Math. Society, 2003.

Conformal deformation of metrics

José F. Escobar (Cornell University)

I will discuss recent results on the problem of comformal deformation of metrics. Special attention will be given to scalar flat metrics because its importance in relativity. A Positive Mass Theorem on manifolds with boundary will be discussed.

Geometry and physics of lightlike curves

Angel Ferrández Izquierdo* (Universidad de Murcia)

Ángel Giménez (Universidad de Murcia)

Pascual Lucas (Universidad de Murcia)

We study some geometrical and physical problems related with lightlike curves in indefinite space forms. We first classify null helices as well as generalized null helices in Lorentzian space forms, and exhibit wide families of interesting examples. Secondly, the localized induction equation (LIE), which is a time evolution equation, is revisited for null curves $\gamma(s,t)$, s being the pseudo-arc parameter, and reformulated in $\mathbb{L}^n$ as $\frac{\partial\gamma}{\partial t} = \frac{\partial 2\gamma}{\partial s2} \wedge \cdots \wedge \frac{\partial^n\gamma}{\partial s^n}$, which will be called the null localized induction equation (NLIE). Then we give explicit solutions of NLIE. Finally, we look for critical points of functionals $\mathcal{F}(\gamma) = \int_\gamma f(k)$ defined on null curves, $f(k)$ standing for a function on the first curvature of γ, and find out solutions in low dimensions.

This work has been partially supported by the DGI(MCYT) grant BFM2001-2871-FEDER.

Asymptotically Simple Space-times

Helmut Friedrich (Max-Planck-Institut fuer Gravitationsphysik)

We discuss the conceptual and practical aspects of the notion of "asymptotic simplicity", describe results about the existence and properties of asymptotically simple solutions to Einstein's field equations, and indicate remaining open problems.

Recent Advances on Osserman Manifolds

Eduardo García-Río (Santiago de Compostela)

The study of the curvature is a central topic in Riemannian and pseudo-Riemannian geometry, as it provides of the simplest algebraic invariant of the metric structure. The Jacobi operators, as being a part of the curvature tensor, became also of interest. Althought the explicit determination of the Jacobi operators is not always possible, many geometric information can be obtained from the knowledge of their eigenvalues and associated eigenspaces. A semi-Riemannian manifold (M, g) is said to be an *Osserman space* if the eigenvalues of the Jacobi operators $R_X = R(\cdot\,, X)X$ are independent both on the point $m \in M$ and the direction $X \in T_mM$, $\|X\| = \pm 1$. It was conjectured by Osserman that such manifolds must be flat or locally isometric to a rank-one symmetric space in the Riemannian category. This was proved in many cases, although the general problem remains still open. The so-called Osserman problem has also been studied in Lorentzian geometry, where it behaves in a completely different way than its Riemannian analog, since a Lorentz manifold is Osserman if and only if it is of constant curvature. Moreover, the existence of nonsymmetric Osserman semi-Riemannian manifolds of any signature (p, q), p, $q \geq 2$ motivated some further investigation on Osserman semi-Riemannian manifolds, with special attention to $(++--)$-metrics. The purpose of this talk is to report on some recent results on the investigation of the Osserman problem. Special attention will be paid to some simple cases occurring when the manifold is locally symmetric or the number of different eigenvalues of the Jacobi operators is assumed to be exactly two.

Spacelike energy of timelike unit vector fields on a Lorentzian manifold

Olga Gil–Medrano (Universidad de Valencia)

Ana Hurtado Cortegana* (Universidad de Valencia)

In a Lorentzian manifold, we define the spacelike energy of a timelike unit vector field as the functional that for a reference frame Z, is given by the integral of the square norm of the restriction of ∇Z to $Z^{\perp}$. We have computed the first and second variation. We have shown that vector fields corresponding to static observers, and to the comoving observer of a GRW are critical . We have also shown that for projective vector fields, the criticality can be described in terms of the Ricci tensor and in particular: every affine reference frame on an Einstein manifold is critical for the spacelike energy .

On the Penrose inequality for general horizons

Edward Malec (Uniwersytet Jagielloński)

Marc Mars Lloret* (Universidad de Salamanca)

Walter Simon (University of Vienna)

For asymptotically flat initial data satisfying certain energy conditions, the "Penrose inequality" conjectures a lower bound for the ADM-mass (or the Bondi mass) in terms of an outermost apparent horizon. The present status of the Penrose inequality will be summarized. First I will recall the proof, due to Geroch and Jang and Wald, of monotonicity of the Geroch-Hawking mass under a smooth inverse mean curvature flow (IMCF) for data with non-negative Ricci scalar. Existence of a suitable weak version of this flow has been obtained by Huisken and Ilmanen, thus proving the Penrose inequality for minimal surfaces in a purely riemannian case. Regarding more general horizons, I will sketch our proof of the Penrose inequality which holds for general apparent horizons and for data satisfying the dominant energy condition, but imposes (i) smoothness of the IMCF and (ii) suitable restrictions on the data. I will show that condition (ii) can be fulfilled, at least locally, by a suitable choice of the initial surface in a given spacetime. Possibilities for relaxing conditions (i) and (ii) will be discussed.

The Bjorling problem in Lorentz-Minkowski spaces

Luis J. Alías (Universidad de Murcia)

Pablo Mira Carrillo* (Universidad Politécnica de Cartagena)

We present a new approach to the study of maximal surfaces in Lorentz-Minkowski space $L3$, as well as some applications and an extension to L^n. This approach is based on the work [1], and relies on the solution to Björling problem in $L3$, which consists on the following: given a spacelike analytic regular curve in $L3$ and an analytic distribution of spacelike tangent planes along this curve, construct all maximal surfaces in $L3$ spanning this configuration.

In this communication we shall explain this method and give several applications. We shall also provide, motivated by [2], an extension of Björling problem to L^n, and use it in that context to obtain several global consequences. For instance, we will show how to produce complete maximal Möbius strips in $L4$.

References:

[1] L.J. Alías, R.M. Chaves, P. Mira, to appear in Math. Proc. Cambridge Philos. Soc., (2003)

[2] L.J. Alías, P. Mira, C.R. Math. Acad. Sci. Paris, 334 (2002), 389–394.

Symplectic methods and index theory in semi-Riemannian geometry

Paolo Piccione (Universidade de Sao Paulo)

(temporary version) We will discuss some recent advances in index theory for symplectic paths and its applications to global semi-Riemannian geometry.

The Bochner technique on Lorentzian manifolds

Alfonso Romero Sarabia (Universidad de Granada)

The Bochner technique is used here to classify several compact Lorentzian manifolds. Some obstructions, both in the compact and noncompact case, to the existence of certain kind of causal vector fields are also obtained.

The qualitative behaviour of plane wave type spacetimes

Miguel Sánchez (Universidad de Granada)

Let $M = M_0 \times R2$ be a plane wave type spacetime endowed with the general metric $\langle \cdot, \cdot \rangle_z = \langle \cdot, \cdot \rangle_x + 2\ du\ dv + H(x,u)\ du2$, where $(M_0, \langle \cdot, \cdot \rangle_x)$ is any (complete) Riemannian manifold and $H(x,u)$ an arbitrary function. The classical exact case $M_0 = R2$, $H(x,u) = \sum_{i,j} h_{ij}(u)x_i x_j (\not\equiv 0)$ is a highly idealized model of plane wave, which has been widely studied by using direct methods. Our purpose is to study relevant geometrical properties (causality, geodesic completeness, geodesic connectedness, conectedness by causal geodesics, focalization of geodesics) in the general case.

Essentially, we show that most of these properties depend on the behaviour of $H(x,u)$ at spatial infinity. Roughly, the quadratic behaviour of $H(x,u)$ in some spatial direction (as in the classical exact case) becomes critical. In fact, if $-H(x,u)$ behaves subquadratically, $-H(x,u) \leq R_0(u)|x|^{2-\epsilon}$ (for some $\epsilon > 0$ and large distance $|x|$ to a fixed point) then the spacetime has some good geometrical properties (for example, M is globally hyperbolic and geodesically connected). But these properties may not hold under any arbitrarily weak superquadratic behaviour $-H(x,u) \leq R_0(u)|x|^{2+\epsilon}$. Then, the quadratic case is specially analyzed.

Black Hole Non-Existence Results in Spacetimes with a Negative Cosmological Constant

Sumati Surya (University of California, Davis)

Interest in spacetimes with a negative cosmological constant has revived in the last decade with the discovery of a large number of blackhole solutions (and also, more recently, with the AdS/CFT correspondence conjecture of string theory). The peculiarity of these new solutions is that they admit horizons with non-spherical topology unlike blackholes in asymptotically flat spacetimes with zero cosmological constant which are constrained to have spherical horizon topology. While the topological censorship theorem provides certain topological constraints on the domain of outer communication of a black hole spacetime (given the Penrose conformal boundary at infinity) it is also of interest to ask whether there are any geometric obstructions to blackhole existence. We will construct such obstructions to blackhole existence for a class of static vacuum spacetimes with negative cosmological constant. Our methods include a generalisation of the null splitting theorem due to Galloway, as well as a suitable modification of the boundary analysis due to Fefferman and Graham.

Quasiconvex foliations and asymptotically flat metrics of non-negative scalar curvature

Brian Smith (Cornell University)

Gilbert Weinstein (University of Alabama at Birmingham)

We prove that a broad subset of the space of asymptotically flat Riemannian metrics of nonnegative scalar curvature on $\mathbb{R}3$ is connected using a new method for prescribing scalar curvature that generalizes a method developed by Bartnik for quasi-spherical metrics.

AdS/CFT and Uniqueness of the AdS Soliton Spacetime

Eric Woolgar (University of Alberta)

In late 1997, Maldacena proposed a specific correspondence between conformal gauge field theory (CFT) in 4 dimensions and string theory on the product space of 5-dimensional Anti-de Sitter (AdS) spacetime with a 5-sphere "internal manifold". The correspondence, both sharpened and generalized by Witten, posits that gauge theories of particle physics may be understood by studying negative Ricci curvature Einstein manifolds (particularly in 5 dimensions) with prescribed Penrose conformal boundary at infinity, a natural problem in general relativity. I will discuss some of the implications of the correspondence for mathematical relativity. I will focus on a specific example, the generalized positive energy conjecture of Horowitz and Myers. In this case, the application of the null splitting theorem of Lorentzian geometry has led Galloway, Surya, and me to construct a uniqueness theorem (in general dimension) for a spacetime known as the AdS soliton.