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Abstracts

Session 24

Mathematical Fluid Dynamics

Organizers:

Diego Córdoba (CSIC, Madrid, and Princeton University)
Susan Friedlander (University of Illinois)
Marco Antonio Fontelos (Universidad Rey Juan Carlos)

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Long-time asymptotics for nonlinear fourth order diffusion equations

José Antonio Carrillo de la Plata (Universidad de Granada)

In this talk a survey on recent results concerning long time behavior of solutions for fourth order diffusion equations is presented. Part of this work has been done in collaboration with G. Toscani and M.J. Cáceres.

We first investigate the large–time behavior of strong solutions to the one-dimensional fourth order degenerate parabolic equation $u_t = -(uu_{xxx})_x$, modeling the evolution of the interface of a spreading droplet. For nonnegative initial values $u_0(x) \in H^1(\mathbb{R})$ we prove explicit and universal algebraic decay in the L^1-norm of the strong solution $u(x,t)$ towards the unique strong source type solution of the equation with the same mass. The method we use is based on the study of the time decay of the entropy introduced by the author for the porous medium equation, and uses analogies between this thin film equation and the porous medium equation.

On the other hand, the previous equation lies in a larger class of equations for which the same method of comparison of entropy production with respect to the porous medium case works. In fact, another interesting equation that is inside this class is $u_t = -(u(\log u)_{xx})_{xx}$. We give necessary conditions over the initial data to have a global in time smooth solution with periodic boundary conditions and we prove the exponential convergence towards the unique steady state with the same mass.

On the dissipative quasi–geostrophic equation

Diego Córdoba Gazolaz (Consejo Superior de Investigaciones Científicas)

We consider the initial value problem for the dissipative 2D Quasi-geostrophic equation and we present a survey of global and local existence results.

On surface water waves

Walter Craig (McMaster University)

This talk will focus on questions of existence, non-existence, and regularity of traveling waves on the surface of an infinite body of water. The classical theory of two dimensional surface profiles will be extended in some cases to three and higher dimensional cases.

Squirt singularities

Charles Fefferman (Princeton University)

Necessary conditions are described for the formation of certain types of singularities for several fluid equations in 2 and 3 dimensions.

Why viscous fluids adhere to rugose walls and some related questions

Enrique Fernández-Cara (Universidad de Sevilla)

This talk deals with a rigorous justification of the following assertion: A viscous fluid cannot slip on a wall covered by microscopic asperities. Indeed, due to the viscous dissipation, the surface irregularities bring to rest the fluid particles in contact with the wall. In mathematical terms, this corresponds to an asymptotic property established for any family of fields that slip on oscillating boundaries and remain uniformly bounded in the H^1-norm. I will also consider other questions, also concerning the behavior of viscous fluids near solid walls with asperities.

[1] Y. Amirat, B. Climent, E. Fernández-Cara, J. Simon: *The Stokes equations with Fourier boundary conditions on a wall with asperities*, Math. Methods Appl. Sci. **24** (2001), no. 5, 255–276.

[2] J. Casado-Díaz, E. Fernández-Cara, J. Simon: *Why viscous fluids adhere to rugose walls (a mathematical explanation)*, J. Diff. Eq., to appear.

Blow up in a 3-d vector model for the Euler equations

Susan Friedlander* (University of Illinois)

Nataša Pavlović (University of Illinois)

We present a 3-d vector dyadic model given in terms of an infinite system of nonlinearly coupled ODE. This "toy" model is inspired by a Littlewood-Paley partition of the nonlinear term in the Euler equations for the motion of an ideal fluid. The model has structural similarities with the Euler equations and it mimics certain important properties of the fluid equations, namely conservation of energy and divergence free velocity. We prove that for certain families of initial data blow-up occurs in the model system in the sense that, for $s > 3/2$, the H^s-norm becomes unbounded in finite time.

Formation of singularities under the localized induction approximation

Susana Gutiérrez de Gracia (Universidad del País Vasco)

We consider the motion of an isolated vortex filament in the three dimensional space under the localized induction approximation. Here, our main topic is to prove the existence of a uniparametric family of smooth self-similar vortex solutions of localized induction approximation that develops a singularity in the shape of a corner in finite time.

We also study some properties of this family of solutions. In particular, we quantify some relevant parameters of the dynamic of these solutions in terms of the curvature of the initial configuration. Moreover, we obtain precise asymptotic developments of the wavelike phenomena that appear along the two asymptotic lines defined by the cusplike deformation.

On weak solutions for generalized Oldroyd model for laminar and turbulent flows of nonlinear viscous—elastic fluid

Mokhtar Kirane (La Rochelle University)

We consider the statement of an initial-boundary value problem for a generalized Oldroyd model describing both laminar and turbulent motions of a nonlinear viscous—elastic fluid. The operator interpretation of a posed problem is presented. The properties of operators forming the corresponding equations are investigated. We introduce approximating equations and prove their solvability. On that base the existence theorem for the operator equation equivalent to the stated initial-boundary value problem is proved.

On the evolution of sharp fronts for the quasi-geostrophic equation

José Luis Rodrigo (Princeton University)

We will present an heuristic derivation of the equation for the evolution of a sharp front for the quasi-geostrophic equation in the periodic setting. We will proof local well-posedness for that equation.