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Abstracts

Session 26

Moduli Spaces in Geometry and Physics

Organizers:

Steven B. Bradlow (University of Illinois)
Oscar García–Prada (Universidad Autónoma de Madrid)

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A new look at the moduli of sheaves

Luis Álvarez-Cónsul (University of Bath)

In this talk I will explain a construction of the moduli space of semistable sheaves on a projective variety, over an algebraically closed field of characteristic zero. The method will be to embed the moduli problem of sheaves into a moduli problem for representations of a quiver, which we associate to our original moduli problem. This embedding underlies and simplifies several algebraic aspects of Carlos Simpson's construction.

Moduli Problems in Sasakian Geometry

Charles Boyer (University of New Mexico)

We discuss moduli spaces of Sasakian structures on certain manifolds. For example, we prove that the moduli space of Sasakian structures on homotopy spheres that are bounded by a parallelizable manifold has an infinite number of components. We also discuss the moduli problem for Sasakian-Einstein structures. Lower bounds on the dimension are given in the case of the k-fold connected sum of $S2 \times S3$ for $1 \leq k \leq 9$.

Some examples of relative SL(2)-character varieties over surfaces

William Goldman (University of Maryland)

For any group G, let $X(G)$ denote the quotient of the $SL(2)$-representation variety of G by $SL(2)$-conjugation. Let S be a compact surface with boundary components $b_1, \ldots, b_n$. The *relative $SL(2)$-character variety of S* is the algebraic family

$$X(\pi_1(S)) \longrightarrow \prod_{i=1}^{n} X(\pi_1(b_i)).$$

We explicitly describe the topology, their real structures, their automorphisms, and Poisson structures for relative character varieties for some surfaces with $\chi(S) = -1, -2$.

Stable principal bundles on projective varieties

Tomás L. Gómez (Universidad Complutense de Madrid)

The moduli space of semistable vector bundles on a curve was constructed in the 60's by Narasimhan and Seshadri, and since then, its generalizations, detailed study and applications have experimented enormous growth. In the next decade, the moduli of semistable principal G-bundles on a curve was constructed by Ramanathan, but its study is not yet as developed as that of vector bundles. In the first part of this talk I will review theorems that have been generalized from vector bundles to principal bundles, and in the second part I will go in more detail about my own recent work. In particular, the study done in collaboration with I. Biswas about the behavior of stability under restriction to subvarieties (Grauert-Müllich, Flenner and Mehta-Ramanathan theorems).

Fourier-Mukai and D branes on Calabi-Yau manifolds

Daniel Hernández Ruipérez (Universidad de Salamanca)

D-branes have been recently a central part of superstring theory. The evolution in the notion of ("B" type) D-brane goes from considering a D-brane as something on which an open string may end to think of it as an object of the (bounded) derived category $D(X)$ of coherent sheaves (Kontsevich, Douglas). In this presentation, open strings with endings in the branes corresponding to two objects in $D(X)$ are interpreted as the morphisms in $D(X)$ between them.

Moreover, D-branes are closely connectd with mirror symmetry and especially Kontsevich's homological mirror symmetry proposal: mirror symmetry should be an equivalence (up to homotopies) between an A_∞-category A_∞ constructed from the derived category of coherent sheaves on a Calabi.Yau and an A_∞-category constructed by Fukaya associated to the isotopy classes of lagrangian submanifolds of the mirror manifold endowed with a flat $U(1)$-bundle. Then symplectic automorphisms of the mirror should correspond to self-equivalences of $D(X)$.

Since the automorphisms of $D(X)$ are the Fourier.Mukai transforms, one can look for the Fourier-Mukai transforms that should correspond to certain symplectic automorphisms of the mirror, like Kontsevich monodromies. We compute tehm in some examples and apply the same tecnhique to show the adiabatic character of T-duality.

Mirror Symmetry and Fukaya Seidel categories

Ludmil Katzarkov (Universirty of California, Irvine)

We will start this talk by introducing Fukaya Seidel categories. Then we will show how they can be used for computing Fukaya categories. We will introduce some recent results by Auroux, Donaldson and myself on singular Lefschetz pencils which combined ideas of category theory - interuniversal geometry allow us to make this approach applicable to smooth 4 dimensional manifolds.

Birational Equivalence of Higgs Moduli

Mridul Mehta (University of Chicago)

We study triples of the form (E, θ, ϕ) over a compact Riemann surface, where (E, θ) is a Higgs bundle and ϕ is a global holomorphic section of the Higgs bundle. Our main result is an explicit description of a birational equivalence which relates geometrically the moduli space of Higgs bundles of rank r and degree d to the moduli space of Higgs bundles of rank $r-1$ and degree d.

Hamiltonian Gromov-Witten invariants coupled to gravity

Ignasi Mundet i Riera (Universitat Politecnica de Catalunya)

We describe a certain compactification of the universal jacobian which fibres over the Deligne-Mumford moduli space of stable curves. As an application we give a construction of Hamiltonian Gromov-Witten invariants coupled to gravity.

Brill-Noether theory and Coherent Systems

Steven Bradlow (University of Illinois)

Oscar García-Prada (Universidad Autónoma de Madrid)

Vicente Muñoz* (Universidad Autónoma de Madrid)

Peter Newstead (University of Liverpool)

We study the moduli space of coherent systems over a complex curve. These are pairs (E, V) formed by a vector bundle E and a fixed dimensional vector space V of holomorphic sections of E. There is a stability condition which depends on a real parameter α. We study how the moduli spaces of α-stable coherent systems vary with α. This theory is applied to study the non-emptiness, irreducibility and dimension of Brill-Noether loci, in some particular cases.

Deformations of Picard Bundles

Peter Newstead (University of Liverpool)

For sufficiently high degree, the symmetric products of an algebraic curve X can be described as bundles over the Jacobian of X with projective space fibres. They are associated with vector bundles, which are known as Picard bundles and have been studied in this form for at least 40 years. (Of course the symmetric products themselves have been studied for much longer.) Around 1980 Kempf and Mukai independently studied the deformations of these bundles; Mukai's paper is especially significant as it is the one in which he introduced what is now known as the Fourier-Mukai transform. Recently Indranil Biswas, Leticia Brambila-Paz and I have made considerable progress on generalising these results to moduli spaces of vector bundles of rank > 1. I will describe our results and attempt to relate them to Mukai's ideas and hopefully to derived categories.

Fourier-Mukai transforms and local systems

Fabio Pioli (Consejo Superior de Investigaciones Científicas)

We study the action of a Fourier-Mukai transform on local systems. In the context of symplectic varieties fibred in Lagrangian tori, we show that this transform is invertible when restricted to rank one local systems supported on Lagrangian submanifolds.

Comments on traditional vanishing theorems

Sundararaman Ramanan (Institute of Mathematical Sciences)

My talk will aim at understanding the formalism that goes into the traditional vansihing theorems. The traditionsl vanishing theorems that I have in mind are of the Bochner kind and depend on the Weitzenbock formula. I will resurrect an old idea of Narasimhan and myself which gives the proper setting for such a formula. I will in fact reinforce it with an actual vanishing theorem in general, of which the traditional vanishing theorems are particular cases. Unfortunately however I have not found a new significant application so far.

The Hilbert compactification

Alexander Schmitt (Universität GH Essen)

In this talk, we will present the Hilbert compactification of the universal moduli space of semistable vector bundles on smooth curves. Based on work by Gieseker, and Nagaraj and Seshadri, this compactification was proposed by Teixidor as an alternative to Pandharipandes's compactification. We will explain that the Hilbert compactification in fact exists as a solution to a well-defined moduli space and has the properties asked for by Teixidor. Finally, we also discuss how a similar approach might be used for studying degenerations of moduli spaces of semistable principal bundles over smooth curves for other reductive groups.