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Abstracts

Session 27

Nonassociative Algebras and Their Applications

Organizers:

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Complex structures in real non-associative algebras

Zalman Balanov (Netanya Academic College)

Let $A = (\mathbb{R}2, *)$ be a real commutative (in general, non-associative) 2-dimensional algebra without 2-nilpotents and $f : A \to A$ be defined by $f(x) = x * x = x2$. Consider the following five statements.

Statement 1. The Riccati equation $\dot{x} = f(x)$, $x \in A$, has a non-zero solution in A bounded on the whole axis.

Statement 2. A polynomial equation $x3 + \alpha x2 + \beta x = 0$ has a non-zero solution in A for all $\alpha, \beta \in \mathbb{R}$ with $\alpha 2 + \beta 2 > 0$.

Statement 3. The topological index $\mathrm{ind}(0, \mathrm{f})$ is equal to 2.

Statement 4. For any non-zero $x \in A$, the map f is quasi-conformal and its Jacobian is positive.

Statement 5. There exists $M \in SL(2, \mathbb{R})$, $L \in GL(2, \mathbb{R})$ such that $x * x = ML^{-1}(Lx)2_{\mathbb{C}}$.

Question. What is a common root of Statements 1–5?

Definition. Let B be a real commutative n-dimensional algebra. We say that B admits a complete complex structure if B contains a two-dimensional subalgebra A in which the equations $x^2 x2 = -x2$ and $y3 = -y$ are non-trivially soluble.

Main Theorem. *Statements 1–5 are equivalent to the existence of a complete complex structure in A.*

Life on the Wedge

Georgia Benkart (University of Wisconsin - Madison)

This talk will survey various constructions of Lie algebras and Lie superalgebras using exterior powers. Recent work on Lie (super)algebras graded by finite root systems and locally finite simple Lie (super)algebras will be featured. Also, connections with the Tits construction of exceptional simple (super)algebras will be discussed.

Structure theory for multiplicatively semiprime algebras

Juan Carlos Cabello Piñar (Universidad de Granada)

The aim of this talk is to give a structure theory for multiplicatively semiprime algebras. The starting point is the concept of ε-closure for an ideal, which will allow us to characterize the multiplicative semiprime algebras as those semiprime algebras that are generalized annihilator algebras for the ε-closure. This fact will motivate the introduction of the concept of ε-radical, which will becomes the cornerstone for an structure theorem of type Yood.

On Locally Finite Split Lie Triple Systems

Antonio Jesús Calderón Martín (Universidad de Cádiz)

The main tool used by Lister (1952) to study finite-dimensional Lie triple systems is the standard embedding, this one being a two-graded Lie algebra $L = L0 \oplus L1$, with L^0 the span of $\{\mathcal{L}(x, y) : x, y \in T\}$, where $\mathcal{L}(x, y)$ denotes the left multiplication operator in T and $L^1 := T$. Later, Faulkner gives in 1980 an alternative approach to the classification of Lie triple systems, also in the finite-dimensional setup, by introducing a Cartan subalgebra $H0$ of the even part of the standard embedding, and a decomposition of T as the direct sum of certain root spaces relative to $H0$.

In the framework of infinite-dimensional Lie algebras, K. H. Neeb, N. Stumme and other authors have successfully developed over the recent years a theory of split and locally finite Lie algebras.

In order to suggest a possible approach to a structure theory of infinite-dimensional Lie triple systems, we introduce and study split and locally finite Lie triple systems, stating that under certain conditions the standard embedding of a split Lie triple system is a split Lie algebra and that the standard embedding of a locally finite Lie triple system is a locally finite Lie algebra. We also give a description of certain locally finite simple split Lie triple systems.

Growing Hearts in Associative Systems and Jordan Cubes

Teresa Cortés Gracia (Universidad de Oviedo)

We will show a process of pasting (or growing) simple hearts to arbitrary associative systems (algebras, pairs, or triple systems). Indeed, given an associative system S, we find a primitive associative system $\tilde{S}$ such that S is a subsystem of $\tilde{S}$, the heart $\mathrm{Heart}(\tilde{S})$ is simple (and nonzero), and the quotient $\tilde{S}/\mathrm{Heart}(\tilde{S})$ is isomorphic to S. When dealing with finite-dimensional systems, the process can be improved to find suitable families of idempotents which interact nicely with S and the heart of $\tilde{S}$. These constructions are used to compare Jordan cubes and associative powers of associative systems.

On algebras satisfying Moufang identities

José Antonio Cuenca Mira (Universidad de Málaga)

Among the algebraic identities satisfied by alternative algebras play a fundamental role the so called Moufang identities. We give some conditions on algebras satisfying the middle Moufang identity that implies alternativiness. Also characterizations of right and left Moufang algebras are given under the same conditions.

Semiregular Associative pairs

Inmaculada De las Peñas Cabrera (Universidad de Málaga)

A new class of associative pairs, called semiregular, is defined and studied in terms of its elements. A new characterization for a ring R to be semiregular is given, that allows us to study the semiregularity of the standard embedding of a pair. It is proved that an associative pair A with Jacobson radical J is semiregular if and only if A/J is regular and idempotents can be lifted modulo J.

Lie-Yamaguti algebras related to G_2

P. Benito Clavijo (Universidad de la Rioja)

Cristina Draper Fontanals* (Universidad de Málaga)

A. Elduque Palomo (Universidad de Zaragoza)

Lie-Yamaguti algebras constitute a class of algebraic systems endowed with a binary and a ternary multiplications, which are related to the reductive homogeneous spaces. No general classification results are known for these systems. In order to get examples of simple Lie-Yamaguti algebras, we describe those whose 'standard embedding' is the simple Lie algebra of type G_2.

3-graded Lie algebras with Jordan finiteness conditions

Antonio Fernández López* (Universidad de Málaga)

Esther García (Universidad de Oviedo)

Miguel Gómez Lozano (Universidad de Málaga)

A notion of socle is introduced for 3-graded Lie algebras (over a ring of scalars Φ containing $\frac{1}{6}$) whose asociated Jordan pairs are non-degenerate. The socle turns to be a 3-graded ideal and it is the linear span of minimal inner ideals (in Zelmanov's sense), each of them being a central extension of the TKK-algebra of a division Jordan pair. Non-degenerate 3-graded Lie algebras having a large socle are essentially determined by TKK-algebras of simple Jordan pairs with minimal inner ideals and their derivations algebras, which are also 3-graded.

Surjective isometries between real JB-triples*

Francisco José Fernández Polo (Universidad de Granada)

In 1997 W. Kaup proved that every surjective real linear mapping from a non exceptional real or complex Cartan factor of rank ¿ 1 to a real JBW*-triple is a triple isomorphism if, and only if, it is an isometry. In fact, W. Kaup left open the cases of two exceptional real Cartan factors. This problem is the starting point of this talk. We will show that every surjective linear isometry between two real reduced Cartan factors is a triple isomorphism. This result, together with Kaup's result allows us to remove the hypothesis of non-exceptionality, solving so the above mentioned problem. Moreover, we will get an extension of Dang's Banach-Stone Theorem to the setting of real JB*-triples.

Finite semifields

Ignacio Fernández Rúa (Universidad de Oviedo)

A *finite semifield* (or finite division ring) D is a finite nonassociative ring with identity such that the set $D^* = D \setminus \{0\}$ is closed under the product and it is a loop (i.e. for any pair of elements $a, b \in D$, $a \neq 0$, there exists a unique solution of the equation $ax = b$ and a unique solution of $xa = b$). Associative finite semifields are finite fields and any multiplicative subgroup of a Galois field is cyclic. However, for an arbitrary finite semifield D the structure of its multiplicative loop D^* is not known. G.P. Wene introduced the concept of *right primitive semifield* for those finite semifields D such that D^* is equal to the set of (right) principal powers of an element of the semifield, and proved that any semifield of 16, 27, 32, 125 and 343 elements is right primitive. Moreover, he showed that all commutative semifields three-dimensional over a finite field of odd characteristic are right primitive, and conjectured that *any finite semifield is right primitive.* We have extended his results to any three-dimensional finite semifield over its center, and have found a counterexample to the above mentioned conjecture.

On an interesting family of Nilpotent Lie Superalgebras

L.M. Camacho (Universidad de Sevilla)

José Ramón Gómez Martín* (Universidad de Sevilla)

R.M. Navarro (Universidad de Sevilla)

The two-step nilpotent Lie algebras play an important role in the Differential Geometry and Theoretical Physics. Probably, among the most important metabelian Lie algebras are the Heisenberg Lie algebras $\mathcal{H}_r$ (of dimension $2r+1$), defined by $\mathcal{C}^1(\mathfrak{g}) = \mathcal{Z}^1(\mathfrak{g})$ and $dim(\mathcal{Z}^1(\mathfrak{g})) = 1$.

In the present work we classify a family of Lie Superalgebras, in arbitrary dimension, which generalize Heisenberg Lie Algebras, in the sense that it is defined by the same conditions. We obtain that there is one of these Superalgebras for each dimension.

Ternary derivations of finite dimensional real division algebras

Clara Jiménez Gestal (Universidad de la Rioja)

Given an algebra A with product xy and three bijective linear maps $\varphi_1, \varphi_2, \varphi_3$, the new algebra $(A, \circ)$, where $x \circ y = \varphi_1^{-1}(\varphi_2(x)\varphi_3(y))$, is called an isotope of A. The relation of isotopy preserves the property of A being a division algebra, so it is natural to classify finite dimensional real division algebras up to isotopy.

The Lie algebra $\mathrm{Tder}(A) = \{(d_1, d_2, d_3) \mid d_1(xy) = d_2(x)y + xd_3(y)\}$ of ternary derivations remains invariant, up to isomorphism, when we move from A to $(A, \circ)$. As a first approach to the problem of classifying finite dimensional real division algebras up to isotopy we study the ternary derivations of these algebras. In this talk we present some results in this direction.

Analytic functions in nonassociative algebras

Yakov Krasnov (Bar-Ilan University)

Let $\mathbb{A}$ be an n-dimensional algebra with a fixed basis $\mathbf{e}_1, \mathbf{e}_2, ..., \mathbf{e}_n$, endowed with the standard vector space operations. The symbol $\circ$ stands for a multiplication rule in $\mathbb{A}$. Every vector $x \in \mathbb{A}$ may be expressed in the coordinate form as $x = x_1\mathbf{e}_1 + ... + x_n\mathbf{e}_n$.

An $\mathbb{A}$-valued polynomial function $u(x)$ is called $\mathbb{A}$-analytic $u(x) \in Hol(\mathbb{A})$ if the following right *Dirac equation* in $\mathbb{A}$ is fulfilled:

$$D \circ u(x) := \mathbf{e}_1 \circ \partial_{x_1} u(x) + ... + \mathbf{e}_n \circ \partial_{x_n} u(x) = 0. \tag{1}$$

We show how one can build all polynomial solution to the Dirac equation (1) in nonassociative algebras $\mathbb{A}$. In $\mathbb{R}2$ we give the full description of nonequivalent analytic function theories as well as their applications to ODE.

Representations of B(1,2)

Concepción López-Díaz María* (Universidad de Oviedo)

I.P. Shestakov (Universidade de Sao Paulo)

We study representations of simple alternative superalgebra $B(1,2)$. The irreducible bimodules and bimodules with superinvolution over this superalgebra are classified, and an analogue of the Kroneker factorization theorem is proved for alternative superalgebras with superinvolution that contain $B(1,2)$.

Jordan pairs and irreducible Lie-Yamaguti algebras

P. Benito (Universidad de La Rioja)

Fabian Martin Herce* (Universidad de La Rioja)

A Lie-Yamaguti algebra (LY-algebra for short) is a vector space V endowed with a binary anticommutative product $x \circ y$ and a triple product xyz such that, among other things, the linear maps $d_{x,y} : z \mapsto xyz$ are derivations of the system. The vector subspace $d_{V,V}$ spanned by such set of maps has the structure of Lie algebra (inner derivation algebra of V), and V can be embedded, in a quite natural way, into a Lie algebra $\mathfrak{L}(V) = d_{V,V} \oplus V$, which is called the standard embedding of V.

The purpose of this talk is to present the classification of the $d_{V,V}$-irreducible LY-algebras V whose standard embedding $\mathfrak{L}(V)$ is a special linear algebra. It is shown that, over algebraically closed fields of characteristic zero, this type of systems are just Lie triple systems or they can be obtained from simple Jordan pairs.

One-sided modules and unital bimodules over Jordan superalgebras

Consuelo Martínez López (Universidad de Oviedo)

At present, the classification of simple finite dimensional Jordan superalgebras has been achieved and we know, for each of them, if they are special or exceptional. So it seems totally natural to consider their representation theory.

Even if we know that a simple Jordan superalgebra is special, we may ask if it is embeddable into a finite dimensional associative superalgebra or what can be said about one-sided modules or unital bimodules. In the same line, it seems natural to try to identify their universal associative enveloping algebra. We will address here some of these questions.

Auto-invariant of some Non-Associative Algebras

Ki-Bong Nam (Universisty of Wisconsin-Whitewater)

The non-associative algebra $\overline{N_{n,m,s}}$ was defined in the paper [1]. The F-algebra automorphism group of $\overline{N_{0,s,0}}$ is found in the paper [2]. The Witt algebra $W^+(1)$ has the same set of $\overline{N_{0,0,1}}$ and the same addition of $\overline{N_{0,0,1}}$ with the commutator $[,]$ using the multiplication of $\overline{N_{0,0,1}}$. We find the automorphism group $Aut_F(\overline{N_{0,0,1}})$ of $\overline{N_{0,0,1}}$ and show that $Aut_F(\overline{N_{0,0,1}})$ is isomorphic to the Lie automorphism group $Aut_{Lie}W^+(1)$. We find some results on $Aut_F(\overline{N_{0,s,0}})$.

References

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A maximal algebra of quotients of a Jordan algebra

Irene Paniello Alastruey (Universidad de Zaragoza)

We introduce a nocion of algebra of quotients for Jordan algebras involving essential inner ideals of denominators, and give sufficient conditions for such an algebra to exist. Moreover, it is proved the existence of a maximal algebra of quotients for a Jordan algebra. Finally we characterize Jordan algebras having an artinian algebra of quotients.

Σ_3 associative algebras and operads

Elisabeth Remm (Université de Haute Alsace)

A Σ_3-associative algebra (or Lie-admissible algebra) is an algebra whose product gives by anticommutation a Lie bracket. We study some particular classes of algebras of this kind, such as associative algebras, Vinberg and Pre-Lie algebras. We define the associated cogebras and present some interesting relations between algebras and cogebras. We study the binary quadratic operads associated to each of these classes of algebras and also their dual operads. This permits us to give a link between the algebras associated to an operad and the algebras associated to the dual operad. We use this duality to construct a tensor functor between the dual operad and the initial operad and apply it to the construction of new families of Vinberg algebras. We define a notion of cohomology for the Σ_3-associative algebras, and come back to the determination of Lie algebras which are affine.

Jordan superalgebras: Substructures and decompositions

A. Elduque (Universidad de Zaragoza)

J. Laliena (Universidad de La Rioja)

Sara Sacristán* (Universidad de La Rioja)

In the classification of maximal subalgebras of finite dimensional simple Jordan superalgebras over an algebraically closed field of characteristic zero, one can find problems like the description of subalgebras B of A^+ or $H(A, *)$, with A a simple associative superalgebra and * a superinvolution, such that the associative algebra generated by B is A. We deal with these problems.

Algebras of quotients of Lie algebras

Mercedes Siles Molina (Universidad de Málaga)

In this work we introduce the notion of algebra of quotients of a Lie algebra. Properties such as semiprimeness, primeness or nondegeneracy can be lifted from a Lie algebra to its algebras of quotients. We construct a maximal algebra of quotients for every semiprime Lie algebra and give a Passman-like characterization of this (unique) maximal algebra of quotients.

On Derivations and Automorphisms of Lie Algebras

Vicente Ramón Varea Agudo (Universidad de Zaragoza)

Throughout L will denote a finite-dimensional Lie algebra over a field F of characteristic zero (unless otherwise stated). This paper This paper concerns with relationship between the structures of the derivation algebra $Der(L)$, the group of automorphisms $Aut(L)$ and L itself. Firstly, we consider several centrality conditions on derivations and automorphisms. A consequence is that the Fitting subgroup of $Aut(L)$ is unipotent and maximal as a nilpotent subgroup, provided that either L is nilpotent (non-abelian) and F is perfect or L has a very restrictive structure. It extends a result due to Leger and Luks. Also, we study Lie algebras without outer nilpotent derivations. Finally, we consider Lie algebras L such that the solvable radical of $Der(L)$ consists of nilpotent derivations. The question on the existence of such Lie algebras was posed by Togo in 1961. Later on, Leger and Luks constructed some of them. By extending this construction and results by Togo, we are able to give different classes of such Lie algebras. In particular, for each even number $n \geq 10$, we show $p(\frac{n}{2})$ such Lie algebras of dimension n with non-isomorphic derivation algebras (here $p(n)$ denotes the partition function). Also, we construct Lie algebras L satisfying the stronger condition that $Der(L)2 = Der(L)$ but $L2 \neq L$.