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Abstracts

Session 28

Nonlinear Dispersive Equations

Organizers:

Gustavo Ponce (University of California Santa Barbara)
Luis Vega (Universidad del País Vasco)

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Two-scale Compensated Compactness

Björn Birnir* (University of California-Santa Barbara)

Nils Svanstedt (University of California-Santa Barbara)

Niklas Wellander (University of California-Santa Barbara)

A general two-scales compensated compactness theorem is stated and proved by combining the well-established concepts of compensated compactness and Nguetsengs two-scale convergence. In particular this yields a two-scales version of the celebrated *div-curl lemma* of Murat and Tartar. A simplification of many results in homogenization theory follows.

Capturing the Semi-Classical Limit of the Focusing Nonlinear Schrodinger Equation

Hector Ceniceros (University of California Santa Barbara)

The FNLS is connected to many applications in science and technology. The structure and dynamics of its so-called semi-classical limit is an open problem. Its numerical investigation is notoriously difficult; extremely high resolution (both in space and time) is required to capture accurately the solution's strong self-focusing and a fairly localized "sea" of solitons and, to make things worse, there is modulational instability. Accurate and adaptive numerical methods designed specifically to study the semi-classical limit of the FNLS will be presented in this talk. Using these robust methods, convincing evidence that the limiting behavior of the FNLS solutions appears to be surprisingly regular for both analytic and non-analytic initial data will be provided.

Nodal solutions of a Schroedinger equation with critical nonlinearity

Monica Clapp* (Universidad Nacional Autonoma de Mexico)

Yanheng Ding (Chinese Academy of Sciences)

We consider a nonlinear Schroedinger equation with critical Sobolev exponent and a potential which is invariant under an orthogonal involution and has a potential well. Using variational methods we show that there is an effect of the topology of the potential well on the number of solutions which change sign exactly once. These solutions localize near the potential well.

Drops: The collapse of a capillary jet

Antonio Córdoba (Universidad Autónoma de Madrid)

The appearance of fluid filaments during the evolution of a viscous fluid jet is a commonly observed phenomenon. In this talk we will present a proof to show that the break-up of such a jet subject to capillary forces is impossible trough the collapse of a uniform filament.

Blow up Behavior of Solutions to the Ricci Flow on $\mathbf{R}^2$

Panagiota Daskalopoulos (Columbia University)

We consider the evolution of the conformally equivalent metric g with $ds^2 = u\,(dx^2 + dy^2)$ under the *Ricci Flow*, which evolves a metric $ds^2 = g_{ij}\,dx^i dx^j$ by its Ricci curvature R_{ij} with $\frac{\partial g_{ij}}{\partial t} = -2\,R_{ij}$. The conformal factor u evolves by the logarithmic fast-diffusion equation $u_t = \Delta \log u$. We use this equivalence to deduce geometric estimates on the solution u near its vanishing time T. We restrict our attention to the *maximal solutions* u which vanish at the exact time $T = \frac{1}{4\pi}\int_R^2 f(x)\,dx$. Geometrically this corresponds to the condition that the conformal metric is complete. Our results consist of upper and lower bounds on the geometric *width* W of the solution and on the *maximum curvature* R. Precise pointwise bounds on the solution u near its vanishing time were derived in the rotationally symmetric case by J.R. King.

Nonrelativistic limits in atomic models

Maria J. Esteban Galarza (Paris-Dauphine)

The Dirac-Fock equations are often used to describe stable configurations of relativistic atoms and molecules. They are totally indefinite nonlinear elliptic equations. Here we will show how passing to the limit in the speed of light one finds the equations of non-relativistic quantum mechanics. Also, this procedure allows us to re-characterize some of the solutions of the Dirac-Fock system. In particular, near the non-relativistic limit we will be able to define a notion of ground-state solution for a problem in which the energy is not bounded from below.

The basis ingredients used here are the variational theory of indefinite problems, spectral theory and the obtention of good enough a priori estimates.

Carleman estimates for the heat and Stokes equations and applications to controllability

Enrique Fernández-Cara (Universidad de Sevilla)

This talk deals with some recent estimates of the Carleman kind that can be deduced for linear parabolic equations and systems. We will analyze some "delicate" situations, where the useful methods seem to fail. For instance, we will consider linearized Navier-Stokes equations with nonsmooth coeffcients, linear heat equations with Fourier boundary conditions, heat equations in moving domains, etc. Then, we will show the way these estimates can be applied to prove the null controllability of some physical systems governed by linear and nonlinear partial differential equations.

References

[1] E. Fernández-Cara and E. Zuazua: *The cost of approximate controllability for heat equations: The linear case*, Adv. Diff. Eqs. Vol. 5 (4–6), April-June (2000), 465–514.

[2] A. Doubova, E. Fernández-Cara, M. González-Burgos: *On the controllability of the heat equation with nonlinear boundary Fourier conditions*, J. Diff. Eq., to appear.

Dispersive Estimates for the Wave Equation with Potential

Vladimir Georgiev* (Università degli Studi di Pisa)

Nicola Visciglia (Scuola Normale Superiore Pisa)

We consider a potential type perturbation for the three dimensional free wave equation. We treat non - negative potentials that satisfy a suitable decay assumption. In this case we generalize the classical space - time Strichartz estimates and we prove some dispersive estimates. Also some results about the perturbed homogeneous Besov spaces associated to the perturbed Laplacian operator are proved.

A Chebyseh collocation method for a Stokes problem

Henar Herrero (Universidad de Castilla-La Mancha)

In the last years, a number of algorithms using spectral Chebyshev collocation methods have been succesfully implemented to solve the incompressible Stokes and Navier-Stokes equations with Dirichlet or Neumann boundary conditions. Convergence has been proved for the Legendre collocation approximation [1-2]. The cases of Dirichlet homogeneous and non homogeneous boundary conditions have been solved theoretically in references [3-4]. However, as far as we know, the problem with Neumann condition has not been theoretically justified. The aim of this paper is the numerical analysis of a collocation method involving the Chebyshev nodes, for mixed homogeneous Dirichlet and Neumann boundary conditions in a Stokes problem.

References
[1] Canuto, C., Hussaini, M.Y., Quarteroni, A., Zang, T.A. Spectral Methods in Fluid Dynamics. Springer-Verlag, Berlin (1988).
[2] C. Bernardi and Y. Maday. Approximations spectrales de problemes aux limites elliptiques. Springer-Verlag, Berlin (1991).
[3] Bernardi, C., Canuto, C. and Maday, Y. SIAM J. Numer. Anal. **25**, 1237 (1988).
[4] C. Bernardi and Y. Maday. SIAM J. Numer. Anal. **26** (4), 769 (1989).

The Cauchy problem for quasilinear Schrodinger equations

Carlos Kenig (University of Chicago)

The Cauchy problem for quasilinear Schrodinger equations,Carlos E.Kenig, Department of Mathematics,University of Chicago,Chicago,IL 60637,USA. We will discuss recent work with Ponce and Vega on the local well-posedness of the Cauchy problem for quasilinear Schrodinger equations.

Wellposedness of the Benjamin-Ono equation for rough initial data

Herbert Koch (Universität Dortmund)

The talk is about well-posedness of the Benjamin-Ono equation in $H^{5/4+\varepsilon}$. It is known that the solution cannot be two times differentiable with respect to the initial data. Thus solutions cannot be contructed by a standard iteration procedure. The main ingredient in our proof is a rough Strichartz estimate for variable coefficient operators. This is joint work with Nikolay Tzvetkov.

Global existence for the critical generalized KdV equation

Felipe Linares (Instituto de Matemática Pura e Aplicada)

We discuss results regarding global existence of solutions for the critical generalized Korteweg-de Vries equation,

$$u_t + u_{xxx} + u4\, u_x = 0, \quad x,\, t \in \mathbb{R}.$$

The theory established shows the existence of global solutions in Sobolev spaces with order below the one given by the energy space $H1(\mathbb{R})$, i.e. solutions corresponding to data $u_0 \in H^s(\mathbb{R})$, $s > 3/4$, with $\|u_0\|_{L2} < \|Q\|_{L2}$, where Q is the solitary wave solution of the equation.

Stabilite des N-solitons pour l'equation de KdV

Yvan Martel (Ecole Polytechnique)

This is a work in collaboration with F. Merle (Universite de Cergy-Pontoise, France) et Tai-Peng Tsai (University of Vancover). We prove by energy methods, and a monotonicity property of the $L2$ mass, the stability of the sum of N solitons for the Kortweg-de Vries equation.

On the unique continuation of solutions to the generalized KdV equation

Gustavo Ponce (University of California-Santa Barbara)

Joint work with C. E. Kenig and L. Vega.

We shall prove that solutions of the generalized Korteweg-de Vries equation are uniquely determined by their values on a semi-line at two different times. In particular, if $u_j = u_j(x,t),\ j = 1, 2$ are real valued solutions of the k-generalized Korteweg-de Vries (k-gKdV) equation

$$p_t u + p3_x u + u^k p_x u = 0, \qquad (x,t) \in \mathbb{R} \times [t_1, t_2],\ \ k \in \mathbb{Z}^+,$$

with $t_1 < t_2$ which are sufficiently smooth and such that for some $b \in \mathbb{R}$

$$u_1(x,t) = u_2(x,t), \quad (x,t) \in (b,\infty) \times \{t_1, t_2\}\, (\text{or}\, (-\infty, b) \times \{t_1, t_2\}),$$

then $u_1 \equiv u_2$.

Long–time and orbital stability for the Vlasov-Poisson System in the stellar dynamics case

Jean Dolbeault (Université de Paris IX-Dauphine)

Oscar Sánchez (Universidad de Granada)

Juan Soler* (Universidad de Granada)

We first study an optimal inequality which relates potential and kinetic energies in an appropriate framework for bounded solutions of the Vlasov-Poisson (VP) system. Optimal distribution functions, which are completely characterized, minimize the total energy. From this variational approach, we deduce bounds for the kinetic and potential energies in terms of conserved quantities (mass and total energy) of the solutions of the VP system and a nonlinear stability result. Then we apply our estimates to the study of the large time asymptotics and observe two qualitatively different regimes. The Galilean invariance of the solutions plays an important role in this analysis.

Finally, using the structure of the variational problems associated with the family of polytropic gas spheres, we propose orbital stability criteria which optimize the nonlinear dynamical criteria known until present.

KP I versus KP I-BBM

Jean-Claude Saut (UniversiteParis-Sud)

The Kadomtsev-Petviashvili I (KP I) equation has been shown to behave badly with respect to Picard iterative methods. This explain the difficulty in solving the Cauchy problem in the natural energy space (a problem which is still open). The aim of the talk is to describe recent results obtained with Nikolay Tzvetkov, on the BBM version (KP I-BBM)of the KP I equation, which has exactly the same validity as a model of water waves. We prove that the KP I-BBM is globally well posed in its energy space. As a consequence,one can prove a very natural orbital stability result of the set of ground state solutions.

Asymptotic behavior to Dissipative Quasi-Geostrophic flow

Maria E. Schonbek* (UC Santa Cruz)

Tomas P. Schonbek (Florida Atlantic University)

I will consider the long time behavior of solutions of dissipative Quasi-Geostrophic flow (QG) with sub-critical powers. The flow under consideration is described by the nonlinear scalar equation

$$\frac{\partial \theta}{\partial t} + u \cdot \nabla \theta + \kappa(-\triangle)^{\alpha}\theta = f, \qquad (1)$$
$$\theta|_{t=0} = \theta_0$$

I will first show how to obtain uniform bounds on the energy of higher derivatives. Then these bounds are used to establish algebraic rates of decay for both the solutions and higher derivatives in different Sobolev spaces.

The incompressible limit in nonlinear elasticity

Thomas C. Sideris* (University of California, Santa Barbara)

Becca Thomases (University of California, Santa Barbara)

Global, small amplitude, classical solutions of the equations of motion for 3D incompressible elastic materials will be constructed as the limit of solutions to the compressible equations.

On global well-posedness for Schroedinger maps in the energy norm

Atanas Stefanov (University of Kansas)

We show global well-posedness of the Cauchy problem for Schrödinger maps, when the data is small and the data belongs to the scaling and energy critical $\dot{H}^1$. We do so by showing that the associated gauged problem (which we call Modified Schrödinger map problem) is globally well-posed, when the data is small in $L2$. This is joint work with A. Nahmod and K. Uhlenbeck.

Bilinear Strichartz Estimates

María C. Vilela (Alfonso X)

We study the bilinear Strichartz estimates adding some positive results to the previously known and giving some counterexamples. As an application, we show that there exits a global solution of the Cauchy problem for the equation $iu_t + \Delta u + \lambda|u|^\alpha u = 0$ in $\mathbb{R}^n$, under some restrictions on α, if the intial value is sufficiently small in some weak L^p space. Finally, we connect this result with the H^s theory and with the existence of self-similar solutions.