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Abstracts

Session 29

Numerical linear algebra

Organizers:

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On the shifted QR iteration applied to a Frobenius matrix

Andrea Bini Dario (Universidad de Pisa)

We show that the shifted QR iteration

$$\begin{cases} A_i - \alpha_i I = Q_i R_i \\ A_{i+1} = R_i Q_i + \alpha_i I \end{cases} \quad i = 0, 1, 2, \ldots$$

applied to a Frobenius matrix

$$A_0 = F = \begin{bmatrix} 0 & & & -a_0 \\ 1 & \ddots & & -a_1 \\ & \ddots & 0 & \vdots \\ & & 1 & -a_{n-1} \end{bmatrix}$$

generates sequences of semiseparable matrices. More precisely, we prove that there exist matrices $\hat{Q}_i$, $\hat{R}_i$, $\hat{A}_i$ of rank at most 1,2, 3, respectively, which coincide with Q_i, R_i, A_i in their upper triangular parts, respectively. A similar result holds if the QR iteration is applied with the quadratic shift. This structural property is used to design an algorithm wich performs a single shifted QR step in $O(n)$ arithmetic operations with $O(n)$ storage. This algorithm improves by a factor of n both the complexity and the storage bounds of the classical QR iteration for Frobenius matrices. The robustness of this approach is investigated and applications to approximating polynomial zeros are shown.

Wavelet Algorithms for High-Resolution Image Reconstruction

Raymond H. Chan (Chinese University of Hong Kong)

Tony F. Chan* (University of California)

Lixin Shen (National University of Singapore)

High-resolution image reconstruction refers to the reconstruction of high-resolution images from multiple low-resolution, shifted, degraded samples of a true image. In this talk, we analyze this problem from the wavelet point of view. By expressing the true image as a function in $L_2(\mathbf{R2})$, we derive iterative algorithms which recover the function completely in the L_2 sense from the given low-resolution functions. These algorithms decompose the function obtained from the previous iteration into different frequency components in the wavelet transform domain and add them into the new iterate to improve the approximation. We apply wavelet (packet) thresholding methods to denoise the function obtained in the previous step before adding it into the new iterate. Our numerical results show that the reconstructed images from our wavelet algorithms are better than that from the Tikhonov least-squares approach.

An Implicitly Restarted Block-Lanczos Method for Large Hermitian Eigenproblems

Daniela Calvetti (Case Western Reserve University)

IRBLEIGS is an implementation of an implicitly restarted block-Lanczos method for computing a few selected nearby eigenvalues and associated eigenvectors of a large Hermitian matrix A. The desired eigenvalues may be extreme or in the interior of the spectrum. The code only requires the evaluation of matrix-vector products with A. In particular, factorization of A is not demanded, nor is the solution of linear systems of equations with the matrix A. This, together with a fairly small storage requirement, makes IRBLEIGS well suited for large-scale problems. The talk describes IRBLEIGS and presents comparisons with other available software.

Speeding up Backfitting of Large Linear Additive Models

Gene Golub (Stanford Universtity)

We consider the *penalized least squares solution*

$$\|Ax - b\|^2 + \alpha\, x^T D\, x = \min!$$

of an overdetermined linear system of size $N \times m$ whose matrix is structured into d block columns (of roughly the same width), which may be imposed by the underlying application or by the need to distribute the system on a parallel computer:

$$\underbrace{\begin{bmatrix} A_1 & \cdots & A_d \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x_1 \\ \vdots \\ x_d \end{bmatrix}}_{x} = b - r\,.$$

D is a positive definite diagonal matrix and $\alpha > 0$ a regularization parameter. We are interested in applications in data mining where the system is huge and where data access and communication time limit the choice of algorithms in the necessarily parallel environment.

In data mining and statistical applications one is particularly interested in the *predicted values* $y :\equiv Ax = b - r$, which can be written as

$$y = f_1 + \cdots + f_d\,, \qquad \text{where} \quad f_i :\equiv A_i x_i \quad (i = 1, \ldots, d).$$

It is easy to derive a structured linear system for these predicted values. Statisticians often solve this system with the block Gauss–Seidel algorithm and call this *backfitting.* We compare backfitting with a number of other options that offer themselves for the solution of the problem.

On the eigenproblem for orthogonal Hessenberg matrices

Bill Gragg (Naval Postgraduate School)

When properly formulated the second stage of Pisarenko's fundamental signal processing algorithm involves computing the eigenvalues and the first elements of the normalized eigenvectors of a unitary (upper) Hessenberg matrix. The nxn matrix is naturally parameterized by n Schur parameter pairs, which are coefficients in the recurrence relation for Szego polynomials, polynomials orthogonal on the unit circle in the complex plane. The eigenvalues and the first eigenvector elements provide Gauss (-Szego) quadrature formulas for integration of functions defined on the unit circle.

We address the practical case when the Schur parameters are real, so the unitary matrix is (real) orthogonal. The eigenvalue problem can then be reduced, mathematically, to a bidiagonal singular value problem of half the size. For numerical stability it is necessary to solve two such problems, but the computational complexity remains O(n2).

We hope to extend these results to also compute the first elements of the normalized eigenvectors (the Gauss-Szego weights) in O(n2) operations, and shall indicate the Demmel type result that small relative errors in the Schur parameter pairs imply small relative errors in the arguments of the eigenvalues ("angles of arrival").

SVD Computation on a Grid

Franklin Luk (Rensselaer Polytechnic Institute)

Grid computing is a new direction in scientific computing. As stated on an IBM web site, "Grid Computing can be defined as applying resources from many computers in a network– at the same time–to a single problem; usually a problem that requires a large number of processing cycles or access to large amounts of data. At its core, Grid Computing enables devices–regardless of their operating characteristics–to be virtually shared, managed and accessed across an enterprise, industry or workgroup. This virtualization of resources places all of the necessary access, data and processing power at the fingertips of those who need to rapidly solve complex business problems, conduct compute-intensive research and data analysis, and engage in real-time."

The singular value decomposition (SVD) is an important problem that is highly compute-intensive. There are SVD applications with large datasets that are distributed over a network. Communication costs become a crucial factor in the overall problem solution. In this talk, we describe our approach to compute the SVD on a grid.

An orthogonal high relative accuracy algorithm for the symmetric eigenproblem

Froilán Martínez-Dopico (Universidad Carlos III)

We propose a new algorithm for the symmetric eigenproblem which computes eigenvalues and eigenvectors with high relative accuracy for the largest class of symmetric, definite and indefinite, matrices known so far. The algorithm is divided in two stages: the first one computes a SVD with high relative accuracy, and the second one obtains eigenvalues and eigenvectors from singular values and vectors. Using the SVD as a first stage is responsible both for the wide applicability of the algorithm and for being able to use only orthogonal transformations, unlike previous algorithms in the literature. Theory, a complete error analysis and numerical experiments are presented.

On the sensitivity of orthonormal bases of invariant subspaces of Hermitian matrices

Julio Moro Carreño* (Universidad Carlos III)

Froilán M. Dopico (Universidad Carlos III)

Explicit formulas are given for the derivatives of orthonormal bases of invariant subspaces of linear Hermitian perturbations $A(t) = A + tE$ of a Hermitian matrix A. A characterization is provided for all possible differentiable families of orthonormal bases via Sylvester equations, together with a procedure allowing to construct differentiable orthonormal bases starting from arbitrary ones.

As a consequence of these results, similar formulas are obtained for derivatives of bases of left and right singular subspaces of arbitrary matrices using the Jordan-Wielandt form.

Comrade matrices and unified algorithms for DCT/DST's

Vadim Olshevsky (University of Connecticut)

The paper provides a full self-contained derivation of fast algorithms to compute discrete Cosine and Sine transforms I - IV. For the Sine I/II and Cosine I/II transforms a unified derivation based on the concept of the comrade matrix is presented. The comrade matrices associated with different versions of the transforms differ in only a few boundary elements; hence, in each case algorithms can be derived in a unified manner. The algorithm is then modified to compute Sine III/IV and Cosine III/IV transforms as well. The resulting algorithms for the versions III/IV are direct and recursive, such algorithms were missing in the existing literature. Finally, formulas reducing Cosine and Sine transforms of the types III and IV to each other are presented.

On the stability of some pivoting strategies for Gauss elimination

Juan Manuel Peña Ferrandez (Universidad de Zaragoza)

We analyse the stability properties of Gaussian elimination when using several pivoting strategies, including scaled partial pivoting strategies and double partial pivoting. The Skeel condition number of the resulting upper triangular matrices and several growth factors are considered. The structure preservation with these pivoting strategies is also presented, considering special classes of matrices such as M-matrices or sign-regular matrices.

Pole Placement Preconditioning

Lothar Reichel (Kent State University)

The partial pole placement problem has received considerable attention in Control Theory, where it is applied to stabilize time-invariant linear control systems. We describe an application of the partial pole placement problem to the construction of preconditioners for linear systems of equations. Numerical examples show that these preconditioners can improve the rate of convergence of the restarted GMRES methods significantly.

A Krylov Subspace Method for Quadratic Matrix Polynomials with Application to Constrained Least Squares Problems

Ren-Cang Li (University of Kentucky)

Qiang Ye* (University of Kentucky)

We present a Krylov subspace type projection method for a quadratic matrix polynomial $\lambda 2I - \lambda A - B$ that works directly with A and B without going through any linearization. We discuss a special case when one matrix is a low rank perturbation of the other matrix. We also apply the method to solve quadratically constrained linear least squares problem through a reformulation of Gander, Golub and von Matt as a quadratic eigenvalue problem and we demonstrate the effectiveness of this approach. Numerical examples are given to illustrate the efficiency of the algorithms.