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Abstracts

Session 31

PDE Methods in Continuum Mechanics

Organizers:

John W. Neuberger (University of North Texas)
Juan Luis Vázquez Suárez (Universidad Autónoma de Madrid)

Index of Abstracts

(In this index, in case of multiple authors, only the speaker is shown)

Anisotropy and microstructure

Nicholas Alikakos* (University of North Texas)

Peter Bates (Brigham Young University)

John Cahn (National Institute of Standards and Technology)

Paul Fife (University of Utah)

Giorgio Fusco (Universitá di LÁquila)

G. Berensal Tanoglu (Izmir Institute of Technology)

We begin by discussing a diffuse interface model, introduced recently by Cahn and his collaborators, for describing the phase transitions in a face-centered cubic crystal structure. In this model anisotropy is not built into the model in an ad hoc manner, but rather follows in a more fundamental way. The equations in the limit as a certain parameter tends to zero lead to a system of sharp interface equations. In this talk we focus on the existence and properties of the basic one-dimensional transition connecting two stable bulk states. We also study the linearized problem about this transition and establish the existence of an increasing number (of the order of 1/(parameter)) of critical eigenvalues. We interpret this result as a manifestation of instability, and indicative of microstructure in or near the interface and relate it to a specific physical phenomenon.

Layer solutions in a halfspace for boundary reactions

Xavier Cabré* (Universitat Politècnica de Catalunya)

Joan Solà–Morales (Universitat Politecnica de Catalunya)

We consider harmonic functions in a halfspace $\mathbb{R}^n_+ = \{(x, y) \in \mathbb{R} \times \mathbb{R}^{n-1} : x > 0\}$ subject to nonlinear Neumann boundary conditions. We study bounded solutions which are monotone increasing (from -1 to 1) in one of the y-variables. We call such functions *layer solutions.*

When $n = 2$, we establish that a necessary and sufficient condition for the existence of a layer solution is that the boundary energy potential (the primitive of the nonlinearity, up to a sign) has at ± 1 two absolute minima at the same height. In addition, we prove uniqueness of the layer solution up to translations in the y-variable. We also show that a solution is increasing in y (i.e., it is a layer solution) if and only if it is a local minimizer in $\mathbb{R}^2_+$.

When $n = 3$, we establish that every layer solution in $\mathbb{R}^3_+$ is in fact a function of only two variables –a symmetry result in the spirit of a conjecture of De Giorgi for reactions in the interior.

Explicit solutions of the denoising problem in image processing

Giovanni Bellettini (Universitá Roma "Tor Vergata' ')

Vicent Caselles* (Universitat Pompeu-Fabra, Barcelona)

Our purpose in this work will be to construct explicit solutions of the so called denoising problem, which appears in image processing. Given the data $f \in L^2(I\!R^2)$, and following Rudin-Osher-Fatemi, the denoised version of f will be the solution $u \in BV(I\!R^2)$ of the variational problem

$$\min_{u \in BV(I\!R^2)} \int_{I\!R^2} |Du| + \frac{1}{2\lambda} \int_{I\!R^2} (u - f)2 \, dx, \tag{1}$$

Our purpose is to find explicit solutions for this problem, i.e., data $f \in L^2(I\!R^2)$ for which we can explicitely compute the solution u of (1). In particular, give f for which the solution u is constructed by the soft-thresholding rule. For that we will have to compute all possible entropy solutions of the following eigenvalue problem for the 1-Laplacian operator:

$$-\mathrm{div}\left(\frac{Du}{|Du|}\right) = u, \qquad u \in L^1_{\mathrm{loc}}(I\!R^2).$$

Capillarity driven spreading of power-law fluids

Marco A. Fontelos (Universidad Rey Juan Carlos, Madrid)

We investigate the spreading over a solid surface of thin liquid films of power-law rheology. Under the lubrication approximation, the Navier-Stokes system modelling the flow leads to a doubly degenerate parabolic fourth order nonlinear partial differential equation. After discussing the basic questions of existence and uniqueness of solutions, we will show that when a nonlinearity exponent related to the rheology of the fluid is larger than one, the governing equation admits self-similar solutions with compact support and moving fronts, representing the spreading of a drop. This is in contrast with the corresponding model for Newtonian fluids where such solutions do not exist contradicting the experimental evidence and giving rise to the so called "contact-line paradox". Hence, we show that Non-Newtonian fluids of power-law rheology are immune to this paradox. The results presented are the result of work in collaboration with S. Betelu (U. of North Texas), D. Aronson (U. of Minnesota), A. Sánchez (U. Rey Juan Carlos, Madrid) and J. L. Vázquez (U. Autónoma de Madrid).

A free boundary problem from nonlocal combustion

Claudia Lederman* (Universidad de Buenos Aires)

Noemi Wolanski (Universidad de Buenos Aires)

We study a singular perturbation problem for a nonlocal evolution operator. The problem appears in the analysis of the propagation of flames, when the high activation energy limit is considered in a combustion model admitting nonlocal effects.

We obtain uniform estimates and we show that limits are solutions to a free boundary problem in a viscosity sense and in a pointwise sense at regular free boundary points.

Homogenization in Free boundary problems

Ki Ahm Lee (Seoul National University)

In this talk, we are going to consider one-phase flame propagation describing premixed laminar flame. We are interested in the case that materials with different reaction rates are distributed periodically in the space. We will discuss the nonvariational stationary solutions, and effective speed of the propagation.

Comparison Principles for Viscosity Solutions of Equations generated by Vector Fields

Frank Beatrous (University of Pittsburgh)

Thomas J. Bieske (University of Michigan)

Juan J. Manfredi* (University of Pittsburgh)

Consider n linearly independent vector fields $\{X_1, X_2, \ldots, X_n\}$. Let $\mathcal{A}(x) = (a_{i,j}(x))$ be a positive definite continuous matrix function. We first adapt the definition of continuous viscosity solutions to the equation

$$\mathcal{L}(u)(x) = \sum_{i,j=1}^{n} a_{i,j}(x) X_i X_j u(x) = 0$$

and extend the comparison principle for semi-continuous functions of Crandall-Ishii-Jensen-Lions to this case. Next, we consider the case where instead of n-vector fields, we have m vector fields with $m < n$. We show that if $\{X_1, X_2, \ldots, X_m\}$ form the horizontal subspace of a Carnot group so that Hörmander's condition

$$\text{rank (Lie algebra)} [\{X_1, X_2, \ldots, X_m\}] = n$$

holds, we still have the comparison principle for semi-continuous functions.

The Total Variation Flow with Measure Initial Data

Fuensanta Andreu (Universitat de Valencia)

Vicent Caselles (Universitat Pompeu-Fabra, Barcelona)

José M. Mazón Ruiz* (Universitat de Valencia)

Salvador Moll (Universitat de Valencia)

We consider the minimizing total variation flow in $\mathbb{R}^N$

$$\frac{\partial u}{\partial t} = \operatorname{div}\left(\frac{Du}{|Du|}\right) \qquad \text{in } Q_T =]0,T[\times\mathbb{R}^N, \tag{2}$$

coupled with the initial condition

$$u(0) = \mu, \qquad \mu \text{ being a Radon measure in } \mathbb{R}^N.$$

We study limit solutions obtained by weakly approximating the initial measure μ by functions in $L^1(\mathbb{R}^N)$, characterizing these limit solutions when the initial condition $\mu = h + \mu_s$ where $h \in L^1(\mathbb{R}^N) \cap L^\infty(\mathbb{R}^N)$, and $\mu_s = \alpha\mathcal{H}^k \llcorner S$, $\alpha \geq 0$, k is an integer and S is a k-dimensional manifold with bounded principal curvatures. In case $k < N-1$ we prove that the singular part of the solution does not move, it remains equal to μ_s for all $t \geq 0$ and its absolutely continuous part is the unique strong solution of the Cauchy problem associated with (2) with initial datum h. In case $k = N-1$ we prove that the singular part of the limit solution is $(1 - \frac{2}{\alpha}t)^+\mu_s$ and we also characterize its absolutely continuous part.

On thermo-elasticity with second sound

Salim Messaoudi (King Fahd University of Petroleum and Minerals, Dhahran)

In thermo-elasticity with second sound, thermal disturbances are modeled propagating as wave-like pulses traveling at finite speed. The physical paradox of infinite speed known in the classical theory of thermo-elasticity is removed by adopting Cattaneo ' s law instead of Fourier' s law. As a consequence a hyperbolic system instead of the coupled hyperbolic-parabolic is obtained. Results concerning existence and decay of solutions for different types of one-dimensional problems have been established by Tarabek and Racke. For the multi-dimensional case, Racke considered a linear problem and proved an existence result as well as a decay result under appropriate conditions on the displacement and the heat flux.

In this talk we consider a semi-linear problem, where a nonlinear source competes with the damping caused by the heat flux, and present a blow up result and discuss the possibility of global solution as well as the boundary stabilization

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Functionals with non standard growth and regularity of minimizers

Giusseppe Mingione (Universitá di Parma)

Functionals with non standard growth of (p, q) type are functionals that are coercive in a Sobolev space that is stricly larger than the space on which they are natually defined. I will talk about regularity of minimizers of such functionals showing sharp conditions for regularity and related counterexamples. Finally, I will talk about examples of scalar minimizers of smooth convex functionals exhibiting fractal singular sets of maximal Hausdorff dimension.

Sobolev Gradients and Variational Problems

John W. Neuberger (University of North Texas)

Sobolev gradients and variational problems Sobolev gradients for variational problems are described. Their construction and use is indicated for problems in elasticity, Ginburg-Landau functionals of superconductivity and other problems. Existence results are given in terms of these gradients.

On the quenching set for a fast diffusion equation. Regional quenching.

Raúl Ferreira (Universidad Carlos III de Madrid)

Arturo de Pablo* (Universidad Carlos III de Madrid)

Fernando Quirós (Universidad Autónoma de Madrid)

Julio D. Rossi (Universidad de Buenos Aires)

We study positive solutions of a very fast diffusion equation, $u_t = (u^{m-1}u_x)_x$, $m < 0$, in a bounded interval, $0 < x < L$, with a quenching type boundary condition $u(0,t) = (T-t)^{1/(1-m)}$. Due to the singular character of the diffusivity at level zero, solutions have a strong tendency towards positivity. On the contrary, the condition on the left boundary forces the solution to be zero in finite time. Our aim is to study the competition between both facts, by describing precisely the set where the solution tends to zero, which coincides with the set where the derivative u_t goes to $-\infty$. We have shown that regional quenching is possible only in the range $m < -1$. In this case, regional quenching always happens if L is large. If L is small it could also be global. In the opposite range $m \geq -1$, we find single-point quenching or global quenching if L is small, and only single-point quenching if L is large.

Besides the quenching set we describe the asymptotic behaviour close to the quenching time. The results are obtained by studying the corresponding blow-up problem for the variable $v = u^{m-1}$, where viscosity solutions must be considered.

Harnack inequality and spectral instantaneous and complete blow-up for some parabolic equations related to Caffarelli-Kohn-Nirenberg inequalities

Ireneo Peral Alonso (Universidad Autónoma de Madrid)

We discuss the relation between Harnack and Hardy inequalities and the nonexistence of solution to problem

$$\begin{cases} u_t - \operatorname{div}(|x|^{-2\gamma}\nabla u) = \lambda \dfrac{u^\alpha}{|x|^{2(\gamma+1)}} + f \text{ in } \Omega \times (0,T), \\ u \geq 0 \text{ in } \Omega \times (0,T) \text{ and } u = 0 \text{ on } \partial\Omega \times (0,T), \\ u(x,0) = u_0(x) \text{ in } \Omega, \end{cases}$$

where $\Omega \subset \mathbb{R}^N$ is a bounded regular domain such that $0 \in \Omega$, $\lambda > 0$, $\alpha \geq 1$, $-1 < \gamma < \dfrac{N-2}{2}$, f and u_0 are positive functions such that $f \in L^1(\Omega \times (0,T))$ and $u_0 \in L^1(\Omega)$. Roughly speaking we prove that:

Harnack plus Hardy implies blow-up.

The case $\gamma = 0$ is a conjecture by H. Brezis that was studied by Baras and Goldstein in the 80's. Some other related problems will be explained by the same kind of arguments.

Thermal avalanche for blow-up solutions of semilinear heat equations

Fernando Quirós* (Universidad Autónoma de Madrid)

Julio D. Rossi (Universidad de Buenos Aires)

Juan Luis Vázquez (Universidad Autónoma de Madrid)

We consider the semilinear heat equation $u_t = \Delta u + u^p$, $1 < p < p_s$, p_s the Sobolev exponent. This problem has solutions with finite-time blow-up. As is well-known, blow-up is complete: if we take approximations by reaction problems with global solution and pass to the limit, this continuation is identically infinite for all $t > T$, where T is the blow-up time. We perform a deeper analysis of how complete blow-up occurs. Actually, the singularity set of a solution that blows up as $t \nearrow T$ propagates instantaneously at time $t = T$ to cover the whole space, producing a catastrophic discontinuity between $t = T-$ and $t = T+$, the avalanche. We describe its formation as a layer appearing in the limit of the approximate problems. The initial structure of the layer is given by the solution of a limit problem, described by an ordinary differential equation. As t proceeds past T the solutions of the approximate problems approach in an outer region a traveling wave. The situation in the inner region depends on the type of approximation: a conical pattern is formed with time when we approximate the power u^p by a flat truncation at level n, while for tangent truncations we get an exponential increase in time and a diffusive spatial pattern.

Free boudary problems for operators with variable coefficients: regularity

Sandro Salsa (Politecnico di Milano)

We present some recent results on the regularity of the free boundary in two phase problems for elliptic operators with variable coefficients. In particular we consider the implication Lipschitz implies further regularity. The theory for Laplace operator has been developped by L.A. Caffarelli in two well known and by now classical papers. These results have been extended by P. Wang to a class of concave fully non linear operators of the type $F(D^2u) = 0$ and by M. Feldman to anisotropic linear operators with constant coefficients and to general fully non linear operators of the type $F(D^2u, Du) = 0$.

Free boundary regularity

Henrik Shahgholian (Royal Institut of Technology of Stockholm)

We discuss recent developments in free boundary regularity. The focus will be on the touching of the free boundary with the fixed boundary.

Cauchy Problem for Ostrovsky Equation

Vladimir Varlamov (University of Texas - Pan American)

The Ostrovsky equation governs propagation of nonlinear waves on the surface of shallow water in the presence of rotation. It is a purely dispersive equation. Unlike its close relatives, Korteweg-de Vries and Kadomtsev-Petviashvili equations, it is nonintegrable by the inverse scattering transform. A Cauchy problem is considered for the Ostrovsky equation. The fundamental solution of the linear problem is constructed by means of the use of special functions and its properties are studied. Solution of the nonlinear Cauchy problem is constructed in the form of a series and its long-time asymptotics is computed.

Nonlinear Diffusion equation and free boundaries

Juan Luis Vázquez Suárez (Universidad Autónoma de Madrid)

We report on progress in the theory of nonlinear diffusion equations, the existence and properties of free boundaries, and pay some regard to the applications.

We consider in particular two problems: (i) the existence of solutions with expanding and shrinking supports for the equations of the type, $u_t = u\,u_{xx} \pm (u_x)2$, appearing in hydrology; (ii) the existence of solutions of equations of the form $u_t = \phi(u_x)_x$, which appear in image processing. We show that the two problems are related and pose a number of open problems.

Part of this work is the result of a collaboration with G. Barenblatt, Berkeley, Ca