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Abstracts

Session 32

Polynomials and Multilinear Analysis in Infinite Dimensions

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Holomorphic functions that attain its maximum modulus at extreme points

María D. Acosta (Universidad de Granada)

Let X be a Banach space and $\mathcal{A}(B_X)$ the Banach space of all complex valued functions defined on the closed unit ball of X that are uniformly continuous and holomorphic on the open unit ball. In the following we take $X = \mathcal{C}(K, Y)$, where K is a compact topological space and Y is a complex Banach space. We show that for a $\mathbb{C}$-rotund Banach space Y, if a mapping $T \in \mathcal{A}(B_X)$ attains its norm, then, T attains the maximum modulus value at an extreme point of B_X if the pair (K, Y) satisfies the so-called "extension property". By using the same ideas, for any infinite-dimensional Banach space Y, we show that the set of functions $f \in B_X$ such that

$$\|f(t)\| = 1, \ \forall t \in K,$$

is a norming subset for $\mathcal{A}(B_X)$. As a consequence, is Y is rotund, then the subset of extreme points of B_X is a norming set for $\mathcal{A}(B_X)$, that is,

$$\|T\| = \sup\{|T(f)| : f \in B_X, f \ \text{ is an extreme point of } \ B_X\}.$$

A strong approximate Morse-Sard theorem in infinite dimensions

Daniel Azagra* (Universidad Complutense de Madrid)

Manuel Cepedello Boiso (Universidad de Sevilla)

We prove that every continuous mapping from a separable infinite-dimensional Hilbert space X into R^m can be uniformly approximated by C^∞ smooth mappings with no critical points. This kind of result can be regarded as a sort of very strong approximate version of the Morse-Sard theorem. Some consequences of the main theorem are as follows. Every two disjoint closed subsets of X can be separated by a one-codimensional smooth manifold which is a level set of a smooth function with no critical points; this fact may be viewed as a nonlinear analogue of the geometrical version of the Hahn-Banach theorem. In particular, every closed set in X can be uniformly approximated by open sets whose boundaries are C^∞ smooth one-codimensional submanifolds of X. Finally, since every Hilbert manifold is diffeomorphic to an open subset of the Hilbert space, all of these results still hold if one replaces the Hilbert space X with any smooth manifold M modelled on X.

Polynomial sequential continuity on $C(K,E)$ spaces

Fernando Bombal* (Universidad Complutense de Madrid)

Ignacio Villanueva (Universidad Complutense de Madrid)

If K is a Hausdorff compact space and E is a Banach space, the following characterization of weak sequential convergence on $C(K,E)$ is well known:

A bounded sequence $(f_n) \subset C(K,E)$ converges weakly to $f \in C(K,E)$ if and only if, for every $t \in K$, the sequence $(f_n(t))$ converges weakly (in E) to $f(t)$. Similarly, a bounded sequence $(f_n) \subset C(K,E)$ is weakly Cauchy if and only if, for every $t \in K$, the sequence $(f_n(t))$ is weakly Cauchy.

In the light of this result, one could ask whether a similar statement would be true when we replace weak convergence in $C(K,E)$ and E by some kind of polynomial convergence. Using a space constructed by Castillo, García and Gonzalo as a counterexample to several polynomial conjectures, we show that no polynomial version of the previous result can be true in general, not even for finite K.

Then, we isolate necessary and sufficient conditions on E for several polynomial versions of the result to be true, when K is a scattered compact Hausdorff space. We show certain relations between the conditions introduced and provide examples of Banach spaces verifying them, together with some related results.

Ideals of polynomials generated by weakly compact operators

Geraldo Botelho (Universidade Federal de Uberlandia, Brazil)

The aim of these notes is to describe the properties and the relationships between the classes of homogeneous polynomials between Banach spaces which are generated by the ideal of all weakly compact operators, namely, the class of weakly compact polynomials and the class of polynomials which the factorization and the linearization methods generate by the ideal of all weakly compact operators. Containment relationships with ideals of absolutely summing polynomials are also investigated.

$\tau_o = \tau_\omega$

Christopher Boyd (University College Dublin)

Let U be an open subset of a locally convex space E. We use $\mathcal{H}(U)$ to denote the space of all holomorphic functions on U. There are a number of different topologies which can be placed on $\mathcal{H}(U)$. We use τ_o to denote the topology of uniform convergence on compact subsets of U. While extending the Cartan-Thullen Theorem to infinite dimensions Nachbin introduced the ported topology τ_ω. In this talk we will look at necessary conditions to ensure $\tau_o = \tau_\omega$ on $\mathcal{H}(U)$. The classes of spaces we consider will include Fréchet spaces, DF spaces, $\mathbf{C}^I$, $\mathbf{C}^{(I)}$ and spaces of the form $E \times F'_b$ where E and F are Fréchet nuclear spaces with a basis.

Numerical Radius and Aron-Berner Extension

Yun-Sung Choi (Pohang University of Science and Technology)

This talk discusses the numerical radius of a polynomial on a Banach space. In particular, it is showed that the Aron-Berner extension preserves the numerical radius of a polynomial. We also discuss the case where the Aron-Berner extension of the composition of two polynomials is identical with the composition of their Aron-Berner extensions.

On weakly sequentially continuous polynomials

Maite Fernández-Unzueta (CIMAT, México)

We will discuss the weakly sequentially continuity property of polynomials on Banach spaces. In particular we will prove that whenever p is an m-homogeneous polynomial on a complex Banach space and $(x_n)_n$ is a bounded sequence such that it converges to zero when evaluated on polynomials of degree less than m, but $p(x_n) = 1$, then there exists a basic sequence $(y_k)_k$, equivalent to a subsequence $(x_{n_k})_k$, such that $p(\sum_{k=1}^{\infty} a_k y_k) = \sum_{k=1}^{\infty} a_k^m$.

Rolle's theorem for the generalized gradient

Juan Ferrera (Universidad Complutense de Madrid)

We prove that in every non reflexive Banach space it is possible to construct a Lipschitz function which vanishes on the sphere $S(0, 2)$, and such that the generalized gradient at the points of the open ball $B(0, 2)$ never contains the 0. Considering that every infinite dimensional reflexive Banach has functions for which Rolle's theorem fails, we obtain that an infinite dimensional Rolle's theorem for the generalized gradient is always false. However, an approximate Rolle's theorem is established.

Norm attaining polynomials at extreme points on $C(K)$

Domingo García (Universidad de Valencia)

We study when every complex continuous norm attaining n-homogeneous polynomial on $C(K, \mathbb{C})$, the Banach space of all complex continuous functions on a compact Hausdorff space K, attains its norm at extreme points. A characterization whenever K is a compact metric space is obtained. It is also discussed when the set of extreme points of the unit ball of $C(K, \mathbb{C})$ is a norming set of the space of all complex continuous n-homogeneous polynomials (joint work with Y.S. Choi, S.G. Kim and M. Maestre).

Spaces with unconditional basis admitting a separating polynomial

Raquel Gonzalo Palomar (Universidad Politécnica de Madrid)

In this talk we present some results concerning spaces with unconditional basis admitting a separating polynomial. In the concrete case of spaces with a 4-homogeneous polynomial, these polynomials can be characterized in terms of certain infinite matrices. Moreover, an infinite matrix which is positive definite and with positive definite entrees defines a 4-homogeneous convex polynomial and induces a space with polynomial norm. With this tool we study the problem of the existence of spaces with unconditional basis and separating polynomial of degree 4 which are not isomorphic to a subspace of the space L_4.

Inequalities for the derivatives of polynomials on Banach spaces

Lawrence A. Harris (University of Kentucky)

In this talk we discuss Bernstein and Markov type inequalities for the Fréchet derivatives of polynomials on Banach spaces. Of special interest is Markov's inequality for higher derivatives. We also give estimates for the derivatives of polynomials with restricted growth on Banach spaces.

Polynomials generated by linear operators

Pablo Galindo (Universidad de Valencia)

Mary Lilian Lourenço* (Universidad de Sao Paulo)

Luiza Amália Moraes (Universidade Federal do Rio de Janeiro)

We study the class of Banach algebra valued n-homogeneous polynomials generated by the n^{th} power of linear operators. We compare it with the finite type polynomials.

Two properties of the Aron-Berner extension of polynomials

Manuel Maestre (Universidad de Valencia)

We give the following Lindenstrauss-like theorem for 2-homogeneous polynomials: For every Banach space X the set of 2-homogeneous continuous polynomials whose Aron-Berner extension to X^{**} attain their norm is a dense subset of the space of all 2-homogeneous continuous polynomials $\mathcal{P}(^2X)$. This is a joint work with R. Aron and D. García.

We also show that for every m-homogeneous continuous polynomial $P : X \longrightarrow X$ the numerical radii of P and of its Aron-Berner extension $\tilde{P}$ coincide. This is a joint work with Y.S. Choi, D. García and S.G. Kim.

Polynomials defined in the dual of a Banach space

José G. Llavona (Universidad Complutense de Madrid)

Luiza Amália Moraes* (Universidade Federal do Rio de Janeiro)

Let $E = F'$ where F is a complex Banach space and $\pi_1 : E'' = E \oplus F^{\perp} \longrightarrow E$ be the canonical projection. Let $P(^nE)$ be the space of the complex valued continuous $n-$homogeneous polynomials defined in E. We characterize the elements $P \in P(^nE)$ whose Aron Berner extension coincides with $P \circ \pi_1$. In connection, the case of weakly continuous polynomials is considered. We prove that if P is a weakly continuous $n-$homogeneous polynomial from E into $\mathbb{C}$, then P is w^*-continuous if and only if $dP(x) \in F \;\; \forall x \in E$. A similar result is proved in case of $n-$homogeneous polynomials whose restrictions to the bounded sets are weakly continuous. We also give an example showing the existence of a weakly sequentially continuous polynomial that is not w^*-sequentially continuous despite $dP(x) \in F \;\; \forall x \in E$.

A characterization of real Hilbert spaces using complex techniques

Gustavo A. Muñoz* (Universidad Complutense de Madrid)

Juan Ferrera (Universidad Complutense de Madrid)

According to a well-known result, every polynomial $P : E \to \mathbb{R}$ defined on a real Banach space E can be extended uniquely to a complex polynomial $\widetilde{P} : \widetilde{E} \to \mathbb{C}$ defined on the complexification $\widetilde{E}$ of E. The ratio $\|\widetilde{P}\|/\|P\|$ depends on how we extend the norm of E to $\widetilde{E}$ and is minimized when the projective tensor norm is consider in $\widetilde{E} = E \otimes \ell_2^2$. This norm is named after Bochnak and it is usually difficult to handle. However it is possible to give several explicit ways to express that norm when E is a real Hilbert space. Moreover, one of these representations of the Bochnak complexification norm of a real Hilbert space characterizes real inner product spaces.

Cotype and absolutely summing homogeneous polynomials in Lp spaces

Daniel Pellegrino (Universidade Federal de Campina Grande)

In this paper we generalize to homogeneous polynomials and multilinear maps a linear result due to Lindenstrauss and Pełczyński for absolutely summing operators. Precisely, if X and Y are two infinite dimensional Banach spaces, X with unconditional Schauder basis (x_n) with coefficient functionals (x_n^*) and moreover $\mathcal{P}_{as(q;1)}(^mX;Y) = \mathcal{P}(^mX;Y)$, we prove that if Y finitely factors the formal inclusion $l_p \to l_\infty$, then $(x_n^*(x)) \in l_t$ for each x in X, with

(a) $t = \frac{mpq}{p-q}$ if $q < p$

(b) $t = mq$ if $q \leq \frac{p}{2}$.

We also obtain similar estimates for the multilinear case. By exploring properties of cotype, these statements enable us to provide various examples of situations in which we have the space of absolutely summing homogeneous polynomials and multilinear maps different from the whole space. Among other results, we prove that if E_1,...,E_n are infinite dimensional $\mathcal{L}_\infty$-spaces then

$$\mathcal{L}_{as(2;2,\ldots,2,\infty)}(E_1, ..., E_n; F) = \mathcal{L}(E_1, ..., E_n; F)$$

if, and only if, F is finite dimensional.

The approximation property on spaces of holomorphic functions

Pilar Rueda (Universidad de Valencia)

The approximation property on a locally convex space E ensures that the Identity map can be approximated uniformly on precompact subsets by finite rank operators. In terms of tensor products what it says is that $E' \bigotimes E$ is dense in the space of continuous linear operators endowed with the topology of uniform convergence on precompact subsets. Aron and Schottenloher characterize the approximation property in terms of holomorphic functions instead of continuous linear operators. Specifically, they prove that a Banach space E has the approximation property if and only if $\mathcal{H}(U) \bigotimes F$ is dense in $(\mathcal{H}(U; F), \tau_0)$ for every locally convex space F, and that this is also equivalent to the fact that $(\mathcal{H}(U), \tau_0)$ has the approximation property for every non-empty open subset U of E. Schottenloher gives a related result for a locally convex domains. In this work, made jointly with C. Boyd and S. Dineen, we obtain analogous results for spaces of holomorphic functions which are weakly uniformly continuous on bounded sets. We show when U is a balanced open subset of a Baire or barrelled metrizable locally convex space E, that the space of holomorphic functions which are weakly uniformly continuous on U-bounded sets has the approximation property if and only if the strong dual E'_b of E has the approximation property.

On the Markov constants of homogeneous polynomials on real normed spaces

Szilárd Gy. Révész (A. Rényi Institute of Mathematics, Hungarian Academy of Sciences)

Yannis Sarantopoulos* (NationalTechnicalUniversity)

Let X and Y be real normed spaces and let $P\colon X \to Y$ be a homogeneous polynomial of degree $\leq m$. In 1997 L. A. Harris has proved that the kth Fréchet derivative of P satisfies the Markov-type inequality $\|\widehat{D}^k P\| \leq c_{m,k}\|P\|$, where the best constant $c_{m,k}$ has been derived as a solution of an extremal problem for polynomials on the real line. Upper and lower estimates for the constant $c_{m,k}$ have been obtained and the exact values, up to $m = 20$, have been computed. In this work we have made an improvement upon these estimates and we have found that the exact order of magnitude for $c_{m,k}$ is the $m \log m$ order of Harris' upper estimate. Our method relies on the technique of potential theory with external fields which we have applied with varying weights and at a border-case situation.

Multiple p-summing operators on Banach spaces

Fernando Bombal (Universidad Complutense de Madrid)

David Pérez-García (Universidad Complutense de Madrid)

Ignacio Villanueva Diez* (Universidad Complutense de Madrid)

We introduce a new class of multilinear p-summing operators, which we call *multiple p-summing*. Using them, we can prove several multilinear generalizations of Grothendieck's "fundamental theorem of the metric theory of tensor products". Several applications and improvements of previous results are given. Among them, we improve a result of H.P. Rosenthal and S.J. Szarek and some results of Y. Meléndez and A. Tonge and Choi, Kim, Meléndez and Tonge.