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Abstracts

Session 34

Recent Developments in the Mathematical Theory of Inverse Problems

Organizers:

Russell Brown (University of Kentucky)
Alberto Ruiz (Universidad Autónoma de Madrid)
Gunther Uhlmann (University of Washington)

Index of Abstracts

(In this index, in case of multiple authors, only the speaker is shown)

Lipschitz stability for the inverse conductivity problem

Giovanni Alessandrini* (Universitá degli Studi di Trieste)

S. Vessella (Universitá degli Studi di Firenze)

It is well known, [1, 2], that the inverse conductivity problem is severely ill-posed in the sense that, when the available a-priori information on the unknown conductivity is given in terms of a regularity bound of any finite order, the best possible stability is of logarithmic type.

We investigate alternative forms of a-priori information under which Lipschitz stability holds.

References

[1] Alessandrini, G. , *Stable determination of conductivity by boundary measurements*, Applicable Anal., 27 (1988), 153-172.

[2] Mandache, N., *Exponential instability in an inverse problem for the Schrödinger equation*, Inverse Problems, 17, (October 2001), 1435-1444.

Uniform estimates for solutions of Helmholtz's equation for the spherical Laplacian

Juan Antonio Barceló (Universidad Politécnica de Madrid)

We characterize the weight functions w for which the uniform estimate

$$\|\Delta_S^{1/2} u\|_{L^2(w)} \leq C(w)\|f\|_{L^2(w^{-1})}$$

holds for u the solution of

$$\Delta u + u = f$$

and u satisfies the Sommerfeld radiation condition. We use Δ_S for the spherical Laplacian.

We apply the above estimate to the study of well-posedness of the Schrödinger equation with lower order perturbation and with conditions close to Mizohata-Takeuchi (see Mizohata, "On the Cauchy Problem", Notes and Reports in Mathematics in Science and Engineering, vol. 3, Academic Press, San Diego). These estimates can also be applied to scattering theory.

A resolution study for imaging and time reversal in random media

Liliana Borcea* (Rice University)

G. Papanicolaou

C. Tsogka

We consider the inverse problem of imaging scatterers (targets), in randomly inhomogeneous media, in a remote sensing regime with significant multipathing of the waves scattered by the inhomogeneities. The imaging is done via an array of transducers, of aperture a, which emits a short pulse and measures the scattered echoes. We study the effect of the random inhomogeneities on the resolution of the images produced with a matched field approach. We derive a theoretical model for the matched field imaging functional, where the effect of the random medium is quantified by a single parameter, the narrow band effective aperture a_e. We give a robust procedure for estimating a_e, which is of interest for at least two reasons: First, knowing a_e allows us to estimate the refocusing resolution of the time reversed, back-propagated field in applications such as secure communications. Second, a_e quantifies in an explicit way the loss of resolution in imaging targets embedded at unknown locations in a noisy medium.

Regularity for the coefficient in the inverse conductivity problem

Russell Brown (University of Kentucky)

I will discuss the minimal smoothness hypotheses under which we are able to recover the conductivity from the Dirichlet to Neumann map.

Concentration of waves in highly heterogeneous media

Carlos Castro (Universidad Politécnica de Madrid)

We present some examples of rapidly oscillating Hölder continuous coefficients for which the corresponding wave equation have a sequence of solutions that propagate as slow as we wish in any direction.

The coefficients we build oscillate arbitrarily fast in the neighborhood of a given point x_0 of the domain or the boundary. This allows us to construct a sequence of high frequency quasi-eigenfunctions for the corresponding eigenvalue problem that concentrate as much as we wish in the neighborhood of x_0. The solutions associated to these quasi-eigenfunctions remain arbitrarily concentrated around x_0 along the time.

These examples show that the well-known observability, controllability and dispersive properties for wave equations with smooth coefficients fail in the class of Hölder continuous coefficients.

Unique continuation for parabolic equations and some applications

Luis Escauriaza* (UPV/EHU)

F.J. Fernández (UPV/EHU)

G. Seregin (Steklov Institute of Mathematics at St. Petersburg)

V. Sverak (University of Minnesota)

L. Vega (UPV/EHU)

We describe some positive results of unique continuation and backward uniqueness for parabolic equations and its application to prove regularity of $L^\infty(0,T;L^3(\mathbb{R}^3))$ solutions of the Cauchy problem for the Navier-Stokes equations.

Recovery of a function from its spherical means

David V. Finch* (Oregon State University)

S. Patch (GE Medical Systems)

Rakesh (University of Delaware)

We study the recovery of a function supported in a bounded connected open set $D \subset R^n$ from the knowledge of its spherical means for spheres centered on a part or the whole boundary of D. Some uniqueness results are proved. For the case when D is a ball, n is odd, and the spherical means are known on the whole boundary, we establish a reconstruction formula similar in form to the Radon inversion formula. Some applications to the wave equation and to thermoacoustic tomography are discussed.

Nonlinear network tomography

Francisco Alberto Grunbaum (University of California at Berkeley)

We consider a multiterminal network whose directed graph allows for cycles and is not necesarily planar. Such networks arise as discrete models of "diffuse or optical tomography" but may be of interest beyond their place of origin. We formulate the inverse problem of recovering the one-step transition probability matrix for the underlying Markov chain from boundary measurements of the distribution of "time of flight". In certain situations we obtain a complete description of the (nonlinear) set of unrecoverable unknowns as well as explicit formulas for the remaining ones in terms of data and these free parameters. We concentrate on "explicit formulas" and do not consider the important issue of building good estimators.

Reconstruction of Label Images

Gabor T. Herman* (City University of New York)

H. Y. Liao (City University of New York)

Let J be a positive integer and L be a finite set. We call the elements of L labels and J-dimensional vectors, whose components are from L, label images. (A typical example is provided by J=NxN and L=0,1, in which case the label images are exactly the binary NxN matrices.)

Let p(y—x) denote, for each label image x, a probability density function for the J-dimensional vectors y over the real numbers and let D denote a finite subset of the set of all binary J-dimensional vectors. We are interested in reconstruction problems of the following type:

Let J, L, p, and D be fixed and given. Assume that, for some unknown x, y has been randomly picked according to p and that we are provided, for every d in D, with the scalar product of d and y. Our task is to estimate x.

A special case of this problem is that of reconstruction in discrete tomography, in which L is a subset of the real numbers and p(y—x)=1 if y=x and is 0 otherwise. (A typical choice for D for the situation described in the first paragraph is such that the set of all given scalar products would consist of the row and column and, possibly, some diagonal sums of the NxN matrix.)

In this talk we report on recent developments in the mathematical theory of the reconstruction of label images in general and of discrete tomography in particular.

Uniqueness of the continuation and inverse problems for elasticity system

Victor Isakov (Wichita State University)

We study the dynamical Lame system and a perturbation of this system by a residual stress. We give Carleman type estimates for this system and use them to obtain uniqueness and stability of the continuation results in the Cauchy problem with the lateral data. We give applications to uniqueness of recontruction of elastic parameters from few lateral boundary measurements in case of some particular initial conditions. This is part joint research with Oleg Imanuvilov, Masahiro Yamamoto, and Jenn-Nan Wang.

Reconstructing conductivities in the plane

Kim Knudsen (Aalborg Uniuversity)

The inverse conductivity problem is the mathematical problem behind a new method for medical imaging called Electrical Impedance Tomography (EIT). The idea in EIT is to recover an unknown conductivity in a body from static electric measurements on the boundary of the body. The mathematical problem is then a typical inverse boundary value problem, where one looks to determine and reconstruct a coefficient in a partial differential equation on a bounded domain Ω from knowledge of the Dirichlet-to-Neumann map on the boundary of the domain.

In this talk I will discuss a recent joint work with A. Tamasan, where we give a direct algorithm for the reconstruction of two-dimensional conductivities in the Sobolev space $W^{1+\epsilon,p}(\Omega),\ \epsilon > 0,\ p > 2$. The method is based on a uniqueness proof due to R. Brown and G. Uhlmann, which uses the $\overline{\partial}$-method of inverse scattering. Moreover, I will describe a numerical implementation of the algorithm, which is joint work with S. Siltanen and J. Mueller. This implementation is based on a fast numerical solution method for pseudo analytic equations.

Hausdorff moments in an inverse problem for the heat equation

Yaroslav V. Kurylev (Loughborough University)

This is joint work with N. Mandache and K.Peat (UK) on theoretical and numerical results in the inverse problem for the heat equation $\partial_t u = \rho^{-1}(x)\Delta u,\ \Omega \subset R^n$. The theoretical part is based on joint results with M.Kawashita and H.Soga (Japan), [1]. We use the spanning property of the products $p_h(x)q_h(x)$, where $p_h(x)$, $q_h(x)$ are harmonic polynomials and the fact that the Dirichlet-to-Neumann map, $p|_{\partial\Omega} \to \partial_n u^p|_{\partial\Omega\times R_+}$, where u^p solves the heat equation and $u(\cdot,t)|_{\partial\Omega} = p_h(\cdot)|_{\partial\Omega}$, $t > 0$, determines $\int_\Omega p_h q_h \rho\, dx$ for any q_h. We apply them to the case of finite observation time and finite number of sources, $p_1, \cdots, p_N$ to construct an approximation to ρ. We develop and test numerically, in $2D$, an algorithm based on these considerations. The results will appear in [2].

[1] Kawashita M., Kurylev Y. and Soga H., SIAM J. Math. Anal., **32**(2000), 522–537.

[2] Kurylev Y., Mandache N. and Peat K. S., *Hausdorff moments in an inverse problem for the heat equation,* to appear in Inv. Probl.

Gromov compactness and stability of inverse spectral problems

Matti Lassas (University of Helsinki)

We consider the stability of an inverse boundary spectral problem for second order elliptic differential operators on a compact manifold. The objective of these problems is to reconstruct the unknown manifold and the operator on it from the boundary spectral data (the boundary, the eigenvalues and the boundary values of the eigenfunctions).

To obtain uniqueness and stability results for this problem we pose a priori conditions e.g. for the Ricci curvature of the manifold and second fundamental form of the boundary. Under these geometric conditions we first establish new regularity results for the metric tensor. Second, we give Gromov type compactness result for manifolds with boundary and finally, we apply these results to obtain new uniqueness and conditional stability of an inverse boundary spectral problem.

This work has been done in collaboration with Michael Anderson, Atsushi Katsuda, Yaroslav Kurylev and Michael Taylor.

Electromagnetic inverse problems

Petri Ola (University of Oulu)

The purpose is to review what is known of time-harmonic inverse scattering and boundary value problems for Maxwell's equations, and present some more recent results and open problems.

Lower order perturbations of the evolution Schrödinger equation

Alberto Ruiz (Universidad Autónoma de Madrid)

A priori estimates for the solutions of the initial value problem of free Schrödinger propagator $e^{it\Delta}$ are very important to prove existence and completeness of the wave operators for perturbations of the Laplace operator. These estimates are related to Kato Δ–smoothness and Δ–supersmoothness. We prove some estimates for spherical derivatives of the solution and obtain well–posedness of the initial value problem for time dependent first order spherical perturbations of Schrödinger equation.

Smooth objective functionals for seismic velocity inversion

Christiaan C. Stolk* (Ecole Polytechnique)

W. W. Symes

In seismic inverse scattering the data are modeled by a high-frequency approximation, together with a linearization in the medium coefficient, which is written as the sum of a smooth background part and an oscillatory perturbation assumed to be small. We study the non-linear reconstruction problem for the background medium. Several estimators for the background medium have been proposed, based on optimization of certain quadratic objective functionals. In this talk we will show that such a functional is smooth (C-infinity) if and only if it is a pseudodifferential bilinear form. This means essentially that it must be of differential semblance type. Smoothness is important for the use of efficient local optimization methods.

Recovery of singularities from backscattering in $\mathbb{R}^2$ and $\mathbb{R}^3$.

Ana Vargas Rey (Universidad Autónoma de Madrid)

This is a joint work with Alberto Ruiz.

Consider an inverse scattering problem associated to a potential q in the Hamiltonian $H := -\Delta + q$. The problem is to obtain information about q from the far field measurements of the scattering solutions of $Hu = k^2 u$. Asymptotically, these solutions are given by

$$u(x,k,\theta) = e^{ikx\cdot\theta} + C|x|^{\frac{1-n}{2}} k^{\frac{n-3}{2}} e^{ik|x|} A(k,\theta,',\theta) + o(|x|^{\frac{1-n}{2}}),$$

where $\theta' = x/|x|$. The backscattering Born approximation for q is q_B defined by

$$\widehat{q_B}(\xi) = A(|\xi|/2, \hat{\xi}, -\hat{\xi}),$$

where $\hat{\xi} = \xi/|\xi|$.

We work in $\mathbb{R}^2$ and $\mathbb{R}^3$. We prove that under some assumptions on the decay of q at infinity, the error $q - q_B$ is smoother that q, i.e., the singularities of q are captured by Q_B. Some previous results of this type in $\mathbb{R}^2$ are due to Ola-Päivärinta-Serov.

Inverse problems in N-body scattering

Andras Vasy (MIT)

I will explain a new result with Gunther Uhlmann on the N-cluster to N-cluster scattering matrix at a fixed energy, namely that for rapidly decreasing potentials between interacting d-dimensional particles (d at least 2, N at least 3) the scattering matrix determines the potential.