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Abstracts

Session 35

Riemannian Foliations

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Morphisms of pseudogroups and foliation maps

Jesús Álvarez (Universidad de Santiago de Compostela)

A morphism of pseudogroups is defined in a more general way than Haefliger's étalé morphisms. Any foliations map induces a morphism between the corresponding holonomy pseudogroups. The main result shows that morphisms between complete pseudogroups of local isometries have a closure, and are complete and smooth along the orbit closures. Here, a closure and completeness are slight adaptations of concepts introduced by Haefliger. This result can be used to study smooth approximation of maps between transversely complete riemannian foliations, which in turn shows the homotopy invariance of the spectral sequence of such foliations. This invariance generalize a result of El Kacimi and Nicolau, showing the topological invariance of their basic cohomology.

Morse inequalities for orbit spaces. A Witten approach.

Manuel Calaza (Universidad de Santiago de Compostela)

Let M be a closed manifold with a smooth action of a compact Lie group G. We have that $H^\star(M/G) = H^\star(\Omega(M/G), d_b)$, where d_b is the restriction of the derivative on $\Omega(M)$. Because the orthogonal projection $\Pi : L^2\Omega(M) \to L^2\Omega(M/G)$ preserves smoothness of differential forms, we have that $\delta_b = \Pi \circ \delta$ is an adjoint to d_b on $\Omega(M/G)$.We check this by using "blow-up" desingularization arguments.

In this context we can try to use the idea of Witten of deforming the exterior derivative by G-invariant Morse functions. We get a modified Laplacian and a Hodge Theorem. We get the Morse inequalities in terms of the traces of operators. As in the original argument, the problem is reduced to local computations in the critical orbits by using the spectral analisis of the harmonic oscilator. In our case it does not seem to be enough.

The solution cames by doing the local espectral analisis in the blowed manifold where the changes experimented by the metric allows an espectral analisis quite different. This idea was suggested by the following example: consider $\mathbb{S}^1$ acting on $\mathbb{CP}^2$ in this way, $z[z_0, z_1, z_2] = [zz_0, z_1, z_2]$, and the Morse function $f([z_0, z_1, z_2]) = |z_0|^2$. We have detected some contributions to the traces not suggested by the clasical results.

Topological characterization of Riemannian foliations

Jesús A. Álvarez López (Universidade de Santiago de Compostela)

Alberto Candel* (CSUN, USA)

A foliation is called Riemannian when its holonomy pseudogroup is given by local isometries of some Riemannian manifold. The topological analogue of the concept of Riemannian foliation is that of equicontinuous foliated space.

We develop the theory of that class of foliated spaces. Using this theory, and that involved in the solution of the local version of Hilbert's 5th problem (Jacobi), we show that certain topological and metric conditions on an equicontinuous foliated space imply that it is a Riemannian foliation.

Related work is due to Sacksteder, Ghys, Kellum, Tarquini.

LS Category of Riemannian Foliations

Hellen Colman (University of Illinois at Chicago)

I will talk about the Lusternik-Schnirelmann category of Riemannian foliations.

The $\overline{\partial}_{\mathcal{F}}$-problem along the leaves

Aziz El Kacimi (Université de Valenciennes)

Let $\mathcal{F}$ be a foliation of codimension n and dimension $2m$ on a differentiable manifold M. We say that $\mathcal{F}$ is *holomorphic* if there exist an open cover $\{U_i\}$ and diffeomorphisms $\phi_i : \mathcal{O}_i \times \mathbf{R}^n \longrightarrow U_i$ (where $\mathcal{O}_i$ is an open set of $\mathbf{C}^m$) such that each coordinate change $\phi_{ij} = \phi_j^{-1} \circ \phi_i : \phi_i^{-1}(U_i \cap U_j) \longrightarrow \phi_j^{-1}(U_i \cap U_j)$, which is of the form $\phi_{ij}(z,t) = (\phi_{ij}^1(z,t), \phi_{ij}^2(t)$, is such that $\phi_{ij}^1(z,t)$ is holomorphic in z for each fixed t. For such foliation we can define its *foliated Dolbeault cohomology* $H^{p*}(M,\mathcal{F})$. Our main result is the following:

Theorem. *Suppose $\mathcal{F}$ is defined by a fibration $\pi : M \longrightarrow X$ where X is an orbifold and suppose that the fibres of π are non compact Riemannian surfaces. Then $H^{01}(M,\mathcal{F}) = 0$.*

The Molino Conjecture for Singular Riemannian Foliations

Gilbert Hector (Institut Girard Desargues, France)

In the late decades of the last century, P.Molino built up an impressive theory for Riemannian Foliations (RF's) and Singular Riemannian Foliations(SFR's). In particular, he proved that closures of leaves of a RF define a SRF, but surprisingly he didn't establish the corresponding claim for SRF's ...leaving it as the so-called "Molino Conjecture". Our contribution will fill this lack as an application of a geometric description of Molino's Transverse Central Sheaf of any SRF. Indeed, we establish the two basic facts:

1) The Transverse Central Sheaf of a SFR is the sheaf of germs of a concrete module of transverse foliated Killing vectorfields.

2) This module may be obtained as the quotient of the corresponding module of the Regular RF obtained by the blowing up procedure. These results give a deep insight in the theory of SFR.They will permit further investigations on the subject like

a) find a minimality criterium for SFR (see the result of X.Masa for RF's),

b) find invariants of SFR's in the DeRham spectral sequence etc......

Characteristic class of Riemannian foliations

Steven Hurder (University of Illinois at Chicago)

We will discuss some rationality properties of the secondary classes of Riemannian foliations, and some relations between the values of the classes and the geometry of Riemannian foliations.

The diffeomorphism group of a Lie foliation

Enrique Macia–Virgós (Universidade de Santiago de Compostela)

The space of leaves $M/\mathcal{F}$ of a Lie foliation $\mathcal{F}$ with dense leaves on a compact manifold M has a natural structure of diffeological space (in the sense of J. M. Soriau). Then the diffeomorphism group $Diff(M/\mathcal{F})$ can be endowed with the so-called functional diffeology, making it a diffeological group (the adequate generalization of the notion of Lie group).

Accordingly to previous results of G. Hector and E. M.-V. (2002), the latter group can be computed from the structural data of the foliation (that is the structural Lie algebra $Lie(G)$ and the holonomy subgroup $\Gamma \subset G$). Namely, $Diff(M/\mathcal{F})$ is isomorphic (in the diffeological sense) to the quotient by Γ of a semidirect product $Aut(G, \Gamma) \times G$, where the first factor is the group of automorphisms of the transverse Lie group G which preserve Γ.

In this talk we show how to explicitly do this computation for any linear flow and for any codimension one linear foliation in the n-dimensional torus, thus generalizing an old result (1985) of P. Donato and P. Iglesias about the linear flows on T^2. Some elementary algebraic tools are needed (e.g. the Pell-Fermat equation). (Joint work with A. Sotelo)

A Coincidence Formula for Foliated Manifolds

Bernd Mümken (Universität Münster)

The coincidence formula for foliated manifolds is a generalization of the classical Lefschetz trace formula. To prove this we establish Künneth formula, Poincaré duality and intersection product in the context of tangential de Rham cohomology and homology of tangential currents.

We apply the formula to get a dynamical Lefschetz formula for foliated flows.

A class of transversely Kahlerian foliations

Jean Jacques Loeb (Université d'Angers)

Marcel Nicolau Reig* (Universitat Autònoma de Barcelona)

In this talk we present a class of Riemannian foliations which come from complex geometry. Let ξ be a linear complex vector field on $\mathbb{C}^n$ and let R denote the (complex) radial vector field. The couple (ξ, R) defines an action of $\mathbb{C} \times \mathbb{C}^*$ on $\mathbb{C}^n$. Under a weak hyperbolicity condition on ξ one can find a union L of invariant linear subspaces of $\mathbb{C}^n$ with the property that the orbit space M of the action restricted to $\mathbb{C}^n - L$ is a compact manifold which inherits a complex structure in a natural way. Moreover, each complex manifold M obtained in this way enjoys the following properties

(1) M is non Kahler,

(2) the dynamics of the vector field ξ are encoded in the geometric properties of M,

(3) there is a holomorphic vector field X on M without zeros and defining a foliation which is transversely Kahlerian.

Conformal foliations

Oscar A. Palmas Velasco (Universidad Nacional Autónoma de México)

We will discuss some results on conformal foliations, i.e., foliations given locally by submersions which preserve angles between horizontal vectors. In particular, we will study conformal foliations with one dimensional fibers on constant curvature spaces.

Basic index theory

Jochen Brüning (Humboldt-Universität zu Berlin)

Franz Kamber (University of Illinois)

Ken Richardson* (Texas Christian University)

Given a Riemannian foliation $(M, \mathcal{F})$ and a $\mathbf{Z}_2$-graded, foliated Hermitian bundle $E \to M$ with associated basic connection ∇, we say that a section s is basic if $\nabla_X s = 0$ for every vector field X tangent to the leaves of $\mathcal{F}$. Let D be a transversally elliptic operator on sections of E that preserves the set of basic sections of E. The basic index of D is defined to be the index of D restricted to the space of basic sections of E. We decompose M as a union of strata that are determined by the action of holonomy groups. We show a partial solution to the problem of finding a topological formula for this index in terms of topological data on the foliation strata of $(M, \mathcal{F})$.

The Basic Intersection Cohomology of Singular Riemannian Foliations

Martintxo Saralegi Aranguren* (Artois University, France)

Robert Wolak (Uniwersytet Jagiellonski)

We study singular riemannian foliations from a cohomological point of view. For this purpose we introduce the basic intersection cohomology (or BIC) which is a finer invariant the usual basic cohomology. On a manifold M foliated by a singular riemannian foliations $\mathcal{F}$ the dimension of leaves of $\mathcal{F}$ defines a stratification making the manifold M a stratified pseudomanifold in the sense of Goresky-MacPherson. This allows us to associate to a basic differential form its perverse degree. The BIC uses basic differential forms of bounded perverse degree. We show that the BIC of a singular riemannian foliation has the Poincaré Duality property which is not true for the basic cohomology of such singular foliations.