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Abstracts

Session 36

Ring Theory and related topics

Organizers:

José Gómez-Torrecillas (Universidad de Granada)
Pedro Antonio Guil Asensio (Universidad de Murcia)
Sergio R. López Permouth (Ohio University)
Blas Torrecillas (Universidad de Almería)

Index of Abstracts

(In this index, in case of multiple authors, only the speaker is shown)

Towards a classification of the isomorphism classes of matrix rings over a fixed ring of scalars

Gene Abrams (University of Colorado at Colorado Springs)

We give necessary conditions which restrict the isomorphism classes of different sized matrix rings over a fixed ring of scalars. We then demonstrate how some partitions of $\mathbf{N}$ arise from such classes. (Preliminary report)

Quotient Finite Dimensionality in Lattices, Grothendieck Categories, and Torsion Theories

Toma Albu (Atilim University, Ankara)

A right R-module M is said to be *quotient finite dimensional* (QFD) if M/N has finite Goldie dimension for each submodule N of M. This concept can be naturally defined for lattices with 0 and 1. We extend a series of results about QFD modules to upper continuous modular lattices by using Lemonnier's Lemma. Applications are given to Grothendieck categories and module categories equipped with a torsion theory.

The results which will be presented have been obtained jointly with *Mihai Iosif* (Bucharest University, Romania) and *Mark L. Teply* (University of Wisconsin-Milwaukee, USA).

Tilting theory and the finitistic dimension

Lidia Angeleri Huegel (Universitá dell'Insubria, Varese)

For a finite-dimensional algebra Λ the little finitistic dimension findimΛ is defined as the supremum of the projective dimensions attained on the category $\mathcal{P}^{<\infty}$ of all finitely generated Λ-modules of finite projective dimension, while the big finitistic dimension FindimΛ is defined correspondingly on the category of all Λ-modules of finite projective dimension. The Finitistic Dimension Conjectures ask when these dimensions coincide (this is known to fail in general), and moreover, whether the little finitistic dimension is always finite.

I will discuss a new approach to these conjectures using infinite dimensional tilting modules. Hereby I will present joint work with Jan Trlifaj.

Coringsand Morita(–like) contexts

Tomasz Brzezinski (University of Wales, Swansea)

Corings are generalisations of coalgebras, defined as bimodules over a non-commutative ring rather than modules over a commutative ring. In particular a ring itself can be viewed as a (trivial) coring. Thus corings allow one to unify ring and module theory with their dual counterparts (coalgebra and comodule theory), and are particularly useful in description of ring extensions. The aim of this talk is to outline a relationship between the properties of bimodules and the properties of corings associated to these bimodules. The guiding principle is a connection between corings and Morita contexts. In particular, we show that data reminiscent of a Morita context (semi-dual version of a Morita context) give rise to a class of corings known as comatrix corings. A comatrix coring, which can be associated to any bimodule which is finitely generated and projective on one side, reflects most closely the properties of this bimodule. In particular separable bimodules give rise to coseparable comatrix corings. On the other hand any (not necessarily comatrix) coseparable coring which has a grouplike element defines a Morita context for a non-unital ring. We describe this context and study its properties.

(Results partially obtained in collaborations with: J Gomez-Torrecillas, L Kadison and R Wisbauer)

Ring hulls

Gary F. Birkenmeier (University of Louisiana at Lafayette)

In this talk we introduce the concepts of a ring hull and of a pseudo ring hull. We consider the existence, properties, and examples of ring hulls and of pseudo ring hulls which satisfy various conditions including right extending, right FI-extending, Baer, and quasi-Baer.

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coGalois groups of torsion free covers

Edgar Enochs (University of Kentucky)

With any torsion free cover there is an associated absolute coGalois group. We study these groups as topological groups and show that they are products of Banach spaces with orthonormal bases over the various rings of p-adic integers.

Quillen's Small Object Argument in the category of quasi-coherent sheaves on a scheme

Edgar Enochs (University of Kentucky)

Sergio Estrada Domínguez * (Universidad de Almería)

Juan R. García Rozas (Universidad de Almería)

Luis Oyonarte (Universidad de Almería)

In this talk we prove the existence of a flat cover and a cotorsion envelope for any quasi-coherent sheaf over a scheme $(X, \mathcal{O}_X)$. Indeed we prove something more general. We define what it is understood by the category of quasi-coherent R-modules, where R is a representation by rings of a quiver Q, and we prove the existence of a flat cover and a cotorsion envelope for quasi-coherent R-modules. Then we use the fact that the category of quasi-coherent sheaves on $(X, \mathcal{O}_X)$ is a thick subcategory of the category of R-modules for such an R to get our result.

Modules whose direct sums are CS and their indecomposable decompositions

José Luis Gómez Pardo (Universidad de Santiago de Compostela)

Every Σ-CS-module is a direct sum of uniform modules but, in contrast with what happens in the injective case, an $\aleph_0$-Σ-CS-module (i.e., a module M such that each countable direct sum of copies of M is CS) need not have an indecomposable decomposition. It is then a natural question to find out if there exists a (least) cardinal $\aleph$ such that each $\aleph$-Σ-CS-module has an indecomposable decomposition. We give an affirmative answer to this question by showing that every $\aleph_1$-Σ-CS-module is a direct sum of uniform modules. A stronger version of the problem is the following: Is there a cardinal $\aleph$ such that each $\aleph$-Σ-CS-module is already Σ-CS? We show that the answer is yes in some particular cases by proving that an $\aleph_1$-Σ-CS-module M wich is either projective or non-M-singular, is actually a Σ-CS-module. We do not know if this is still true for arbitrary $\aleph_1$-Σ-CS-modules, but we also show that, for any given ring R, there exists a cardinal $\aleph$ such that each $\aleph$-Σ-CS right R-module is already a Σ-CS-module.

Simple modules over small von Neumann regular rings

Dolors Herbera (Universitat Autònoma de Barcelona)

Let R be a von Neumann regular ring with infinite index of nilpotency that, in addition, is $\aleph_0$-injective or $\aleph_0$-continuous. We show that if R is a V-ring then it must have a simple module whose dimension over its endomorphism ring is at least $2^{2^{\aleph_0}}$. Hence *small*, $\aleph_0$-injective or $\aleph_0$-continuous, regular rings with infinite index of nilpotency are not V-rings. The main step in the proof of the result relies on a counting argument based on a Lemma by Tarski, the method is patterned in ideas first used by Osofsky.

In a more elementary way, we show that if R is a simple regular ring that is a countably generated algebra over a commutative field K then none of its simple modules can be injective unless it is semisimple artinian. This leads us to study simple modules over countable dimensional simple ultramatricial K-algebras. We show that in these examples all division rings that are finite dimensional over K can be realized as endomorphism rings of simple modules.We also describe the injective hulls of some simple modules. Over this class of rings $\mathrm{Ext}^1_R(V_1, V_2) \neq 0$ for any pair of simple modules, this implies that modules of socle-height 2 are of wild representation type.

Sigma-cotorsion rings

Ivo Herzog (Ohio State University)

A right R-module is said to be cotorsion if $\mathrm{Ext}^1_R(F, M) = 0$ for every flat right R-modules F_R. The module M is called a Σ-cotorsion module if for every index set I, the coproduct $M^{(I)}$ of I-many copie of M is also cotorsion. It is not difficult to see that a ring R is Σ-cotorsion as a right module over itself if and only if it is right prefect. I will report on several other characterizations of Σ-cotorsion rings.

Cyclic homology of Hopf algebras

Pascual Jara* (Universidad de Granada)

Dragos Stefan (University of Bucharest)

Let H be a Hopf algebra over a field k and let $\mathcal{CM}(H)$ be the category of crossed representations over H. For example H can be regarded as an object in $\mathcal{CM}(H)$ with respect to the adjoint action and canonical coaction. We will denote this crossed representation by ${}_{ad}H^{\Delta}$. Another example is $HH_*(B, A)$, where A/B is an H–Galois extension and $HH_*(B, A)$ is the Hochschild homology of B with coefficients in A. The main aim of our talk is to show that one can associate a cyclic object $Z_*(M)$ to every crossed representation M of H. This construction defines an exact functor $Z_*(-)$ from $\mathcal{CM}(H)$ to the category of cyclic objects, which behaves nicely with respect to direct sums and filtrations of crossed representations. These properties of $Z_*(-)$ allows us to compute the cyclic homology $HC_*(M)$ of $Z_*(M)$. For example, we show that $HC_*({}_{ad}H^{\Delta})$ is the usual cyclic homology (defined by Connes) of H. Also, for an H–Galois extension A/B we prove that $HC_*(HH_0(B, A))$ is the relative cyclic homology of A. It is well–known that G–strongly graded algebras are kG–Galois extensions. By specializing our results to this particular case we are able to compute the cyclic homology of strongly graded algebras. Since kG is G–strongly graded, our result includes the computation of cyclic homology of group algebras, due to Burghelea.

Coinvariant theory for quantum matrices

Mátyás Domokos (Renyi Institute of the Hungarian Academy of Sciences)

Thomas Lenagan* (University of Edinburgh)

Analogues of the conjugation action of the general linear group on the space of matrices are defined in terms of coactions of the quantum general linear group on the algebra of quantum matrices. Coinvariants for these coactions are found, and shown to form commutative polynomial subalgebras of the algebra of quantum matrices. Quantum traces of the quantum powers of a quantum matrix are defined, and an analogue of Newton's formula is obtained. As a consequence, these quantum traces commute, answering a question in the physics literature.

When is a smash product semiprime ?

Christian Lomp (Universidade do Porto)

Cohen and Fishman raised the question whether the smash product of a semisimple Hopf algebra and a semiprime module algebra is semiprime. In this talk we show that the smash product of a commutative semiprime module algebra over a semisimple cosemisimple Hopf algebra is semiprime. In particular we show that the central H-invariant elements of the Martindale ring of quotients of a module algebra form a von Neumann regular and self-injective ring whenever A is H-semiprime and H has a bijective antipode. For a semiprime Goldie PI H-module algebra A with central invariants we show that A# H is semiprime if and only if the H-action can be extended to the classical ring of quotients of A if and only if every non-trivial H-stable ideal of A contains a non-zero H-invariant element.

On multiplicative invariants of finite groups

Martin Lorenz (Temple University)

Multiplicative actions of a finite group G are G-actions on the Laurent polynomial algebra $k[x_1^{\pm 1}, \ldots, x_n^{\pm 1}]$ (over some commutative base ring k) that stabilize the lattice of monomials in $\{x_1^{\pm 1}, \ldots, x_n^{\pm 1}\}$. We will discuss some recent results on the structure of the invariant algebra $k[x_1^{\pm 1}, \ldots, x_n^{\pm 1}]^G$ focusing in particular on the Cohen-Macaulay property.

Degeneration, rigidity and irreducible components of Hopf algebras

Abdenacer Makhlouf (Université de Haute Alsace)

The aim of my talk is to discuss the concepts of degeneration, deformation and rigidity of Hopf algebras and to apply them to the geometric study of the varieties of Hopf algebras. The main result is the description of the n-dimensional rigid Hopf algebras and the irreducible components for $n < 14$ and $n = p^2$, p being prime number. This talk will be organized as follows : first I summarize the basic concepts of degeneration, deformation and rigidity with examples and some useful properties. The second part will be devoted to the geometric description of the algebraic varieties of n-dimensional Hopf algebras .

Nonunital rings, categories and topology

Leandro Marín (Universidad de Murcia)

Let R be an associative ring. We will denote $R-\mathsf{DMod}$ the category of left R-moduies such $R\otimes_R M \simeq M$. We will also denote $\mathsf{CMod}-R$ the category of right R-modules such that $M \simeq \mathrm{Hom}_R(R, M)$, in both cases with the canonical homomorphisms. These categories generalize the usual categories of unital modules for unital rings.

Let X be an infinite set and $\mathbb{Z} < X >$ the free noncommutative algebra over the elements in X. The subring $\sum_{x\in X} x\mathbb{Z} < X >$ will be denoted $\mathbb{Z} < X >_0$.

The study of right continous functors (right exact functors that preserve colimits) between $R-\mathsf{DMod}$ and $R'-\mathsf{DMod}$ can be reduced to the particular case of right continuous functors between $\mathbb{Z} < X >_0 -\mathsf{DMod}$ and the category of abelian groups. In this talk we will connect the structure of these functors and the topological space $X^{\mathbb{N}}$ (X is endowed with the discrete topology and $X^{\mathbb{N}}$ with the product topology). These results will show us a strong connection between the categories $\mathsf{CMod}-\mathbb{Z} < X >_0$ and $\mathsf{RCFunc}(\mathbb{Z} < X >_0 -\mathsf{DMod}, \mathsf{Ab})$.

Full functors in nature

Stefaan Caenepeel (Free University of Brussels)

Claudia Menini* (University of Ferrara)

Gigel Militaru (University of Bucharest)

We introduce and study a special class of full functors, namely (nf)-functors i.e. *naturally full functors*. We will give some easy criterium to decide when a pair of adjoint functors are (nf)-functors and as an application, several examples of (nf)-functors will be constructed. We will also study when a Frobenius pairs of functors are (nf)-functors.

Prime and Irreducible Preradicals over Associtive Rings

Federico Francisco Raggi Cárdenas (Universidad Nacional Autónoma de México)

We give definitions of prime and irreducible preradicals,as well as, prime modules. We study some relations between these concepts using the lattice structure of the class of all preradicals developd in previous papers. We give some characterizations of rings that have certain conditions on prime or irreducible radicals.

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c-Injectivity Over Principal Ideal Domains

Catarina Santa-Clara * (Universidade de Lisboa)

Patrick F. Smith (University of Glasgow)

Let R be any ring and let X and M be R-modules. The module X is called *M-c-injective* if, for every closed submodule K of M, every homomorphism $\phi : K \to X$ can be lifted to M. (Recall that a submodule K of M is *closed* (in M) if it has no proper essential extension in M.) The module M is called *self-c-injective* if M is M-c-injective. We shall present some results concerning self-c-injective modules over Principal Ideal Domains.

The fixed subalgebra of the group of automorphisms of a commutative algebra

Manuel Saorín Castaño (Universidad de Murcia)

In this talk we shall address the following problem. Suppose $A = K[X_1, ..., X_n]/I$ is a finite dimensional commutative algebra over a field K, where I is a two-sided ideal of $K[X_1, ..., X_n]$ contained in $(X_1, ..., X_n)2$. When is it true that the subalgebra $A^{Aut_K(A)}$ of A, given by the elements fixed by the action of all automorphisms of A, coincides with the field K?. We shall approach the problem first in case K is algebraically closed of characteristic 0 and, for connections with Differential Geometry, in the case when $K = \mathbf{R}$ is the real field. We will show that the crucial point here is either the rank of $Aut_K(A)$ (as an algebraic group), when $K = \bar{K}$, or the rank of $Aut_{\mathbf{C}}(A \otimes_{\mathbf{R}} \mathbf{C})$, when $K = \mathbf{R}$ is the real field. Generally speaking, when that rank is positive, the subalgebra $A^{Aut_K(A)}$ 'tends to coincide' with the ground field K whereas the opposite tends to be true when that rank is zero.

Is there a one-sided quantum group?

Earl Taft (Rutgers University)

There exist bialgebras which have a left antipode but no right antipode. We try to construct such a left Hopf algebra in the framework of quantum groups. By asking that a left antipode be an algebra morphism, we are forced to add more relations, so that the result is a Hopf algebra with a two-sided antipode. It has the unusual feature that it remains non-commutative when q=1. So the question in the title remains open.

Baer Modules

Syed Tariq Rizvi (Ohio State University)

In this paper we introduce the notion of a Baer module and study the connections of the Baer property with that of the extending property in a general module theoretic setting. It is shown that a finitely generated abelian group is Baer precisely if it is semisimple or torsionfree. The closely linked notion of a quasi-Baer module is also introduced and its connections to the FI- extending property are investigated. We establish several properties and applications of Baer and quasi-Baer modules.

Dual Krull Dimension and Quotient Finite Dimensionality

Mark Teply (University of Wisconsin-Milwaukee)

A right R-module M is said to be quotient finite dimensional (QFD) if M/N has finite Goldie dimension for each submodule N of M. This concept can be naturally defined for lattices with 0 and 1. We obtain some results that relate dual Krull dimension and the QFD property for upper continuous modular lattices. Applications are made to modules. In particular we answer an open question of Albu and Rivzi about modules with dual Krull dimension. (This paper is joint work with M. Iosif and T. Albu.)

Torsion theoretic properties of $\sigma[M]$

Robert Wisbauer (Universität Düsseldorf)

Let M be any left module over an associative ring R and denote by $\sigma[M]$ the full subcategory of R-Mod whose objects are submodules of M-generated modules. The structure of M is closely related to the internal properties of the category $\sigma[M]$. However, the class $\sigma[M]$ is by definition also a hereditary pretorsion class in R-Mod and the talk is about (external) properties of this class from the torsion theoretic point of view. We focus on properties which are of particular interest for the category of comodules over a coring C considered as subcategory of the modules over the dual coring C^*.