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Abstracts

Session 38

Variational Problems for Submanifolds

Organizers:

Frank Morgan (Williams College)
Antonio Ros (Universidad de Granada)

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Plane like minimal surfaces in periodic media

Luis Caffarelli (Univ. of Texas at Austin)

Rafael de la Llave* (Univ. of Texas at Austin)

Given a metric in Euclidean space which is periodic under integer translations, we show that there is a number A such that given any strip of width A – in any orientation – there is a minimal surface contained in the strip. We also discuss generalizations to other manifolds, and to PDE's and problems in statistical mechanics.

Complete embedded maximal surfaces in L^3 with isolated singularities

Francisco José López Fernández (Universidad de Granada)

In this lecture we study the space of complete embedded maximal spacelike surfaces with a finite number $n+1$ of singularities. We prove that any such surface is a entire graph with $n+1$ conical singularities, and the corresponding moduli space is an analytical manifold of dimension $3n+4$.

Geometric results in classical minimal surface theory

William H. Meeks III (University of Massachusetts)

In the past decade there has been some exceptionally beautiful results proven concerning the classical theory of minimal surfaces. These results include the recent work of Collin and Meeks-Rosenberg which provide the theoretical basis for describing analytically all properly embedded minimal surfaces in three-dimensional Euclidean space in terms of meromorphic data on closed Riemann surfaces. It follows from this analysis and previous results that the plane, catenoid and helicoid are the only properly embedded minimal surfaces of finite topology and genus zero. There has also been important progress made on understanding the topology and conformal structure of properly embedded minimal surfaces. I will discuss the recent proof of Frohman and Meeks of the "Topological Classification Theorem" for properly embedded minimal surfaces. Time permitting, I will include in my talk some recent joint results with Perez and Ros and with Rosenberg.

Dirac Operator and Hypersurfaces

Sebastián Montiel Gómez (University of Granada)

In the last years, some geometers of submanifolds have been led to study diverse Dirac operators which arise from extrinsic geometry of surfaces, and some Riemannian geometers specialists in this operator have started to consider its particular features on submanifolds.

This talk deals with the relation between the spectral properties of the classical Dirac operator and the geometry of hypersurfaces. We will give lower bounds for the eigenvalues of the Dirac operator of a hypersurface bounding a compact domain in a Riemannian spin manifold in terms of its mean curvature and of some conformal extrinsic invariants. We will give applications to classical geometry of hypersurfaces and to the theory of Einstein manifolds with boundary.

Proper minimal surfaces in $\mathbb{R}^3$

Santiago Morales (University of Granada)

Proper minimal surfaces in $\mathbb{R}^3$ have peculiar properties that are not shared by general minimal surfaces; especially in the embedded case.

It has been proved that, under additional conditions, this family of surfaces has strong restrictions on their conformal structures. For instance, Huber and Osserman proved that if M is a complete minimal surface with finite total curvature, then M has the conformal type of a compact Riemann surface minus a finite number of points. In particular it is parabolic, that is, M is not compact and M does not carry a negative non-constant subharmonic function. In the same context, Collin, Kusner, Meeks, and Rosenberg have proved that if M is a properly immersed minimal surface in $\mathbb{R}^3$, then $M(+) = \{(x_1, x_2, x_3) \in M \ : \ x_3 \geq 0\}$ is parabolic.

These results have motivated a conjecture that asserted: $f : M \to \mathbb{R}^3$ is a complete proper minimal immersion where M is a Riemannian surface without boundary and with finite genus, then M is parabolic.

This talk is about a counterexample to the conjecture.

Theorem: There exists $\chi : D \to \mathbb{R}^3$, a conformal proper minimal immersion defined on the open unit disk.

Double Bubble Problems

Frank Morgan (Williams College)

The classical isoperimetric problem seeks the least-area way to enclose a region of prescribed volume. The double bubble problem seeks the least-area way to enclose and separate two regions of prescribed volumes. I'll discuss the recent proofs in R3 and R4, partial results in S3, H3, G3, and T3, some history, and open problems.

Willmore surfaces of high normal bundle degree

Franz Pedit (University of Massachusetts)

Using methods from quaternionic holomorphic geometry, we show that compact Willmore surfaces of "high" (compared to the genus of the surface) normal bundle degree are either minimal surfaces with planar ends or twistor projections of holomorphic curves in $CP3$. A special case are the results of Bryant, Ejiri and Montiel on Willmore spheres.

The periodic isoperimetric problem

Antonio Ros Mulero (University of Granada)

The periodic isoperimetric problem is one of the nicest open questions in classical differential geometry: given a discrete group G of isometries of the Euclidean three space, it consists of describing, among G-invariant regions with prescribed volume fraction, those whose boundary has least area (modulo G). For instance, the classical P Schwarz triply periodic minimal surface is conjectured to have least area (per cubic cell) among surfaces dividing the 3-space in two $Pm3m$-invariant regions with equal volume fractions ($Pm3m$ is the group of symmetries of the cubic lattice $\mathbb{Z}^3$). In this talk, we will present some recent progresses in this field. In particular, we will give sharp isoperimetric inequalities in the case G is a cubic space group.

On some isoperimetric problems in $\mathbb{R}^n$

César Rosales (Universidad de Granada)

Two of the most natural *isoperimetric problems* to consider in a smooth domain $\Omega \subset \mathbb{R}^n$ arise when one tries to find, for any $V_0 > 0$, a minimum for the functional $F \mapsto \mathcal{P}(F)$ over the class of all the measurable sets $F \subseteq \Omega$ satisfying $|F| = V_0$. Here, $|F|$ denotes the Lebesgue measure of F and $\mathcal{P}(F)$ refers either to the *whole perimeter* of F or to the *relative perimeter* of F in Ω —for which only $\partial F \cap \Omega$ contributes to compute perimeter, while $\partial F \cap \partial\Omega$ does not—. When the functional under consideration is the relative perimeter the problem is usually called a *free boundary problem.* In this talk we deal with the above problems in the particular case in which Ω coincides with a convex domain C or with its complementary $\mathbb{R}^n - \overline{C}$. First we treat questions related to the *existence* and *regularity* of the solutions (called *isoperimetric regions*). In case the existence is guaranteed for any value of volume we discuss some topological and geometric properties of the isoperimetric regions, such as the connectedness or the convexity. In few cases is also possible to characterize completely the isoperimetric solutions. For example, if Ω is a convex cone different from a halfspace, then the solutions to the free boundary problem are balls centered at the vertex intersected with the cone.

Harmonic Diffeomorphisms onto the Hyperbolic Plane

Scott Pauls (Dartmouth College)

Michael Wolf* (Rice University)

We discuss recent progress on the problem of understanding the space of harmonic diffeomorphisms from the hyperbolic plane onto the hyperbolic plane.