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Harmonic Analysis and Applications – On the occasion of the 70th anniversary of R. Wheeden

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TALKS

On the dynamics of the first exotic contact form by J.Gonzalo and F.Varela along a flow in its kernel by Vittorio Martino

Abbas Bahri
Rutgers University, USA
abahri@math.rutgers.edu

The standard contact structure α_0 of S^3 has a vector-field v defining a Hopf fibration in its kernel. Legendre transform w.r.t v can be performed. Symmetric Hamiltonian problems are thereby transformed into their Lagrangian counterparts. It was believed that the existence of such a v was special to this framework. This belief turns out to be wrong. V. Martino has produced a vector-field in the kernel of the first contact form α by J.Gonzalo and F.Varela such that $d\alpha(v,.)$ is also a contact form with the same orientation than α. This provides a new textbook example in Contact Form Geometry. We will describe in our talk the first contact form of J.Gonzalo and F.Varela and the vector-field v in its kernel by V.Martino; we will study the related dynamics and the related Reeb vector-fields periodic orbit problems at the light of the homology for contact forms/structures that we have defined in our work.

Products and paraproducts of functions in $\mathcal{H}^1$ and BMO

Aline Bonami
Universitè d'Orleans, USA
aline.bonami@gmail.com

The interest in products of two functions, one in $\mathcal{H}^1$ and the other one in BMO, has started in a joint work with Iwaniec, Jones and Zinsmeister. We will speak of a joint work in progress with Sandrine Grellier and Luong Dang Ky, which gives improvements compared to [**BIJZ**]. Firstly one can find two continuous bilinear operators S and T such that $S(f,g)+T(f,g)=fg$, which map $\mathcal{H}^1 \times BMO$ respectively into L^1 and $\mathcal{H}^\Phi_w$. Here the product fg is given a meaning in the distribution sense and $\mathcal{H}^\Phi_w$ is an adapted weighted Hardy-Orlicz space. The second improvement concerns this last space, which can be replaced by a smaller one, in relation with the space of multipliers of BMO. The decomposition is obtained through paraproducts related to a wavelet decomposition of the product, in the same spirit as in Dobyinski [**D**]. When the ordinary product is replaced by a bilinear operator that presents cancellations, as for instance in the div-curl lemma, the L^1 component is in $\mathcal{H}^1$.

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The Jacobian determinant revisited in Sobolev spaces

HAIM BREZIS
Rutgers University, USA
brezis@math.rutgers.edu

Self-improving properties of inequalities of Poincaré type on s-John domains

SENG-KEE CHUA
National University of Singapore, Singapore
matcsk@nus.edu.sg

Poincaré inequalities over quasimetric balls with given exponents and weights are self-improving in the sense that they imply global inequalities of a similar kind, but with improved exponents and larger classes of weights. The principal theorem of this talk is about self-improving results on s-John domains which is a consequence of our companion paper "Self-improving properties of inequalities of Poincaré type on measure spaces and applications" in J. Funct. Anal., 255 (2008), 2977-3007. We have reduced our assumption on the measure to be just reverse doubling on the domain instead of the usual assumption of doubling. Moreover, while the primary case considered in the literature is $p \leq q$, we will also study the case $1 \leq q < p$. However as the original theorem itself is very long and complicated, we will concentrate on just Euclidean spaces and $q > p$.

The Kato Problem for Degenerate Elliptic Operators

DAVID V. CRUZ-URIBE
Trinity College - Hartford, USA
David.CruzUribe@trincoll.edu

Given a weight w in the Muckenhoupt class A_2, let $\mathbf{A}$ be an $n \times n$ matrix of complex-valued measurable functions such that for some $0 < \lambda < \Lambda < \infty$, and all ξ, $\nu \in \mathbb{R}^n$,

$$\begin{cases} \lambda w\left(x\right)|\xi|^2 \leq \operatorname{Re}\left\langle \mathbf{A}\xi, \xi\right\rangle, \\ |\left\langle \mathbf{A}\xi, \eta\right\rangle| \leq \Lambda w(x)|\xi||\eta| \end{cases}$$

Since w and w^{-1} can be unbounded, $\mathbf{A}$ is a degenerate elliptic matrix. We define the second order elliptic operator $\mathcal{L}_w = -w^{-1} \div \mathbf{A}\nabla$. Our goal is to show that the Kato problem for uniformly elliptic operators extends to the degenerate case. More precisely, that the domain of $\mathcal{L}_w^{1/2}$ is $H^1(w)$, and for all $f \in H^1(w)$,

$$\|\mathcal{L}_w^{1/2} f\|_{L^2(w)} \approx \|\nabla f\|_{L^2(w)}.$$

In this talk we will discuss the proof of this result in the special case that the heat kernel of the semigroup $e^{-t\mathcal{L}_w}$ satisfies Gaussian bounds:

$$|W_t(x,y)| \leq \frac{C_1}{t^{n/2}} \exp\left(-C_2 \frac{|x-y|^2}{t}\right).$$

The proof is similar in outline to the proof in the uniformly elliptic case due to Hofmann, Lacey and McIntosh, but with significant differences since the presence of weights creates numerous obstacles.

We will also discuss recent work on the Kato problem for higher order elliptic operators and in the general case when Gaussian bounds do not hold. We will also discuss weighted L^p inequalities. Using the techniques of Auscher and Martell, we will show that in certain cases, the degenerate operator satisfies *unweighted* estimates.

Based on a joint work with Cristian Ríos and José María Martell.

TBA

Bruno Franchi
Università di Bologna, Italy
franchib@dm.unibo.it

Porous media: the Muskat problem in 3D

Francisco Gancedo
University of Chicago, USA
fgancedo@math.uchicago.edu

The Muskat problem considers the filtration of two incompressible fluids throughout porous media. In this talk we shall discuss the relevance of the nonlinear singular integral operators involved in the problem, as well as the Rayleigh-Taylor condition and the topology of the initial data, in order to have local-existence.

Multilinear multipliers and rough singular integrals

Loukas Grafakos
University of Missouri at Columbia, USA
grafakosl@missouri.edu

We present a version of the Coifman-Meyer multiplier theorem for multilinear operators in which the number of derivatives required is small related to the dimension of the ambient space. Applications of this result are given in the context of rough singular integrals.

A refraction problem and Mongre-Ampère type equations

Cristian E. Gutiérrrez
Temple University, USA
gutierre@temple.edu

The physical phenomena of refraction and reflection occur simultaneously: if a light ray strikes a boundary separating to media with different refractive indices, then the ray splits into an internally reflected ray and a refracted (or transmitted) ray, each one having certain intensity. A precise description these intensities is given by the Fresnel formulas, a consequence of Maxwell's equations. In this talk, I will give some physical background to understand this phenomena, next present a new model taking into account the splitting of energy, and then show existence of surfaces separating two homogeneous materials transmitting radiation in a prescribed way. The problem gives rise to a Monge-Ampère type equation.

Harmonic Analysis related to Schrödinger operators

Eleonor Harboure
Universidad Nacional del Litoral, Argentina
harbour@santafe-conicet.gov.ar

Universal blow-up profiles for the energy critical non-linear wave equation

Carlos Kenig
University of Chicago, USA
cek@math.uchicago.edu

We describe the possible profiles at the blow-up time for solutions to the energy critical focusing nonlinear wave equation, whose energy norm is close to the one of the standing wave solution.

Two weight inequalities for singular integrals: Recent results

Michael Lacey
Georgia Institute of Technology, USA
lacey@math.gatech.edu

We will survey recent results on two weight inequalities for singular integrals, namely inequalities of the form

$$\|T(uf)\|_{L^2(v)} \leq C\|f\|_{L^2(u)}$$

for two weights u, v, and singular integral T. We will discuss (1) characterizations of the weak-type inequalities for the maximal truncations (2) L^p characterizations, when one weight is doubling (3) and new sufficent conditions for the L^2 inequality. Joint work with Eric Sawyer and Ignacio Uriate-Tuero, among others.

Subharmonic functions in subriemannian settings

Ermanno Lanconelli
University of Bologna, Italy
lanconel@dm.unibo.it

Let L be a linear second order partial differential operator in $\mathbb{R}^N$, with nonnegative characteristic form. Assume L is hypoelliptic, and endowed with a stricly positive fundamental solution Γ. We show some characterisations of the L-subharmonic functions in terms of mean value operators on the level set of Γ. When $L = \Delta$ is the classical Laplace operator, our result gives back, in particular, classical theorems by Blaschke, Privaloff, Radò, Reade and Saks.

This is joint work with Andrea Bonfiglioli.

Some progress on discrete restriction problem

Xiaochun Li
University of Illinois at Urbana-Champaign, USA
xcli@math.uiuc.edu

We will talk on some recent progress on the discrete restriction problem. This problem plays an important role in the study of non-linear Schrodinger equation on torus. This is a joint work with Yi Hu.

Discrete Littlewood-Paley analysis and applications to boundedness of singular integrals on Hardy spaces

Guozhen Lu
Wayne University, USA
guozhen.lu@gmail.com

Analytic capacity and capacities associated with scalar signed Riesz kernels

Joan Mateu
Universitat Autònoma de Barcelona, Spain
mateu@mat.uab.cat

The real and the imaginary parts of the Cauchy kernel are scalar Riesz kernels of homogeneity -1. One can associate to each of them a notion of capacity related to bounded potentials. In this talk we will prove that the capacities are comparable to the classical analytic capacity. Higher dimensional versions of the above also will be discussed. The results we present in the talk are a joint work with L. Prat and J. Verdera.

Geometric discrepancy in two dimensions

Jill Pipher
Brown University, USA
jpipher@math.brown.edu

Discrepancy theory originated with some apparently simple questions about sequences of numbers. The discrepancy of an infinite sequence is a quantitative measure of how far it is from being uniformly distributed. An infinite sequence $\{a_1, a_2, ...\}$ is said to be uniformly distributed in $[0, 1]$ if

$$\lim_{n\to\infty}(1/n|\{a_1, a_2, ...\} \cap [s, t]|) = t - s.$$

The discrepancy of a sequence with respect to its first n entries is defined as

$$D(\{a_k\}, n) := \sup_{s<t} |n(t - s) - |\{a_1, a_2, ..., a_n\} \cap [s, t]||.$$

Van der Corput asked: exist a sequence such that $D(\{a_k\}, n)$ is bounded by a constant for all n? Van Aardenne-Ehrenfest showed in 1945 that the answer is no. The sharp lower bound for all sequences is $\log n$. If a sequence $\{a_k\}$ is uniformly distributed, then it is also the case that for all (Riemann) integrable functions f on $[0, 1]$,

$$\lim_{n\to\infty}(\frac{1}{n}\sum_{k=1}^{n} f(a_k)) = \int_0^1 f(x)\, dx.$$

Uniformly distributed sequences can provide good numerical schemes for approximating integrals, with precise theoretical bounds on the rate of convergence. These bounds depend on the discrepancy of the sequence. Roth showed that the discrepancy problem for sequences has an equivalent geometric formulation in terms of a discrepancy measures of n-point distributions in the unit square, with axis-parallel rectangles taking the place of intervals. This formulation initiated an investigation of discrepancy of point distributions in IRn, with respect to rectangles, polygons, and circles by Roth, Schmidt, and then Beck, Chen, Travaligni and many others. Recent important contributions to discrepancy theory in higher dimensions are due to Bilyk, Lacey and Vagharshakyan. Many fundamental questions remain. I will discuss some work on discrepancy of n-point distributions in the unit square mea- sured with respect to various families of sets, which is joint with D. Bilyk, X. Ma, C. Spencer.

Spectral multipliers for Hodge Laplacians on the Heisenberg group and flag singular integrals

Fulvio Ricci
Scuola Normale Superiore di Pisa, Italy
f.ricci@sns.it

The functional calculus on the left-invariant riemannian Laplace-Beltrami operator on the Heisenberg group brings in singular integral operators with kernels exhibiting two different kinds of homogeneity: euclidean homogeneity locally and sub-riemannian homogeneity at infinity.

Recent work developed jointly with D. Müller and M. Peloso shows that the coexistence of the two types of homogeneity becomes even more apparent in the analysis of the riemannian Hodge Laplacian acting on differential forms. Here, both homogeneities show up even in the local

analysis, giving rise to singular kernels that near the origin are instances of "two-flag kernels", introduced and studied by A. Nagel, E. Stein, S. Wainger and myself.

Degenerate quasilinear problems related to Monge Ampére equations

Cristian Ríos
University of Calgary, Canada
crios@math.ucalgary.ca

Motivated by its applications to the regularity of degenerate elliptic Monge-Ampére equations, we study a class of quasilinear equations with infinite degenerate ellipticity. Under certain restrictions linking the non-linearity and the degeneracy of the operator, we obtain existence, uniqueness, and regularity of continuous weak solutions. We also show that the restrictions imposed can not be relaxed without including further hypotheses. This is joint work with Eric Sawyer from McMaster University, and Richard Wheeden from Rutgers University.

Stable area-stationary surfaces in the sub-Riemannian Heisenberg group

Manuel Ritoré
Universidad de Granada, Spain
ritore@ugr.es

In this talk we shall survey recent progress on the classification of critical points and second-order minima of the sub-Riemannian area functional in the first Heisenberg group.

TBA

Eric Sawyer
MacMaster University, USA
sawyer@mcmaster.ca

Characterizations of rectifiable sets in $\mathbb{H}^n$

Raul Serapioni
Università degli Studi di Trento, Italy
serapion@science.unitn.it

According to Federer's definition, k-dimensional rectifiable sets in $\mathbb{R}^n$ are contained, up to a negligible set, in the countable union of Lipschitz images of subsets of $\mathbb{R}^k$. In Euclidean spaces it is equivalent to use coverings with countable unions of k-dimensional C^1 submanifolds or to require the almost everywhere existence of approximate tangent spaces or of tangent measures. Also in groups it is possible to follow the pattern of the three definitions, provided we have good intrinsic notions of Lipschitz maps, of C^1 submanifolds or of approximate tangent subgroups

and tangent measures. But the complete equivalence of the three definitions is still an open question.

We discuss some partial equivalence of the three definitions inside Heisenberg groups.

Variation norm estimates for Carleson's operator and applications

Christoph Thiele
University of California at L.A., USA
thiele@math.ucla.edu

Carleson's theorem on almost everywhere convergence of Fourier series follows from a priori bounds on the maximal partial Fourier sum operator. We strengthen these bounds to variation norm bounds. This gives quantitative convergence results for Fourier series, and we will also discuss further applications to ergodic theory and nonlinear Fourier analysis. The work is joint with Richard Oberlin, Andreas Seeger, Terrence Tao, and Jim Wright.

Recent progress on weighted estimates involving certain linear and multilinear maximal functions

Rodolfo H. Torres
University of Kansas, USA
torres@math.ku.edu

Maximal functions are efficient tools to control (in appropriate senses) singular integral operators. Usually end-point and weighted estimates involving maximal functions, which often capture particular geometric and analytic features of a situation, translate into results for other operators which are much more difficult to handle in a direct way. This well-established approach for linear operators applies also in the multilinear setting, where new maximal functions have started to emerge.

We will present some recent results having weighted estimates and maximal functions as a common theme. First, we will describe sharp weighted estimates for (linear) fractional integral operators (joint work with M. Lacey, K. Moen, and C. Perez) which are ultimately derived from a version of the end-point Muckenhoupt-Wheeden conjecture. Then, we will review some work on multilinear weighted estimates (joint work with A. Lerner, S. Ombrosi, C.Perez and R. Trujillo-Gonzalez) and present some newer results about the multilinear strong maximal functions (joint work with L. Grafakos, L. Liu and C. Perez) as well as some other multilinear operators.

Global Green's function estimates for some linear and nonlinear elliptic problems

Igor E. Verbitsky
University of Missouri, USA
igor@math.missouri.edu

Bilateral estimates will be presented for the kernel of the Neumann series associated with a general integral operator with positive kernel satisfying a quasimetric property. Global estimates of Green's function associated with the fractional Schrödinger operator $Lu = (-\Delta)^{\alpha/2}u - qu$

$(0 < \alpha \leq 2)$ will be deduced as a consequence. Here q is a general nonnegative measurable function (or measure).

Analogous estimates will be discussed for some nonlinear operators: the p-Laplacian with a natural growth term, $-\Delta_p u - qu^{p-1}$, $1 < p < \infty$, as well as more general quasilinear and fully nonlinear elliptic operators, in particular $F_k[-u] - qu^k$ where F_k is the k-Hessian operator.

Some applications to ground states, removable singularities, and existence of solutions for equations with natural growth of gradient terms will be discussed.

This talk is based on joint work with Michael Frazier, Benjamin Jaye, Fedor Nazarov, and Nguyen Cong Phuc.

On the A_2 conjecture and corona decomposition of weights

Alexander Volberg
Michigan State University, USA
volberg@math.msu.edu

Using corona decomposition of weights we prove that week and strong weighted estimates for any Calderón-Zygmund operator T are virtually equal and then by extrapolation methods we deduce the following estimate

$$\|T(f)\|_{L^2(w)} \leq C\varphi([w]_{A_2})\|f\|_{L^2(w)}$$

where $\varphi(t) = t\log(e+t)$. The work is joint with Carlos Perez and Sergei Treil.

Stability and convergence of the Calderón reproducing formula

Michael Wilson
Michigan State University, USA
wilson@cems.uvm.edu

We show that, if $1 < p < \infty$ and w is a Muckenhoupt A_p weight, then, for all $f \in L^p(w)$, the familiar Calderón reproducing formula converges to f in the $L^p(w)$ sense, where the convergence is taken in a very natural way. We show that the convergence is stable with respect to small translation and dilation errors in the underlying kernels, and we give a quantitative estimate of the rate of convergence (or, more properly, the degree of approximation). We show analogous convergence and stability results for the real-variable Hardy space $H^1(\mathbb{R}^d)$. We show that the Calderón reproducing formula converges weak-$*$ in $BMO(\mathbb{R}^d)$ (something it does *not* do in L^∞), and does so in a stable fashion.

Fourth order equations in CR geometry

Paul Yang
Princeton University, USA
yang@Math.Princeton.edu

Our motivation to consider fourth order equations in CR geomety comes from conformal geometry where the study of Paneitz operators has yielded geometric consequences. In CR geometry of dimension three, the analogue of the Yamabe problem and the embedding problem turns out to require an understanding of the fourth order operator studied by C. Fefferman and Hirachi. The positivity of this operator is a key property relevant to both questions.

POSTERS

One singular elliptic equations involving a concave term and critical Caffarelli-Kohn-Nirenberg exponent

MOHAMMED BOUCHEKIF
University of Tlemcen, Algeria
m_bouchekif@yahoo.fr

In this work, we establish the existence of multiple positive solutions for singular elliptic equations involving a concave term and critical Caffarelli-Kohn-Nirenberg exponent.

An $\mathbb{L}^p-\mathbb{L}^q$ Version of Morgan's Theorem For The Dunkl-Cherednik Transform

SIDI LAFDAL HAMAD
University of Hassan II, Morocco
hamadlafdal@yahoo.fr

In this note, we give an $\mathbb{L}^p-\mathbb{L}^q$-version of Morgan's theorem for Dunkl-Cherednik transform $\mathcal{F}$ on the Euclidean vector space $\mathfrak{a}$. More precisely, we prove that for all $1 \leq p, q \leq +\infty$, $\alpha > 2$, $\beta = \frac{\alpha}{\alpha-1}$, $a > 0$ and $b > 0$, then for all measurable functions on $\mathfrak{a}$, the conditions $e^{a\|x\|^\alpha} f \in L^p(\mathfrak{a})$, $e^{b\|y\|^\beta}\mathcal{F} \in L^q(\mathfrak{a})$ and $(a\alpha)^{1/\alpha}(b\beta)^{1/\beta} > (\sin(\frac{\pi}{2}(\beta-1))^{1/\beta}$ imply $f = 0$, where $L^p(\mathfrak{a})$ is the Lebesgue space associated with the Dunkl-Cherednik transform.

The Riesz potential as a multilinear operator into BMO_β spaces

SILVIA I. HARTZSTEIN
Universidad Nacional del Litoral, Argentina
shartzstein@gmail.com

For $\alpha > 0$, we consider the standard Riesz type fractional integral defined by convolution with the kernel $K_\alpha(y_1, \ldots, y_k) = (|y_1| + \cdots + |y_k|)^{\alpha - kn}$, on the tensor product of the entries and restrict it to the diagonal of $(\mathbb{R}^n)^k$, (n and $k \geq 2$ fixed positive integers). More precisely, for $x \in \mathbb{R}^n$,

$$I_{\alpha,k}\vec{f}(x) = \int_{\vec{y}\in(\mathbb{R}^n)^k} \frac{f_1(y_1)\ldots f_k(y_k)}{(|x-y_1| + |x-y_2| + \cdots + |x-y_k|)^{(kn-\alpha)}} d\vec{y} \tag{1}$$

for a k-dimensional vector field $\vec{f} = (f_1, f_2, \ldots, f_k) \in L^{p_1} \times \cdots \times L^{p_k}$ and $\vec{y} = (y_1, \ldots, y_k) \in (\mathbb{R}^n)^k$. Kenig and Stein showed in [**KS**] that, when $\sum_{i=1}^k \frac{1}{p_i} - \frac{\alpha}{n} > 0$, the target space of this operator is L^q with $\frac{1}{q} = \sum_{i=1}^k \frac{1}{p_i} - \frac{\alpha}{n}$.

We consider the opposite situation, when $\sum_{i=1}^k \frac{1}{p_i} - \frac{\alpha}{n}$ vanishes or is negative, and show that the target space is the space BMO_β defined through mean oscillations, with $\beta = \alpha - \sum_{i=1}^k \frac{n}{p_i}$, recovering from the linear case, the result proved in [**HSV**]. Since BMO_β is a space of classes modulo polynomial of order $[\beta]$ we redefine the fractional integral in order to guarantee the convergence of the integral when $\sum_{i=1}^k \frac{1}{p_i}$ is small.

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L^p estimates for a singular integral operator motivated by Calderón's second commutator

Eyvindur Palsson

Cornell University, USA

eap48@cornell.edu

When Calderón studied his commutators, in connection with the Cauchy integral on Lipschitz curves, he ran into the bilinear Hilbert transform by dropping an average in his first commutator. He posed the question whether this new operator satisfied any L^p estimates. Lacey and Thiele showed a wide range of L^p estimates in two papers from 1997 and 1999. By dropping two averages in the second Calderón commutator one bumps into the trilinear Hilbert transform. Finding L^p estimates for this operator is still an open question.

On my poster I will discuss L^p estimates for a singular integral operator motivated by Calderoón's second commutator by dropping one average instead of two.

Higher-order multilinear Poincare and Sobolev inequalities in Carnot groups

Kabe Moen

Washington University in St. Louis, USA

moen@math.wustl.edu

We introduce the notions of higher-order weighted multilinear Poincare and Sobolev inequalities in Carnot groups. When $p < 1$ the classical Poincare inequality and higher-order versions fail; we provide a substitute for products of functions in the setting of Carnot groups. As an application, we establish weighted Leibniz-type rules in Campanato-Morrey spaces.

Based on a joint work with V. Naibo.

Bellman Approach to the Behavior of the Hardy-Littlewood Maximal Operator on A_∞

Winston Ou
Scripps College, USA
winston.ou@gmail.com

We apply the improved Bellman method to obtain Monge-Ampere equations which, when solved, give precise bounds on the behavior of the Hardy-Littlewood maximal function on the class of Muckenhoupt A_∞ weights.

Based on a joint work with L. Slavin.

A notion of strong linear independence of the set of integer translates of an L^2 function

Maciej Paluszyński
University of Wrocław, Poland
Maciej.Paluszynski@math.uni.wroc.pl

We consider a subspace of $L^2(\mathbb{R})$ spanned by integer shifts of one function ψ, and formulate a condition on the family $\{\psi(\cdot - n)\}_{n=-\infty}^{\infty}$, which is equivalent to the weight function $\sum_{k\in\mathbf{Z}} |\hat{\psi}(\xi + k)|^2$ being positive a.e. It has been conjectured, that this condition is the L^2 linear independence, that is if a L^2 convergent infinite linear combination is 0 then all coefficients are 0. The actual condition is stronger: The infinite linear combination is only assumed to be L^2 convergent in the sense of Cesàro averages.

Discrete approximations of Riesz equilibrium measure

Igor Pritsker
Oklahoma State University, USA
igor@math.okstate.edu

We study approximation of the Riesz equilibrium measure associated with an arbitrary compact set of positive capacity in $\mathbb{R}^d$, $d \geq 2$, by the counting measures of points located near this set. Using the energy estimates, it is possible to find rates of such approximation in the classical Newtonian and logarithmic cases. In particular, we quantify the weak* convergence of discrete measures to the equilibrium measure, and give the estimates of convergence rates for discrete potentials to the equilibrium potential. These results are applied to the problems of Schur on the arithmetic means of zeros of polynomials with integer coefficients. We prove a generalization of the Erdös-Turán discrepancy theorem. Further applications include the estimates of growth for Fekete polynomials, convergence rates for discrete energy approximations to Robin's constant, and the distribution of Fekete points.

Smooth sets in Euclidean spaces

Daniel Seco
Universitat Autònoma de Barcelona, Spain
dseco@mat.uab.cat

We study some properties of smooth sets in the sense defined by Hungerford. We generalize and sharpen Hungerford's Theorem on the Hausdorff dimension of the boundaries of smooth sets in Euclidean spaces and show the invariance of the definition under a class of automorphisms of the ambient space.

Transference results for bilinear multipliers

Saurabh Shrivastava
Indian Institute of Technology Kanpur, India
saurabhk@iitk.ac.in

In this presentation we will discuss certain transference results for bilinear multiplier operators defined on Euclidean groups $\mathbb{R}$, $\mathcal{T}$ and $\mathbb{Z}$. In particular, bilinear analogues of some of the well known results of de Leeuw and Jodeit will be discussed.

On the Perturbation Theory and the Inverse Problem for the Periodic Multidimensional Schrödinger Operator

Oktay Veliev
Dogus University, Turkey
oveliev@dogus.edu.tr

We develop the asymptotic formulas, obtained in my previous papers, for the Bloch eigenvalues and the Bloch functions of the Schrödinger operator with the periodic potentials. Then using these formulas, we determine constructively a family of spectral invariants of this operator from the given Bloch eigenvalues. Some of these invariants generalize the well-known invariants and others are entirely new. The new invariants are explicitly expressed by Fourier coefficients of the potential which present the possibility of determining the potential constructively by using the Bloch eigenvalues as input data. Finally, we give an algorithm for the unique determination of the potential of the three-dimensional Schrödinger operator from the given Bloch eigenvalues for a special class of the periodic potential.
