

THE CREAM PACKAGE: A FAST GENERAL PURPOSE UNIVERSAL ALGEBRA PACKAGE FOR GAP

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ABSTRACT. The Cream package for GAP provides fast algorithms to execute common algebraic operations on finite universal algebras with unary or binary operations. Among others, it computes automorphisms, congruences, endomorphisms and subalgebras. As a one-size-fits-all package, it only relies on universal algebra theorems, without taking advantage of specific theorems about particular algebras or the knowledge of specific representations.

In this talk we introduce the package, detail some of its algorithms and their implementation, and show examples of applications.

INTRODUCTION

The determination of algebraic properties of mathematical structures, such as their automorphism groups, is one of the oldest algebraic problems. As P.J.Cameron observed: [?]:

In the famous Erlanger Programme, Klein proposed that geometry is the study of symmetry; more precisely, the geometric properties of an object are those which are invariant under all automorphisms of the objects. (...) According to Artin, “the investigation of symmetries of a given mathematical structure has always yielded the most powerful results.”

This importance equally applies if the structure under consideration is an algebra itself. In particular, automorphisms, endomorphisms, congruences, and subalgebras are of interest in order to study classes of algebras.

However, the computation of these properties is a difficult task for general algebras. Therefore the majority of existing tools target specific classes such as groups to take advantage of domain specific theorems.

In contrast, the Cream package for GAP provides fast algorithms for finite algebras of unary or binary type without further restrictions. In this talk we will introduce the package.

1. ALGORITHMS

We give some examples of the algorithms used in Cream. We will also address several aspects of their implementations that contribute to the efficiency of the overall package.

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2. APPLICATIONS

We present several example applications of the Cream package.

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