ON THE USEFULNESS OF COMPUTER ALGEBRA SYSTEMS

MANUEL DELGADO

Abstract. We aim to give a brief overview of computer algebra systems, with emphasis on open-source ones. At the same time, we aim to illustrate its use in creating potentially good questions and discarding non-interesting ones.

Introduction

There exist a few general-purpose computer algebra systems. These require a large amount of computer capabilities. Specialized computer algebra systems, devoted to a specific part of mathematics, are more common.

We plan to refer some of the general-purpose systems. Emphasis will be put on SageMath[1], which is a free, open-source mathematics software system built on top of many existing specialized open-source packages, for instance, NumPy, Maxima, GAP or R . Besides its usefulness for research, one should remark on its possible use in teaching. Extensive documentation is available. A notable book by Zimmermann et al. [4] is composed of a part accessible to high school and undergraduate students, while other parts are suitable for graduate students and researchers.

Among the many specialized computer algebra systems one finds GAP [3], a system for computational discrete algebra, with particular emphasis on Computational Group Theory. We will use it as an example for going into some details.

1. GAP

GAP (which stands fo Groups, Algorithms and Programming) provides a programming language, a large library of functions implementing algebraic algorithms written in the GAP language. GAP is used in research and teaching for studying groups and their representations, rings, vector spaces, algebras, combinatorial structures, and more. The system, including source, is distributed freely.

GAP distributes user contributed programs, called packages. Package authors remain responsible for their maintenance. We will use for our illustrations the package numericalsgps [2], a package to compute with numerical semigroups.

2. Numerical semigroups

We will briefly introduce numerical semigroups: subsets of the non-negative integers (N) that are closed under addition and have a finite complement in N. Despite their apparent simplicity, numerical semigroups are fascinating mathematical objects at the origin of many mathematical challenges. We shall refer to some of them.

The author has been partially supported by CMUP — UIDB/00144/2020.

[1] https://www.sagemath.org/

3. Use of the GAP packages numericalsgps and intpic

Now we are in a position to use the package numericalsgps for computing some examples. As visual capabilities are very useful for those who like visual thinking, we will accompany part of the examples with images produced with intpic [1], another GAP package. To illustrate the use of software in creating potentially good questions and discarding non-interesting ones, we will use numericalsgps to probe some existing conjectures for some large families of numerical semigroups.

References

[1] M. Delgado, "intpic", *a GAP package for drawing integers* (v. 0.3.0).
[2] M. Delgado, P. A. García-Sánchez and J. Morais, "NumericalSgps", *a GAP package for numerical semigroups*.
[3] The GAP Group, GAP – Groups, Algorithms, and Programming. `https://www.gap-system.org/`
[4] P. Zimmermann et al. *Computational Mathematics with SageMath.* `https://www.gap-system.org/`

Manuel Delgado; CMUP, Departamento de Matemática, Faculdade de Ciências, Universidade do Porto, Rua do Campo Alegre s/n, 4169–007 Porto (Portugal)
Email address: `mdelgado@fc.up.pt`