IDENTITY CHECKING IN PLACTIC-LIKE MONOIDS

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The study of the identities satisfied by a semigroup S is of great importance, since, by Birkhoff's Theorem, the equational theory of S defines the variety generated by it. The characterization of the identities satisfied by S is naturally connected to the identity checking problem CHECK-ID(S) [1]. It is well-known that, for any finite semigroup S, the problem CHECK-ID(S) is decidable, since there are only finitely many substitutions of the variables occurring in the identity by elements of S. Furthermore, CHECK-ID(S) is in the complexity class coNP. However, in the case of infinite semigroups, the brute-force approach used in the finite case does not work, and only recently there have been results on the computational complexity of identity checking for infinite semigroups, beyond undecidability and trivial or "easy" decidability in linear time [2, 3, 4].

The ubiquitous plactic monoid [5], also known as the monoid of Young tableaux, has deep connections to several areas of mathematics, in particular, to the theory of symmetric functions. The plactic monoids of finite rank are known to satisfy non-trivial identities [6], but no "global" identity which is satisfied regardless of rank [7]. In contrast, monoids related to the plactic monoid, such as the hypoplactic monoid (the monoid of quasi-ribbon tableaux, connected with quasisymmetric functions), sylvester monoid (the monoid of binary search trees) and Baxter monoid (pairs of twin binary search trees, connected with Baxter), satisfy global identities, and the shortest identities have been characterized [8].

This talk will focus on results on the hypoplactic monoid [9] and on the sylvester and Baxter monoids [10]. We characterize their equational theories, and show that the identity checking problem for these monoids is decidable in polynomial time. We also give a finite equational basis for the varieties generated by these monoids, thus showing that they have finite axiomatic rank.

REFERENCES

[1] KHARLAMPOVICH, O. G. AND SAPIR, M. V., *Algorithmic problems in varieties.* Internat. J. Algebra Comput. **5.4-5**, pp. 379–602 (1995).

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[2] Chen, Y. and Hu, X. and Kitov, N. V. and Luo, Y. and Volkov, M. V., *Identities of the Kauffman monoid K_3*. Comm. Algebra **48.5**, pp. 1956–1968 (2020).
[3] Daviaud, L. and Johnson, M. and Kambites, M., *Identities in upper triangular tropical matrix semigroups and the bicyclic monoid.* J. Algebra **501**, pp. 503–525. (2018)
[4] Kitov, N. V. and Volkov, M. V., *Identities of the Kauffman Monoid K_4 and of the Jones Monoid J_4*. Fields of logic and computation III, Vol. **12180**, Lecture Notes in Comput. Sci. Springer, Cham, pp. 156–178 (2020).
[5] Lascoux, A. and Schützenberger, M.-P., *Le monoïde plaxique.* Noncommutative Structures in Algebra and Geometric Combinatorics (Naples, 1978), in: Quad. "Ricerca Sci.", **109**, pp. 129—156 (CNR, Rome, 1981).
[6] Johnson, M. and Kambites, M., *Tropical representations and identities of plactic monoids.* Trans. Amer. Math. Soc. **374**, No. 6, 4423–4447 (2021).
[7] Cain, A. J. and Klein, G. and Kubat, Ł. and Malheiro, A. and Okniński, J.. *A note on identities in plactic monoids and monoids of upper-triangular tropical matrices.* arXiv:1705.04596 (2017).
[8] Cain, A. J. and Malheiro, A., *Identities in plactic, hypoplactic, sylvester, Baxter, and related monoids.* Electronic Journal Of Combinatorics, **25**, No. 3 (2018).
[9] Cain, A. J. and Malheiro, A. and Ribeiro, D., *Identities and bases in the hypoplactic monoid.* Communications in Algebra, **50**, no. 1, 146–162 (2022).
[10] Cain, A. J. and Malheiro, A. and Ribeiro, D., *Identities and bases in the sylvester and Baxter monoids.* arXiv: 2106.00733 (2021).

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