

LINEAR NON-DEGENERACY AND UNIQUENESS OF THE BUBBLE SOLUTION FOR THE CRITICAL FRACTIONAL HENON EQUATION IN \mathbb{R}^N

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INTRODUCTION

In this talk we show a linear non-degeneracy result of positive radially symmetric solutions of the critical Henon equation associated to the fractional Laplacian operator in the N -dimensional Euclidean space. As a consequence a uniqueness result of those solutions with Morse index equal to one is obtained. In particular, we get that the ground state solution is unique. Our approach follows some ideas developed in the deep, and celebrated, papers done by R. Frank and E. Lenzmann [2] and R. Frank, E. Lenzmann, L. Silvestre [3], but, of course, our proofs are not based on ODE arguments as occurs in the local case. Our non-degeneracy result extends, in the radial setting, some known theorems proved by J. Dávila, M. del Pino and Y. Sire [1] and by F. Gladiali, M. Grossi and S.L.N. Neves [4]. However, due to the nature of the fractional operator and the weight in nonlinearity, we also argue in a different way than these authors do.

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