RELAXATION FOR OPTIMAL DESIGN PROBLEMS WITH PERIMETER PENALISATION

ANA CRISTINA BARROSO, JOSÉ MATIAS, AND ELVIRA ZAPPALE

Abstract. We discuss some relaxation and integral representation results for certain optimal design problems with a perimeter penalisation.

Introduction

We obtain a measure representation for a functional arising in the context of optimal design problems under linear growth conditions (see Barroso, Matias and Zappale [1]). Starting from an energy $F(\chi, u)$, which has a bulk term depending on the symmetrised gradient of u, as well as a perimeter term, the functional in question is the relaxation of $F(\cdot, \cdot)$ with respect to a pair (χ, u), where χ is the characteristic function of a set of finite perimeter, corresponding to the optimal shape, and u is a function of bounded deformation. The perimeter term, which penalises the interface between the two regions $\{\chi = 1\}$ and $\{\chi = 0\}$, is added to ensure compactness of minimising sequences.

In Barroso and Zappale [2] and [3], a similar investigation was undertaken in the case of non-standard p-q growth conditions on the original bulk energy densities, which now depend on the full gradient of the u variable, and where the energy also includes a perimeter penalisation term. In this setting, we showed in [2] that one of the relaxed functionals under consideration only admits a weak measure representation, whereas for the other a strong measure representation holds. Under some convexity assumptions, we provided a partial characterisation of the corresponding measures, a full representation was obtained in the one-dimensional setting.

In [3] we further identified some conditions under which the relaxation process gives rise to no concentration effects. In this case, we showed that the integral representation in question is composed of a term which is absolutely continuous with respect to the Lebesgue measure, and a perimeter term, but has no additional singular term.

References

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The first author has been partially supported by the Fundação para a Ciência e a Tecnologia (Portuguese Foundation for Science and Technology) through the project UIDB/04561/2020.

The second author has been partially supported by Fundação para a Ciência e a Tecnologia (Portuguese Foundation for Science and Technology) through the project UID/MAT/04459/2020.

The third author has been partially supported by INdAM GNAMPA, through the GNAMPA project 2020.

The talk at the 8IMM 2022 has been given by the first author.

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Ana Cristina Barroso; Departamento de Matemática and CMAFcIO, Faculdade de Ciências da Universidade de Lisboa, Portugal
Email address: acbarroso@ciencias.ulisboa.pt

José Matias; Departamento de Matemática and CAMGSD, Instituto Superior Técnico, Portugal
Email address: jose.c.matias@tecnico.ulisboa.pt

Elvira Zappale; Dipartimento di Scienze di Base ed Applicate per l'Ingegneria, Sapienza - Università di Roma, Italy
Email address: elvira.zappale@uniroma1.it