LIE ALGEBROIDS AND VARIATIONAL CALCULUS

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Abstract. Several variational principles introduced in recent years can be treated in a unified way by using the concept of Lie algebroid. A general variational problem will be established which includes the standard Hamilton principle, the Poincaré variational principle, the Euler-Poincaré variational principle, the Clebsh variational principle, among others, as particular examples.

Variational principles are a fundamental tool in Classical Mechanics and Field Theories. In recent years several variational principles have been introduced to derive an adequate set of differential equations describing the evolution of a system in the presence of a symmetry group or related constructions. They consists on the prescription of a subset of the infinitesinal admissible variations, from where a set of differential equations is obtained by imposing that the differential of the action functional at a critical curve vanishes over the prescribed set of infinitesimal variations.

Examples of such equations are, for instance, Poincaré equations [1], Euler-Poincaré equations [2, 3], Lagrange-Poincaré equations [4, 5], or Clebsh equations [6, 7].

We will show that all these variational principles can be obtained in a unified way by considering the set of admissible curves on a Lie algebroid with an adequate Banach manifold structure [8]. Reduction and reconstruction results will be interpreted in terms of morphisms of Lie algebroids. The generalization to Field Theories will be considered [9].

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