A NOTION OF JACOBIAN FOR BV MAPS AND A NEW APPROACH TO TOPOLOGICAL SINGULARITIES

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Abstract. We introduce a notion of Jacobian determinant for some R^2-valued maps of bounded variation on a bounded planar domain. This notion extends the Distributional Determinant and allows for some applications in elasticity. We will focus on classical singularities appearing in Ginzburg-Landau and dislocation models. This is a joint work with Riccardo Scala and Lucia De Luca.

Introduction

We introduce a weak notion of 2×2-minors of gradients of a suitable subclass of BV functions. In the case of maps in $BV(\mathbb{R}^2;\mathbb{R}^2)$ such a notion extends the standard definition of Jacobian determinant to non-Sobolev maps.

We use this distributional Jacobian to prove a compactness and Γ-convergence result for a new model describing the emergence of topological singularities in two dimensions, in the spirit of Ginzburg-Landau and core-radius approaches. Within our framework, the order parameter is an SBV map u taking values in $\mathbb{S}s^1$ and the energy is made by the sum of the squared L^2 norm of ∇u and of the length of (the closure of) the jump set of u multiplied by $\frac{1}{\epsilon}$. Here, ϵ is a length-scale parameter. We show that, in the $|\log \epsilon|$ regime, the Jacobian distributions converge, as $\epsilon \to 0^+$, to a finite sum μ of Dirac deltas with weights multiple of π, and that the corresponding effective energy is given by the total variation of μ.

References

[1] Alicandro, R., Ponsiglione, M. Ginzburg-Landau functionals and renormalized energy: A revised Γ-convergence approach. *J. Funct. Anal.* **266** (2014), 4890–4907.

[2] Alicandro, R., De Luca, L., Garroni, A., Ponsiglione, M.Metastability and dynamics of discrete topological singularities in two dimensions: a Γ-convergence approach. *Arch. Rational Mech. Anal.* **214** (2014), 269–330.

[3] Ambrosio, L., Fusco, N., Pallara, D. *Functions of bounded variation and free discontinuity problems.* Clarendon Press Oxford, 2000.

[4] Jerrard R.L., Soner H.M. The Jacobian and the Ginzburg-Landau energy. *Calc. Var. Partial Differ. Equ.* **14** (2002),151–191.

[5] Scala, R., Van Goethem, N. Currents and dislocations at the continuum scale. *Methods Appl. Anal.* **23** (2016), 1–34.

[6] Scala, R., Van Goethem, N. A variational approach to single crystals with dislocations. *SIAM J. Math. Anal.* **51** (2019), 489–531.

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