SOME RESULTS ABOUT SCHRÖDINGER BRIDGES AND HARMONIC FUNCTIONS VALUED IN METRIC SPACES WITHOUT CURVATURE RESTRICTIONS

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Abstract. We introduce the dynamical Schrödinger problem, defined for a wide class of entropy and Fisher information functionals, as a geometric problem on abstract metric spaces. Under very mild assumptions (in particular, without any curvature restrictions) we prove a generic Γ-convergence result towards the geodesic problem as the noise parameter $\varepsilon \downarrow 0$. Our novel technique is based on *adaptive perturbations by gradient flows*. We then study the dependence of the entropic cost on the parameter ε.

A similar method allows us to prove the following result. If $u : \Omega \subset \mathbb{R}^d \to \mathrm{X}$ is a harmonic map valued in a metric space X and $\mathsf{E} : \mathrm{X} \to \mathbb{R}$ is a convex function, in the sense that it generates an EVI_0-gradient flow, we prove that the pullback $\mathsf{E} \circ u : \Omega \to \mathbb{R}$ is subharmonic. In addition, we establish generalized maximum principles, in the sense that the L^q norm of $\mathsf{E} \circ u$ on $\partial\Omega$ controls the L^p norm of $\mathsf{E} \circ u$ in Ω for some well-chosen exponents $p \geq q$, including the case $p = q = +\infty$. In particular, our results apply when E is a geodesically convex entropy over the Wasserstein space, and thus settle some conjectures of Y. Brenier.

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