FREE RESOLUTIONS AND GENERALIZED HAMMING WEIGHTS

EDGAR MARTÍNEZ-MORO

Abstract. In this talk, we explore the relationship between free resolution of some monomial ideals and Generalized Hamming Weights (GHWs) of binary codes. More precisely, we look for a structure smaller than the set of codewords of minimal support that provides us some information about the GHWs. We prove that the first and second generalized Hamming weight of a binary linear code can be computed (by means of a graded free resolution) from a set of monomials associated to a binomial ideal related with the code. Moreover, the remaining weights are bounded by the Betti numbers for that set.

The study of GHWs for linear codes is a hot topic in Coding Theory since it can be applied in information theory. However, there are just a few families of codes for which the complete generalized weight hierarchy is known. In 2013, Johnsen and Verdure showed how the GHWs of a linear code could be computed from a minimal graded free resolution of a monomial ideal associated with the set of codewords of minimal support of the code, denoted by MC. In this work, given a linear code C, we look for a structure smaller than MC that provides us some information about the GHWs. In particular, for a long time the we have conjectured that a Gröbner test-set (which is a subset of MC) determines the GHWs of a linear code. This conjecture was supported by computational evidence but unfortunately this is not true, as it will be shown. Although this is not true, we are able to prove that at least the first and second GHWs of a binary linear code can be computed using this set. Moreover, we obtain an upper bound for the other GHWs.

References

[1] García-Marco, I.; Márquez-Corbella, I.; Martínez-Moro, E.; Pitones, Y. Free Resolutions and Generalized Hamming Weights of Binary Linear Codes. *Mathematics 2022, 10, 2079.* https://doi.org/10.3390/math10122079

Edgar Martínez-Moro; Universidad de Valladolid, Castilla, Spain
Email address: edgar.martinez@uva.es

This work has been partially supported by part by Grant PGC2018-096446-B-C21 funded by MCIN/AEI/10.13039/501100011033 and by "ERDF A way of making Europe".