Differentiability properties of Zygmund functions

Juan J. Donaire

Universitat Autònoma de Barcelona.

The Zygmund space is defined as the set of bounded functions $f : \mathbb{R}^n \to \mathbb{R}$ such that

$$\sup_{x,h\in\mathbb{R}^n}\frac{|f(x+h)+f(x-h)-2f(x)|}{\|h\|}<\infty.$$

This space has a close connection to the Bloch space and, consequently to the radial behaviour of the derivative of conformal maps.

In this sense, it has a special relevance to determine the size of the set of points where the ratio $\frac{|f(x+h)-f(x)|}{\|h\|}$ is bounded as $h \to 0$.

We will provide some new results concerning this kind of problems and the significant difference between the one dimensional case and the higher dimensional ones and the key role played by the probabilistic techniques used to prove them.

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