k_p-Density of finite rank operators in the space of p-compact operators

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Abstract

A Banach space X is said to have the *approximation property* (AP) if the identity map I_X on X can be approximated by finite rank operators uniformly on every compact subset of X. A well-known result due to Grothendieck [1] states that X has the AP if and only if, for every Banach space Y, the space $\mathcal{F}(Y,X)$ of all finite rank operators from Y to X is $\|\cdot\|$-dense in the space $\mathcal{K}(Y,X)$ of all compact operators (here, $\|\cdot\|$ denotes the usual operator norm). This characterization draws heavily on the visualization of relatively compact sets as those sitting inside the convex hull of norm null sequences. Following this idea, a strengthened form of compactness has been recently introduced by Sinha and Karn [2] as follows. If $1 \leq p < \infty$ and p' is the conjugate index of p (i.e., $1/p + 1/p' = 1$), a set $K \subset X$ is said to be *relatively p-compact* if there exists a p-summable sequence (x_n) in X such that

$$K \subset \left\{ \sum_n a_n x_n \colon \sum_n |a_n|^{p'} \leq 1 \right\} := p\text{-co}(x_n).$$

The corresponding notion of approximation property of order p is then defined in an obvious way: X is said to have the *p-approximation property* (p-AP) if I_X can be approximated by finite rank operators uniformly on every p-compact subset of X.

Relying on the notion of p-compactness, an operator $T \in \mathcal{L}(Y,X)$ is said to be *p-compact* if T maps bounded subsets of Y to relatively p-compact subsets of X. In the space $\mathcal{K}_p(Y,X)$ of all p-compact operators from Y to X we have the natural norm

$$k_p(T) = \inf \left\{ \left(\sum_n \|x_n\|^p \right)^{1/p} \; : \; T(B_Y) \subset p\text{-co}(x_n) \right\}.$$

Now, we say that a Banach space X has the *k_p-approximation property* (k_p-AP) if $\mathcal{F}(Y,X)$ is k_p-dense in the space $\mathcal{K}_p(Y,X)$ for every Banach space Y. In this talk, we separate the p-AP from k_p-AP. Moreover, we show some equivalent conditions to the k_p-AP and give some applications.

References

[1] A. Grothendieck, *Produits tensoriels topologiques et espaces nucléaires*, Memoirs Amer. Math. Soc. **16** (1955).

[2] D. P. Sinha and A. K. Karn, *Compact operators whose adjoints factor through subspaces of ℓ_p*, Studia Math. **150** (2002), 17–33.