About some class of Roumier type functions

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Abstract

Let $(M_k)_{k=0}^{\infty}$ be a logarithmically convex sequence of positive numbers such that:

1. $\exists H_1 > 1 \ \exists H_2 > 1 \ \ \forall k, m \in \mathbb{Z}_+ \quad M_{k+m} \le H_1 H_2^{k+m} M_k M_m$;
2. $\forall \varepsilon > 0 \ \exists a_\varepsilon > 0 \ \forall k \in \mathbb{N} \ \ M_k \le a_\varepsilon \varepsilon^k k!$;
3. $\exists \gamma \in (0,1) \ \exists b_1 > 0 \ \exists b_2 > 0 \ \forall k \in \mathbb{N} \ \ M_k \ge b_1 b_2^k k!^\gamma$.

Let C be an open convex acute cone in $\mathbb{R}^n$ with the apex at the origin, b be a convex continuous and positively homogeneous of degree one function on $\overline{C}$, the clousure of the cone C,

$$U = \{\xi \in \mathbb{R}^n : \ - < \xi, y > \le b(y), \forall y \in C\},$$

We consider the space $G_M(U)$ of infinitely differentiable, $C^\infty(U)$–functions, such that for each $m \in \mathbb{N}$ and each $\varepsilon > 0$

$$p_m(f) = \sup_{x \in U, \alpha \in \mathbb{Z}_+^n} \frac{|(D^\alpha f)(x)|(1 + \|x\|)^m}{\varepsilon^{|\alpha|} M_{|\alpha|}} < \infty \ .$$

The equivalent description of $G_M(U)$ and the description of strong dual to $G_M(U)$ in the terms of Fourier-Laplace of functionals will be given.