

# Discrete Hilbert transform on sparse sequences

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## Abstract

Let  $H((a_n)) = \sum_n a_n * v_n / (z - t_n)$  be a discrete Hilbert transform. We are looking for necessary and sufficient conditions for  $H$  being bounded operator from  $l^2(v_n)$  to  $L^2(\mu, C)$  where  $\mu$  is some measure. The special interest for us is the case when  $\mu$  is a discrete measure. For fast increasing sequences  $|t_n|$  we are able to find such conditions. They are similar to classical Muckenhoupt condition.

Discrete Hilbert transform naturally appears when we study Hilbert spaces of entire functions with Riesz basis of reproducing kernels (Paley-Wiener spaces, de Branges spaces, some Fock-type spaces).

From our results we can obtain the description of all Carleson measures (and, in particular, Bessel sequences) and complete interpolating sequences for such spaces.

As an application we verify Feichtinger conjecture for such spaces (and reproducing kernels) and give a counterexample for Baranov's conjecture about Bessel sequences in de Branges spaces.