Bounded symbols of truncated Toeplitz operators

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Abstract

A truncated Toeplitz operator A_ϕ, $\phi \in L^2$, is the map on the space $K_\theta = H^2 \ominus H^2$ defined by

$$A_\phi : f \to P_{K_\theta}(\phi f), \ f \in K_\theta \cup L^\infty.$$

The symbol of a truncated Toeplitz operator is not uniquely determined, $A_\phi = 0$ if $\phi \in \theta H^2 + \overline{\theta H^2}$. Clearly, if $\phi \in L^\infty$, then ϕ is bounded and $\|A_\phi\| \leq \|\phi\|_\infty$. In 2007 D. Sarason asked a natural question: *Does every bounded truncated Toeplitz operator admit a bounded symbol?* In 2009 A. Baranov, I. Chalendar, E. Frician, J. Mashreghi and D. Timotin gave a negative answer to this question. In my talk I present a description of spaces K_θ , in which every every bounded truncated Toeplitz operator admits a bounded symbol. This property turns out to be equivalent to a factorization property for a certain class of pseudocontinuable functions, the latter is fulfilled for so-called one-component inner functions ϕ. The talk is based on a joint work with Anton Baranov and Vladimir Kapustin.