Completeness of translates of entire functions and hypercyclic operators

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Abstract

In this communication we introduce new classes of hypercyclic operators on the space $H(\mathbb{C})$ of all entire functions with uniform convergence topology.

Let X be a topological vector space. A continuous linear operator $\Phi : X \to X$ is said to be hypercyclic, if there exists $x \in X$, such that it's orbit $\mathrm{Orb}(\Phi, x) = \{\Phi^n x,\ n = 0, 1, 2, ...\}$ is dense in X. We obtain the following results.

Theorem 1 *Let $T : H(\mathbb{C}) \to H(\mathbb{C})$ be a continuous linear operator such that $TS_\lambda f - S_\lambda Tf = \lambda S_\lambda f$, $\forall \lambda \in \mathbb{C}$, $\forall f \in H(\mathbb{C})$. Then the system $\{S_\lambda g, \lambda \in \Lambda \subset \mathbb{C}\}$ is complete in $H(\mathbb{C})$, if $g \in \ker T \setminus \{0\}$ and Λ contains a limit point.*

Corollary 2 *Let $T : H(\mathbb{C}) \to H(\mathbb{C})$ be a continuous linear operator such that 1) $TS_\lambda f - S_\lambda Tf = \lambda S_\lambda f$, $\forall \lambda \in \mathbb{C}$, $\forall f \in H(\mathbb{C})$; 2) $\ker T \neq \{0\}$. Then T is hypercyclic.*