

Roots and commutativity of Toeplitz operators on The Bergman space

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Abstract

One of the major questions in the theory of Toeplitz operators on the Bergman space over the unit disk \mathbb{D} in the complex plane \mathbb{C} is a complete description of the commutant of a given Toeplitz operator, that is the set of all Toeplitz operators that commute with it. In [2], we have obtained a complete description of the commutant of Toeplitz operator T with any quasihomogeneous symbol $\phi(r)e^{ip\theta}$, $p > 0$ in case it has a Toeplitz p -th root S with symbol $\psi(r)e^{i\theta}$, namely, commutant of T is the closure of the linear space generated by powers S^n which are Toeplitz. But the existence of p -th root was known until now only when $\phi(r) = r^m$, $m \geq 0$.

In this talk we shall first introduce the notion of a p -th root of a quasihomogeneous Toeplitz operator. Then we will present our main result which is the existence of p -th roots for a much larger class of symbols, for example, it includes such symbols for which

$$\phi(r) = \sum_{i=1}^k r^{a_i} (\ln r)^{b_i}, 0 \leq a_i, b_i \text{ for all } 1 \leq i \leq k.$$

References

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