Spectral properties of a class of nonselfadjoint operators with real spectrum and Krein and Cartwright classes of entire functions

Dmitry Yakubovich

Abstract

In this work, we study spectral properties of a one-dimensional singular perturbation A of an unbounded selfadjoint operator A_0 with compact resolvent. The domains of these two operators may differ, but we assume that their graphs differ in a one-dimensional space. We assume that the spectrum of A is real. Our approach is based some previus work by V. Kapustin. We show that for any operator A of our class there exist an inner function $\Theta(z)$ and an outer function $\varphi(z)$ in the upper half plane $\mathbb{C}_+$ with $\frac{\varphi}{z+i} \in H^2$ and

$$\Theta = \frac{\varphi}{\overline{\varphi}} \qquad \text{a.e. on } \mathbb{R}$$

such that A is unitarily equivalent to the operator $T = T_{\Theta,\varphi}$ which acts on the model space $K_\Theta = H^2(\mathbb{C}_+) \ominus \Theta H^2(\mathbb{C}_+)$, with the domain defined as

$$\mathcal{D}(T) = \{f \in K_\Theta : \qquad \exists\, c = c(f) \in \mathbb{C} : zf - c\varphi \in K_\Theta\},$$

and

$$Tf = zf - c\varphi, \qquad f \in \mathcal{D}(T).$$

Criteria for completeness of eigenvectors and for the possibility to remove the whole spectrum by an adequate perturbation of the type considered in terms of the sparsity of the spectrum of the unperturbed operator will be given. The proofs use entire functions of Hermite–Biehler, Krein and Cartwright classes.

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